論文2003-40SC-2-2

시간지연 퍼지 시스템의 보장비용 및 H_∞ 필터링

(Guaranteed Cost and H_∞ Filtering for Delayed Fuzzy Dynamic Systems)

李 甲 來 *, 曺 喜 洙 **, 朴 烘 培 ***

(Kap Rai Lee, Hee Soo Cho, and Hong Bae Park)

요 약

본 논문은 시간지연을 갖는 퍼지 시스템에 대한 보장 비용과 H_∞ 외란감쇄 성능을 갖는 퍼지 필터링 문제를 다룬다. 본 연구는 외란감쇄에 대한 확장 L_2 노음 제한조건과 LQ 비용함수의 성능 상한치 제한조건을 만족하는 필터링 설계 방법이다. Lyapunov 함수를 이용하여 필터의 존재성에 대한 충분조건을 유도하고 선형행렬부등식(LMI: linear matrix inequality)으로 나타낸다. 필터 설계는 LMI 해를 구함으로써 바로 구할 수 있다. 제안한 방법의 설계 과정을 설명하기 위한 시뮬레이션 예제를 또한 나타낸다.

Abstract

This paper presents a method for designing guaranteed cost fuzzy filter with a desired H_∞ disturbance rejection constraint of delayed fuzzy dynamic systems. This method not only guarantees an induced L_2 norm bound constraint on disturbance attenuation, but also minimizes an upper bound on a linear quadratic performance measure. A sufficient condition for the existence of guaranteed cost fuzzy filter with H_∞ constraint is then presented in terms of linear matrix inequalities(LMIs). A simulation example is given to illustrate the design procedures and performances of the proposed methods.

Keywords : guaranteed cost filtering, H_∞ filtering, delayed fuzzy systems

I. Introduction

In the past few years, there has been rapidly growing interest in fuzzy control of nonlinear systems, and there have been many successful applications. Recently, stability analysis and systematic design are among the most important issues for fuzzy control systems and there have been significant research efforts on these issues[1~6].

These methods are conceptually simple and straightforward. The nonlinear system is represented by a Takagi-Sugeno(T-S) fuzzy model. And then, the control design is carried out on the basis of the fuzzy model via the so-called parallel distributed compensation(PDC) scheme. Tanaka et al. [4],[5] presented stability analysis for a class of fuzzy dynamic systems. Ma et al. [6] presented the analysis and design of the fuzzy controller and fuzzy

* 正會員, 平澤大學校 情報科學部

(Dept. of Information Science, Pyongtaek University)

** 正會員, 서울地方中小企業廳

(Small & Medium Business Administration)

*** 正會員, 慶北大學校 電子電氣컴퓨터學部

(School of Electrical Engineering, and Computer Science, Kyungpook National Univ.)

接受日字:2002年9月24日, 수정완료일:2003年2月11日

observer on the basis of T-S fuzzy model using separation property. Chen[9] and Lee et al.[10] presented the design method of fuzzy H_∞ controller to satisfy an H_∞ norm bound constraint on disturbance attenuation. Chen et al [12] also considered guaranteed-cost control with a desired H_∞ disturbance rejection constraint.

Since time delay is frequently a source of instability and encountered in various engineering systems, the H_∞ control problem for delayed systems has received considerable attention over the last few decades. In fuzzy control systems, Cao et al.[13] presented stability analysis and synthesis of delayed fuzzy dynamic systems. Lee et al. [14] also presented output feedback fuzzy H_∞ control of delayed fuzzy dynamic systems.

For practical control systems, a fuzzy control with guaranteed cost control performance under a desired H_∞ disturbance rejection constraint is more appealing for nonlinear systems.

This paper presents a method for designing guaranteed cost fuzzy filter with a desired H_∞ disturbance rejection constraint of delayed fuzzy dynamic systems. This methods not only guarantees an induced L_2 norm bound constraint on disturbance attenuation, but also an upper bound on a linear quadratic performance measure. A sufficient condition for the existence of guaranteed cost fuzzy filter with H_∞ constraint is then presented in terms of linear matrix inequalities(LMIs). A simulation example is given to illustrate the design procedures and performances of the proposed methods.

Ⅱ. Problem formulation

The continuous fuzzy dynamic model, proposed by Takagi and Sugeno, is described by fuzzy IF-THEN rules which represented local linear input-output relations of nonlinear system. Consider a nonlinear system with time-varying delayed states that can be described by the following T-S fuzzy model with time-varying delayed states:

Plant Rulei:

IF $z_1(t)$ is M_{i1} and ⋯ and $z_g(t)$ is M_{ig}

THEN $\dot{x}(t)=A_i x(t)+A_{di}x(t-d_1(t))+B_i w(t)$

$$\begin{aligned} y(t) &= C_i x(t)+C_{di}x(t-d_2(t))+D_i w(t), \\ e_z(t) &= C_{zi}x(t), \\ e_\infty(t) &= C_{\infty i}x(t), \quad i=1,2,\cdots,r \\ x(t) &= \psi(t), \quad t\in[-\max(d_1(0),d_2(0)),0] \end{aligned} \tag{1}$$

where M_{ij} is the fuzzy set, $x(t)\in R^n$ is the state vector, $\psi(t)\in R^n$ is the continuous initial value function, $w(t)\in R^p\in L_2(0,T)$ is the square-integrable noise signal, $y(t)\in R^m$ is the measurement, $e_L(t)\in R^{q_1}$, and $e_\infty(t)\in R^{q_2}$ is the signal to be estimated, r is the number of IF-THEN rules, $z_1\sim z_g$ are some measurable system variables, i.e., the premise variables, and all matrices are constant matrices with appropriate dimensions, $d_i(t), i=1,2,$ are the time-varying delays with following assumptions:

$$0\le d_i(t) < \infty, \quad \dot{d}_i(t)\le\beta_i < 1, \quad i=1,2. \tag{2}$$

Given a pair of $(y(t),u(t))$ by using a center average defuzzifer, product inference, and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t))\{A_i x(t)+A_{di}x(t-d_1(t))+B_i w(t)\} \\ y(t) &= \sum_{i=1}^{r} h_i(z(t))\{C_i x(t)+C_{di}x(t-d_2(t))+D_i w(t)\} \\ e_z(t) &= \sum_{i=1}^{r} h_i(t)C_{zi}x(t) \\ e_\infty(t) &= \sum_{i=1}^{r} h_i(t)C_{\infty i}x(t) \\ x(t) &= \psi(t), \quad t\in[-\max(d_1(0),d_2(0)),0] \end{aligned} \tag{3}$$

where

$$\begin{aligned} w_i(z(t)) &= \prod_{j=1}^{n} M_{ij}(z_j(t)) \\ h_i(z(t)) &= w_i(z(t))/\sum_{i=1}^{r} w_i(z(t)) \\ z(t) &= [z_1(t)\ z_2(t)\ \cdots\ z_g(t)]^T \end{aligned}$$

where $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed that

$$w_i(z(t)) \geq 0, \quad i=1,2,\cdots,r \qquad \sum_{i=1}^{r} w_i(z(t)) > 0 \tag{4}$$

for all t. Then we can obtain the following conditions:

$$h_i(z(t)) \geq 0, \quad i=1,2,\cdots,r \qquad \sum_{i=1}^{r} h_i(z(t)) = 1 \tag{5}$$

for all t. As a fuzzy H_∞ filter of the fuzzy system (1), we consider the following structure:

Filtering Rule i:

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_g(t)$ is M_{in}

THEN
$$\dot{\hat{x}}(t)=F_i\hat{x}(t)+G_iy(t), \quad i=1,2,\cdots,r$$
$$\hat{e}_z(t)=C_{zi}\hat{x}(t)$$
$$\hat{e}_\infty(t)=C_{\infty i}\hat{x}(t)$$
$$\hat{x}(0)=0, \tag{6}$$

where the matrix F_i and G_i are to be determined. The final output of this fuzzy filter is

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))\{F_i\hat{x}(t)+G_iy(t)\}, \quad \hat{x}(0)=0 \tag{7}$$

From (3) and (7), we obtain the following estimation error system

$$\dot{\zeta}(t)= \mathcal{A}(z)\zeta(t)+\mathcal{A}_1(z)x(t-d_1(t)) + \mathcal{A}_2(z)x(t-d_2(t))+\mathcal{B}(z)w(t) \tag{8}$$
$$\zeta(t)= [\phi(t)^T \ \phi(t)^T]^T, \quad t\in[-\max(d_1(0), d_2(0)), 0]$$
$$\tilde{e}_z(t) = \mathcal{C}_z(z)\zeta(t)$$
$$\tilde{e}_\infty(t) = \mathcal{C}_\infty(z)\zeta(t)$$

with the following notations

$$\mathcal{A}(z) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\ \mathcal{A}_{ij}, \tag{9}$$
$$\mathcal{C}_z(z)= \sum_{i=1}^{r} h_i(z(t))\ \mathcal{C}_{zi},$$
$$\mathcal{C}_\infty(z)= \sum_{i=1}^{r} h_i(z(t))\ \mathcal{C}_{\infty i},$$
$$\mathcal{A}_1(z) = \sum_{i=1}^{r} h_i(z(t))\ \mathcal{A}_{1i},$$
$$\mathcal{A}_2(z)= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\ \mathcal{A}_{2ij},$$
$$\mathcal{B}(z) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\ \mathcal{B}_{ij},$$

where $\zeta(t)=[(x(t)-\hat{x}(t))^T \ \ x^T(t)]^T$, $\tilde{e}(t)=e(t)-\hat{e}(t)$,

$$\mathcal{A}_{ij}=\begin{bmatrix} F_i & A_i-G_iC_j-F_i \\ 0 & A_i \end{bmatrix}, \tag{10}$$
$$\mathcal{A}_{2ij}=\begin{bmatrix} -G_iC_{d_j} \\ 0 \end{bmatrix},$$
$$\mathcal{A}_{1i} = [A_{di}^T \ \ A_{di}^T]^T,$$
$$\mathcal{B}_{ij}=[(B_i-G_iD_j)^T \ \ B_i^T]^T,$$
$$\mathcal{C}_{zi}=[C_{zi} \ \ 0],$$
$$\mathcal{C}_{\infty i}=[C_{\infty i} \ \ 0].$$

In general, the guaranteed cost filtering is more appealing to achieve a desired filtering performance. Therefore, we consider the following cost function without considering $w(t)$.

$$J_2 := \int_0^T \|\tilde{e}_z(t)\|^2 dt < \delta \tag{11}$$

Since H_∞ control is control design to efficiently eliminate the effect of $w(t)$, for given γ we define H_∞ control performance of the system (10) as the quantity

$$J_\infty := \int_0^T \|\tilde{e}_\infty(t)\|^2 dt \leq \gamma^2\Big[\int_0^T \|w(t)\|^2 dt + x(0)^T Q_0 x(0)^T + \sum_{i=1}^{2}\int_{-d_i(0)}^{0} x(\tau)^T Q_i x(\tau)d\tau\Big] \tag{12}$$

for all $T>0$ and all $w\in L_2[0 \ \ T]$, where $\|\cdot\|$ denotes the Euclidean norm. The weighting matrix

Q_i, $i=0,1,2$, in (12) are measure of the initial state uncertainty at $t\le 0$ relative to the uncertainty in $w(t)$. A large value of Q_i indicates that the state at $t\le 0$ is very close to zero.

This paper addresses designing fuzzy filter (7) for the system (3) such that the estimation error system is globally exponentially stable and cost function (11) is satisfied with a desired H_∞ constraint (12).

Ⅲ. Fuzzy filter design

Define a Lyapunov functional

$$V(\zeta,t)=\zeta^T(t)\,P\,\zeta(t)+\sum_{i=1}^{2}\int_{t-d_i(t)}^{t}x(\tau)^T S_i x(\tau)d\tau, \quad (13)$$

where $P>0$ and $S_i>0$. Then there exist positive scalars δ_1 and δ_2 such that $\delta_1\|\zeta\|^2\le V(\zeta,t)\le\delta_2\|\zeta\|^2$. If there exists scalar $\alpha>0$ such that $V(\zeta,t)\le-\alpha\|x\|$, then the unforced system of (8) is globally exponentially stable[15]. From assumption (2)

$$\begin{aligned}V(\zeta,t)&\le \zeta^T(t)\,P\,\zeta(t)+\zeta^T(t)P\zeta(t)\\&+\sum_{i=1}^{2}\{x(t)^TS_ix(t)-x(t-d_i(t))^T(1-\beta_i)S_ix(t-d_i(t))\}\\&:=V_a(\zeta,t),\end{aligned}\quad (14)$$

Lemma 1. Consider the system (3) with assumption (2), and let $R_i>0$, $i=0,1,2$ be a given initial state weighting matrix and $\gamma>0$ a given scalar. If there exist matrices $P>0$, $S_1>0$ and $S_2>0$, and positive scalar α satisfying the following inequalities

$$\Omega_{ii}<0,\quad i=1,2,\cdots,r \quad (15)$$

$$\Omega_{ij}+\Omega_{ji}<0,\quad i<j<r, \quad (16)$$

$$[I_n\ \ I_n]P[I_n\ \ I_n]^T-\gamma^2Q_0<0 \quad (17)$$

$$S_i-\gamma^2Q_i<0,\ i=1,2 \quad (18)$$

then the corresponding estimation error system is globally exponentially stable and achieves H_∞ control performance for all $w\in L_2[0,T]$. In here

$$\Omega_{ij}=\begin{bmatrix}\mathcal{A}_{ij}^TP+P\mathcal{A}_{ij}+\mathcal{S}+S_a+\mathcal{C}_{\infty i}^T\mathcal{C}_{\infty i} & P\mathcal{A}_{1i} & P\mathcal{A}_{2ij} & P\mathcal{B}_{ij}\\ * & -\mathcal{S}_1 & 0 & 0\\ * & * & -\mathcal{S}_2 & 0\\ * & * & * & -\gamma^2 I\end{bmatrix}, \quad (19)$$

where $*$ represents the elements below the main diagonal of a symmetric matrix and

$$\mathcal{S}=\begin{bmatrix}0 & 0\\ 0 & S_1+S_2\end{bmatrix},\quad S_a=\begin{bmatrix}0 & 0\\ 0 & \alpha I\end{bmatrix} \quad (20)$$
$$\mathcal{S}_i=(1-\beta_i)S_i,\ i=1,2.$$

Proof : Consider $V_a(\zeta,t)\le-\alpha\|x\|^2$ and the following condition

$$J_a(t):=V_a(\zeta,t)+\tilde{e}^T(t)\,\tilde{e}(t)-\gamma^2w^T(t)w(t)\le 0. \quad (21)$$

From closed loop system (8)

$$J_a(t)\le\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\begin{bmatrix}\zeta(t)\\x(t-d_1)\\x(t-d_2)\\w(t)\end{bmatrix}^T\begin{bmatrix}\mathcal{A}_{ij}^TP+P\mathcal{A}_{ij}+\mathcal{S}+S_a & P\mathcal{A}_{1i} & P\mathcal{A}_{2ij} & P\mathcal{B}_{ij}\\ * & -\mathcal{S}_1 & 0 & \\ * & * & -\mathcal{S}_2 & 0\\ * & * & * & -\gamma^2 I\end{bmatrix}\begin{bmatrix}\zeta(t)\\x(t-d_1)\\x(t-d_2)\\w(t)\end{bmatrix}$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j(\mathcal{C}_{\infty i}\zeta(t))^T\mathcal{C}_{\infty j}\zeta(t). \quad (22)$$

By considering the following condition

$$\begin{aligned}\tilde{e}(t)^Te(t)&=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j(\mathcal{C}_{\infty i}\zeta(t))^T\mathcal{C}_{\infty j}\zeta(t)\\&\le\sum_{i=1}^{r}h_i\zeta^T(t)\ \mathcal{C}_{\infty i}^T\ \mathcal{C}_{\infty i}\zeta(t)\end{aligned} \quad (23)$$

(22) can be presented as follows

$$J_a(t)\le\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\begin{bmatrix}\zeta(t)\\x(t-d_1)\\x(t-d_2)\\w(t)\end{bmatrix}^T\Omega_{ij}\begin{bmatrix}\zeta(t)\\x(t-d_1)\\x(t-d_2)\\w(t)\end{bmatrix}. \quad (24)$$

The inequality $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Omega_{ij} < 0$ is equivalent to

$$\sum_{i=1}^{r} h_i(z(t))h_i(z(t))\Omega_{ii} + \sum_{i<r}^{r} h_i(z(t))h_j(z(t))\{\Omega_{ij}+\Omega_{ji}\} \le 0. \tag{25}$$

Thus if (15) and (16) are satisfied, then $V_a(\zeta,t) \le d\|x\|^2$and (21) is satisfied. From (21)

$$\int_0^T \|\tilde{e}(t)\|^2 dt - \gamma^2 \int_0^T \|w(t)\|^2 dt \le \zeta^T(0)P\zeta(0) + \sum_{i=1}^{2}\int_{-d_i(0)}^{0} x^T(\tau)S_i x(\tau)d\tau \tag{26}$$

because $V(\zeta,T)>0$. It follows from initial condition (8) that

$$\begin{aligned}
&\int_0^T \|\tilde{e}(t)\|^2 dt - \gamma^2\Big[\int_0^T \|w(t)\|^2 dt + x_0^T Q_0 x_0 \\
&+ \sum_{i=1}^{2}\int_{-d_i(0)}^{0} x^T(\tau)Q_i x(\tau)d\tau\Big] \le \zeta^T(0)P\zeta(0) \\
&- \gamma^2 x_0^T Q_0 x_0 + \sum_{i=1}^{2}\int_{-d_i(0)}^{0} x^T(\tau)S_i x(\tau)d\tau \\
&- \gamma^2 \sum_{i=1}^{2}\int_{-d_i(0)}^{0} x^T(\tau)Q_i x(\tau)d\tau \le x_0^T \\
&\{[I_n \quad I_n]P[I_n^T \quad I_n^T]^T - \gamma^2 Q_0\}x_0 \\
&+ \sum_{i=1}^{2}\int_{-d_i(0)}^{0} x^T(\tau)[S_i - \gamma^2 Q_i]x(\tau)d\tau.
\end{aligned} \tag{27}$$

Thus if (15)-(18) are satisfied, H_∞ control performance (12) is satisfied. □

Lemma 2. Consider the system (3) with assumption (2). If there exist matrices $P>0$, $S_1>0$, $S_2>0$, $R_1>0, R_2>0$ and positive scalar ρ satisfying the following inequalities

$$\Psi_{ii} < 0, \quad i = 1,2,\cdots,r \tag{28}$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad i<j<r, \tag{29}$$

$$-\rho + \zeta^T(0)\,P\,\zeta(0) < 0 \tag{30}$$

$$-R_i + N_i^T S_i N_i < 0, \quad i = 1,2 \tag{31}$$

then the corresponding estimation error system achieves guaranteed cost function δ and the guaranteed cost bound δ is $\rho + tr(R_1) + tr(R_2)$.

In here

$$\Psi_{ij} = \begin{bmatrix} \mathcal{A}_{ij}^T P + P\mathcal{A}_{ij} + \mathcal{S} + \mathcal{C}_{zi}^T \mathcal{C}_{zi} & P\mathcal{A}_{1i} & P\mathcal{A}_{2ij} \\ * & -\mathcal{S}_1 & 0 \\ * & * & -\mathcal{S}_2 \end{bmatrix}, \tag{32}$$

$$\int_{-d_i(0)}^{0} \phi(\tau)\phi(\tau)^T d\tau = N_i N_i^T, \quad i = 1,2 \tag{33}$$

where

$$\mathcal{S} = \begin{bmatrix} 0 & 0 \\ 0 & S_1 + S_2 \end{bmatrix}, \quad \mathcal{S}_i = (1-\beta_i)S_i, \quad i = 1,2, \tag{34}$$

and $*$ represents the elements below the main diagonal of a symmetric matrix.

Proof : From Lyapunov functional (13), we obtain

$$\begin{aligned}
J_2 &= \int_0^{t_f} \{\tilde{e}_z^T(t)\,\tilde{e}_z(t)\}dt \\
&= V(0) - V(t_f) + \int_0^{t_f} [\tilde{e}_z^T(t)\,\tilde{e}_z(t) + \dot{V}(t)]dt \\
&\le V(0) + \int_0^{t_f} [\tilde{e}_z^T(t)\,\tilde{e}_z(t)) + \dot{V}(t)]dt
\end{aligned} \tag{35}$$

If

$$\dot{V}(t) + \tilde{e}_z^T(t)\,\tilde{e}_z(t) < 0 \tag{36}$$

we obtain the upper bound of the cost function J_2 as follows

$$J_2 \le V(0) = \zeta^T(0)P\zeta(0) + \int_{-d_1(0)}^{0} x^T(\tau)S_1 x(\tau)d\tau + \int_{-d_2(0)}^{0} x^T(\tau)S_2 x(\tau)d\tau\ . \tag{37}$$

The condition (36) is equivalent to the existence of $P>0$ satisfying

$$\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \begin{bmatrix} \zeta(t) \\ x(t-d_1) \\ x(t-d_2) \end{bmatrix}^T \\
&\begin{bmatrix} \mathcal{A}_{ij}^T P + P\mathcal{A}_{ij} + \mathcal{S} & P\mathcal{A}_{1i} & P\mathcal{A}_{2ij} \\ * & -\mathcal{S}_{11} & 0 \\ * & * & -\mathcal{S}_{22} \end{bmatrix} \begin{bmatrix} \zeta(t) \\ x(t-d_1) \\ x(t-d_2) \end{bmatrix} \\
&+ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (\mathcal{C}_{zi}\zeta(t))^T \mathcal{C}_{zj}\zeta(t) < 0.
\end{aligned} \tag{38}$$

By considering the following condition

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (\mathcal{C}_{zi}\zeta(t))^T \mathcal{C}_{zj}\zeta(t) \le \sum_{i=1}^{r} h_i \zeta^T(t) \mathcal{C}_{zi}^T \mathcal{C}_{zi}\zeta(t) \tag{39}$$

it can be easily shown that the inequality (38) is equivalent to

$$\sum_{i=1}^{r} h_i(z(t))h_i(z(t))\Psi_{ii} + \sum_{i<r}^{r} h_i(z(t))h_j(z(t))\{\Psi_{ij}+\Psi_{ji}\}\le 0. \tag{40}$$

From (40) we got (28) and (29). The (30) and (31) are related to the upper bound of J_2 performance measure. The first term of right hand side in (37) is equivalent to (30). The second term of right hand side in (37) has the following relations

$$\int_{-d_1(0)}^{0}\phi^T(\tau)S_1\phi(\tau)d\tau = \int_{-d_1(0)}^{0} tr\{\phi(\tau)^T S_1\phi(\tau)\}d\tau = tr\{N_1N_1^TS_1\} = \sum_{i=1}^{2} tr\{N_1^TS_1N_1\} < tr\{R_1\}. \tag{41}$$

This is equivalent $-R_1 + N_1^TS_1N_1 < 0$. The third term of right hand side in (37) has the following relations

$$\int_{-d_2(0)}^{0}\phi^T(\tau)S_2\phi(\tau)d\tau = \int_{-d_2(0)}^{0} tr\{\phi(\tau)^T S_2\phi(\tau)\}d\tau = tr\{N_2N_2^TS_2\} = tr\{N_2^TS_2N_2\} < tr\{R_2\}. \tag{42}$$

From (41) and (42), (31) is obtained and $\rho + tr(R_1) + tr(R_2)$ is an upper bound of guaranted cost. □

The next Theorem 1 presents a solution to the guaranteed cost filtering problem with H_∞ constrain for the delayed fuzzy model in terms of LMIs from lemma 1 and lemma 2.

Theorem 1. Consider the system (3), and let $R_i>0,\ i=0,1,2$ be a given initial state weighting matrix $\gamma>0$ a given scalar. If there exist common matrix $X>0,\ Z>0,\ S_1>0, S_2>0,\ R_1>0, R_2>0$ and matrices $W_i,\ Y_i,\ i=1,2,\cdots,r,$ and positive scalar $\alpha,\ \rho$ satisfying the following LMIs:

$$\Phi_{ii}<0,\quad \Lambda_{ii}<0,\qquad i=1,2,\ldots,r, \tag{43}$$

$$\Phi_{ij}+\Phi_{ji}<0,\ \Lambda_{ij}+\Lambda_{ji}<0,\ i<j<r,\ i=1,2,\cdots,r \tag{44}$$

$$-\rho + \phi^T(0)X\phi(0) + \phi^T(0)Z\phi(0) < 0, \tag{45}$$

$$-R_i + N_i^TS_iN_i < 0,\ i=1,2 \tag{46}$$

$$X + Z - \gamma^2 Q_0 \le 0 \tag{47}$$

$$S_i - \gamma^2 Q_i \le 0,\ i=1,2, \tag{48}$$

then there exists a fuzzy filter (7) such that the estimation error system (8) achieves the guaranteed cost function (11) and H_∞ constraint (12).

In here

$\delta = \rho + tr(R_1) + tr(R_2)$,

$$\Phi_{ij} = \begin{bmatrix} W_i + W_i^T + C_{\infty i}^T C_{\infty i} & XA_i - Y_iC_j - W_i & XA_{d_i} & -Y_iC_{d_j} & XB_i - Y_iD_j \\ XA_i - Y_iC_j - W_i & \Delta_i + \alpha I & ZA_{di} & 0 & ZB_i \\ A_{di}^TX & A_{di}^TZ & -\mathfrak{S}_1 & 0 & 0 \\ -C_{dj}^TY_i & 0 & 0 & -\mathfrak{S}_2 & 0 \\ B_i^TX - D_j^TY_i & B_i^TZ & 0 & 0 & -\gamma^2 I \end{bmatrix}, \tag{49}$$

$$\Lambda_{ij} = \begin{bmatrix} W_i + W_i^T + C_{zi}^TC_{zi} & XA_i - Y_iC_j - W_i & XA_{di} & -Y_iC_{di} \\ XA_i - Y_iC_j - W_i & \Delta_i & ZA_{di} & 0 \\ A_{di}^TX & A_{di}^TZ & -\mathfrak{S}_1 & 0 \\ -C_{dj}^TY_i & 0 & 0 & -\mathfrak{S}_2 \end{bmatrix}, \tag{50}$$

where

$$\Delta_i = ZA_i + A_i^TZ + S_1 + S_2,\quad \mathfrak{S}_r = (1-\beta_r)S_r,\ r=1,2. \tag{51}$$

Furthermore, filter gains are given by

$$F_i = X^{-1}W_i,\qquad G_i = X^{-1}Y_i. \tag{52}$$

Proof: Let $P=\text{diag}\{X, Z\}$, where X and Z are symmetric positive definite matrices to be found. Denoting $W_i = XF_i$ and $Y_i = XG_i$, $i=1,2,\cdots,r,$ and considering (10), we obtained (43)-(48) from lemma 1 and lemma 2. □

It has been seen that the filter design problem of the fuzzy system (3) can be transformed into a linear algebra problem. This set of LMIs constitutes a finite-dimensional convex feasibility problem. There are several efficient algorithms to solve the above convex LMIs problem [17,18]

Ⅳ. Design example

We will design a fuzzy filter for the following nonlinear system;

$$\begin{aligned}\dot{x}_1(t) &= -5.125x_1(t)-0.5x_1(t-d(t))-2x_2(t)\\ &\quad -6.7x_2^3(t)-0.2x_2(t-d(t))\\ &\quad -0.67x_2^3(t-d(t))+w(t)\\ \dot{x}_2(t) &= x_1(t)\\ y(t) &= x_2(t)-0.01x_2(t-d(t))+0.1w(t)\\ e(t) &= 2x_1(t)\end{aligned} \tag{53}$$

where time-varying delay and initial state is

$$d(t)=1+0.5\cos(0.1t) \tag{54}$$
$$\phi(t)=[e^t \;\; -e^t]^T,\ t\le 0$$

$x_1(t)$ is estimated using a fuzzy filter and assume that $x_2(t)$ is observable. It is also assumed that

$$x_1(t)\in[-1.5 \;\; 1.5],\qquad x_2(t)\in[-1.5 \;\; 1.5]. \tag{55}$$

Using the same procedure as in [6], the nonlinear term can be represented as

$$-6.7x_2^3(t)=M_{11}\cdot 0\cdot x_2(t)-(1-M_{11})\cdot 15.075\,x_2(t). \tag{56}$$

By solving the equation, M_{11} is obtained as follows:

$$\begin{aligned}M_{11}(x_2(t)) &= 1-\frac{x_{2(t)}^2}{2.25}\\ M_{12}(x_2(t)) &= \frac{x_{2(t)}^2}{2.25}.\end{aligned} \tag{57}$$

M_{11} and M_{12} can be interpreted as membership functions of fuzzy set. By using these fuzzy sets, the nonlinear system can be presented by the following T-S fuzzy model

Plant Rule 1: IF $x_2(t)$ is M_{11} THEN}

$$\begin{aligned}\dot{x}(t) &= A_1x(t)+A_{d1}x(t-d(t))+B_1w(t)\\ y(t) &= C_1x(t)+C_{d1}x(t-d(t))+D_1w(t)\\ e_z(t) &= C_{z1}x(t),\\ e_\infty(t) &= C_{\infty 1}x(t)\end{aligned} \tag{58}$$

Plant Rule 2: IF $x_2(t)$ is M_{12} THEN}

$$\begin{aligned}\dot{x}(t) &= A_2x(t)+A_{d2}x(t-d(t))+B_2w(t)\\ y(t) &= C_2x(t)+C_{d2}x(t-d(t))+D_2w(t)\\ e_z(t) &= C_{z2}x(t)\\ e_\infty(t) &= C_{\infty 2}x(t)\end{aligned} \tag{59}$$

where $x(t)=[x_{1(t)} \;\; x_{2(t)}]^T$,

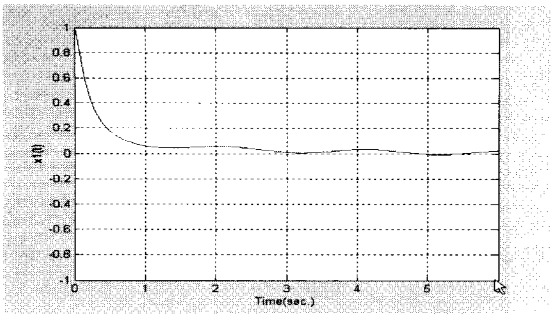
$$A_1=\begin{bmatrix}-5.125 & -2\\ 1 & 0\end{bmatrix},\quad A_{d1}=\begin{bmatrix}-0.5 & -0.2\\ 0 & 0\end{bmatrix},$$

$$A_2=\begin{bmatrix}-5.125 & -17.075\\ 1 & 0\end{bmatrix},\quad A_{d2}=\begin{bmatrix}-0.5 & -1.71\\ 0 & 0\end{bmatrix} \tag{60}$$

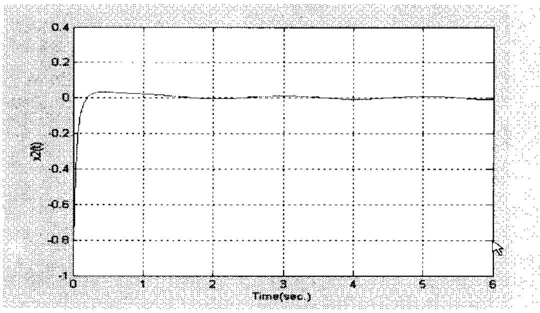
$B_1=B_2=[1 \;\; 0]^T$, $C_1=C_2=[0 \;\; 1]$,
$C_{d1}=C_{d2}=[0 \;\; -0.01]$, $D_1=D_2=0.1$,
$C_{\infty 1}=C_{\infty 2}=\mathrm{diag}[1.5 \;\; 1.5]$, $C_{z1}=C_{z2}=[1 \;\; 0]$.
Let $\gamma=3$, $\beta_1=\beta_2=0.05$, $R_0=\mathrm{diag}[2.4 \;\; 2.4]$, $R_1=R_2=\mathrm{diag}[2.7,\; 2.7]$, then filter gains obtained from Theorem 1 are

$$F_1=\begin{bmatrix}-6.2112 & -3.8335\\ -1.9171 & -29.2087\end{bmatrix},$$
$$F_2=\begin{bmatrix}-4.9771 & -4.0592\\ -2.1546 & -29.4116\end{bmatrix}$$
$$G_1=\begin{bmatrix}-2.7060\\ 29.1623\end{bmatrix},\quad G_2=\begin{bmatrix}-15.1443\\ 20.3392\end{bmatrix} \tag{61}$$

The upper bound of guaranteed cost is 72.8945. The simulation results of nonlinear systems with time-varying delays are shown in <Fig. 1(a)> and <Fig. 1(b)>, respectively.



(a) $x_1(t) - \hat{x}_1(t)$



(b) $x_2(t) - \hat{x}_2(t)$

그림 1. 시간지연 비선형시스템에 대한 시뮬레이션 결과
Fig. 1. The simulation result of nonlinear system with time delays.

For these simulation, the noise signal $w(t)$ is

$$w(t) = 0.1 \cdot \cos(\pi t) \quad (62)$$

and the initial value of the state is assumed by

$$[x_1^T(t) \quad x_2^T(t)]^T = [1 \quad -1]^T, \quad t \leq 0. \quad (63)$$

The designed fuzzy filter estimates the states of the nonlinear system without the steady state errors.

V. Conclusion

In this paper, we have developed guaranteed cost fuzzy H_∞ filter design method for delayed fuzzy dynamic systems. We have obtained sufficient conditions for the existence of fuzzy filters such that the estimation error system is globally exponentially stable and achieves guaranteed cost and H_∞ performance simultaneously. The filter design has utilized the concept of parallel distributed compensation and the filter gains can also be directly obtained from the LMI solutions.

Reference

[1] K. Tanaka and M. Sugeno, "Stability analysis and design of fuzzy control systems," *Fuzzy Sets and Syst.*, Vol. 45, No. 2, pp. 135~156, 1992.

[2] C. L. Chen, P. C. Chen, and C.-K. Chen, "Analysis and design of fuzzy control systems," *Fuzzy Sets and Syst.*, Vol. 57, pp. 125~140, 1993.

[3] H. O. Wang, K. Tanaka, and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: Stability and design issues," *IEEE Trans. Fuzzy Syst.*, Vol. 4, No. 1, pp. 14~23, Feb. 1996.

[4] K. Tanaka and M. Sano, "A robust stabilization problem of fuzzy control systems and ts application to backing up control of a truck-trailer," *IEEE Trans. Fuzzy Syst.*, Vol. 2, No. 2, pp. 119~134, May 1994.

[5] K. Tanaka, T. Ikeda, and H. O. Wang, "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H_∞ control theory, and linear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, Vol. 4, No. 1, pp. 1~13, Feb. 1996.

[6] X. J. Ma, Z. Q. Sun, and Y. Y. He, "Analysis and design of fuzzy controller and fuzzy observer," *IEEE Trans. Fuzzy Syst.*, Vol. 6, pp. 41~51, Feb. 1998.

[7] B. S. Chen, C. H. Lee, and Y.-C. Chang, "H_∞ tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach," *IEEE Trans. Fuzzy Syst.*, Vol. 4, No. 1, pp. 32~43, Feb. 1996.

[8] Z. Han and G. Feng, "State feedback H_∞ controller design of fuzzy dynamic systems using LMI techniques," in *Proc. FUZZ-IEEE,*

Anchorage, AK, May 1988, pp. 538~544.

[9] K. R. Lee, E. T. Jeung and H. B. Park, "Robust fuzzy H_∞ control for uncertain nonlinear systems via state feedback : an LMI approach," *Fuzzy Sets and Syst.*, Vol. 120, No. 1, pp. 123~134, 2001..

[10] B. S. Chen, C. S. Tseng, and H. J. Uang, "Robustness design of nonlinear dynamic systems via fuzzy linear control," *IEEE Trans. Fuzzy Syst.*, Vol. 7, pp. 571~585, Oct. 1999.

[11] A. Jadbabie, M. Jamshidi, and A. Titli, "Guarantwd cost design of continuos-time Takagi-Sugeno fuzzy controllers via linear matrix inequalities," in *Proc. FUZZ-IEEE, Anchorage, AK, May 1988*, pp. 268~273.

[12] B. S. Chen, C. S. Tseng, and H. J. Uang, "Mixed H_2/H_∞ fuzzy outout feedback control design for nonlinear dynamic systems : an LMI approach," *IEEE Trans. Fuzzy Syst.*, Vol. 8, No. 3, pp. 249~265, June. 2000.

[13] Y. Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, Vol. 8, No. 2, pp. 200~211, April. 2000.

[14] K. R. Lee, J. H. Kim, E. T. Jeung and H. B. Park, "Output feedback robust H_∞ control of uncertain fuzzy dynamic systems with time-varying delay," *IEEE Trans. Fuzzy Syst.*, Vol. 8, No. 6, pp. 657~664, Decem. 2000.

[15] M. Vidysagar, *Nonlinear Systems Analysis.* Englewood Cliffs, NJ: Prentice-Hall, 1993.

[16] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, 1994.

[17] Y. Nesterov and A. Nemirovski, *Interior Point Polynomial Methods in Convex Programming: Theory and Applications*, SIAM, Philadelphia, 1994.

[18] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox: For Use with MATLAB*, The Math Works Inc., 1995.

저 자 소 개

李 甲 來(正會員) 第 39卷 SC編 5號 參照
현재 평택대학교 정보과학부 전임강사

朴 烘 培(正會員) 第 36卷 S編 5號 參照
현재 경북대학교 전자전기공학부 교수

曺 喜 洙(正會員)
1993년 : 경북대학교 전자공학과 공학사. 1995년 : 포항공대 공학석사. 2003년 : 경북대학교 공학박사. 1996년~현재 서울지방중소기업청. <주관심분야 : 견실제어, 퍼지제어 및 응용, 모델링 등>