

# Bidding Strategy Determination by Defining Strategic Vector

Dong-Joo Kang\*, Balho H. Kim\*\*, Koo-Hyung Chung\*\* and Young-Hwan Moon\*

**Abstract** - This paper presents a schematic process based on the method of eliminating dominated strategies to obtain the optimal bidding strategy pursuing the Nash equilibrium point. The proposed approach is demonstrated for a bidding game in a generation competitive market with 2-dimensional bidding strategy vectors constituting a price-quantity strategy curve.

**Keywords:** strategy, dominated strategy, nash equilibrium, bidding strategy vector

## 1. Introduction

As the electric power industry moves from the conventional monopolistic or vertically integrated environment to privatized, deregulated, and competitive circumstances, each market participant is interested in maximizing its own profits rather than minimizing system-wide costs [1,2]. As a result, the conventional least-cost methods of scheduling entire generation resources cannot efficiently handle this kind of competitive electricity marketplace [3,6,7]. Here, we consider the Poolco system without considering network configurations for the modeling of a competitive energy market [4,5]. The cost-based pool (CBP) model currently operated by KPX is the cost-based bidding system in which each generator bids their energy at its fuel cost. In this case, the cost function is the unique supply function of a generator. But in Price Based Pool or Two-Way Bidding Pool markets, generators bid their energy at its price, which is not fixed but varies with generator's profit uplifted to fuel cost. Each strategy composed of generation quantity and price is defined as a strategy vector (quantity, price). All the strategy vectors subject to one generator build a strategy curve, also denoting the supply curve. This paper shows the strategy determination process by defining a supply curve consisting of strategy vectors.

## 2. Formation of Bidding Strategy Curve

The energy offered by a generator is specified as a discrete quantity value,  $G_0, G_1, G_2, G_3, G_4$ , as illustrated in Fig 1. When a generator participates in the bidding process, he offers the price, including profit added to marginal fuel cost, illustrated as a bar graph. The magnitude of the profit

uplift component is not fixed but changes as a generator's alternate strategic choice.

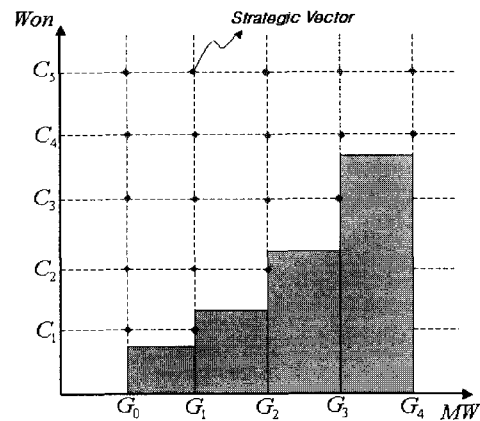


Fig. 1 Cost Function and Strategic Price Vector

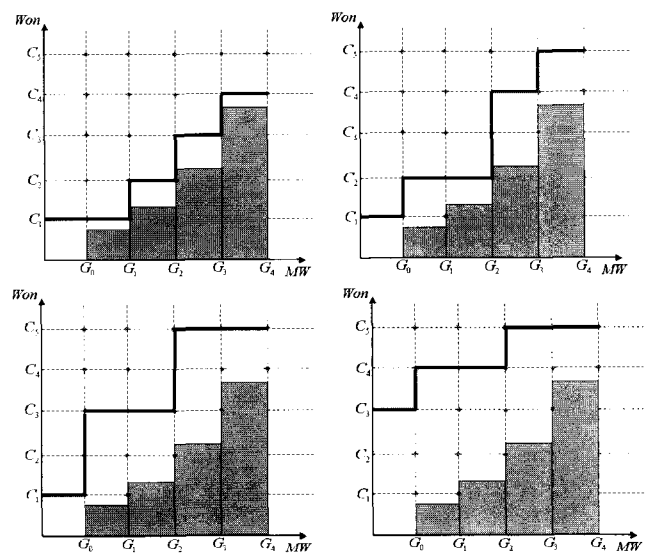


Fig. 2 Strategic Supply Curves

The strategic price-quantity vector is in a higher position than the cost vector, which is the corner point of cost bar

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graph as indicated in Figure 1. These price vectors are connected and constitute various supply curves as shown in Fig. 2.

### 3. Dominant Strategy and Nash Equilibrium

Several methods are available for finding the equilibrium point under perfect information. This paper uses the method of successive elimination of dominated strategies. A player (a generator in this paper) in bidding game may have any number of pure strategies available. We call one of these strategies its dominant strategy if it procures more profit than all of the other strategies, no matter what other generators do. Dominant strategies, when they exist, are extremely useful in helping us solve any game. Because a dominant strategy guarantees the player the highest possible payoff compared to the other strategies, the player should choose the dominant strategy if he has one. But finding the Nash equilibrium point in all games can be difficult. In larger games, some of a player's strategies may be dominated even though no single strategy dominates all of others. If the players find themselves in this type of game, they may be able to obtain equilibrium by removing dominated strategies from consideration as possible choices. Removing dominated strategies reduces the size of the game, and then the reduced game may still have dominated strategies. A process of successive and iterative elimination of dominated strategies reduces the size of a game until no further reductions can be made. If this process ends in a unique outcome, that outcome is the Nash equilibrium of the game and the strategies that yield it are the equilibrium strategies for each player.

### 4. Case Study

We assume the pool market introducing generation sector competition in which two generators, A and B, play the bidding game to maximize their benefit.

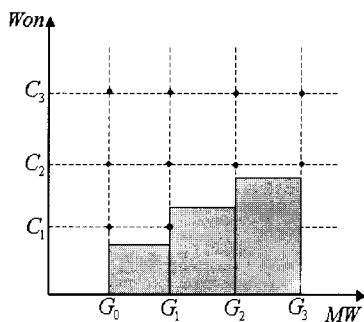


Fig. 3 The cost function of Generator A

The cost function and strategic supply curves of generator A are illustrated in Figs. 3 and 4, respectively, and the cost function and strategic supply curves of generator B are illustrated in Figs. 5 and 6, respectively.

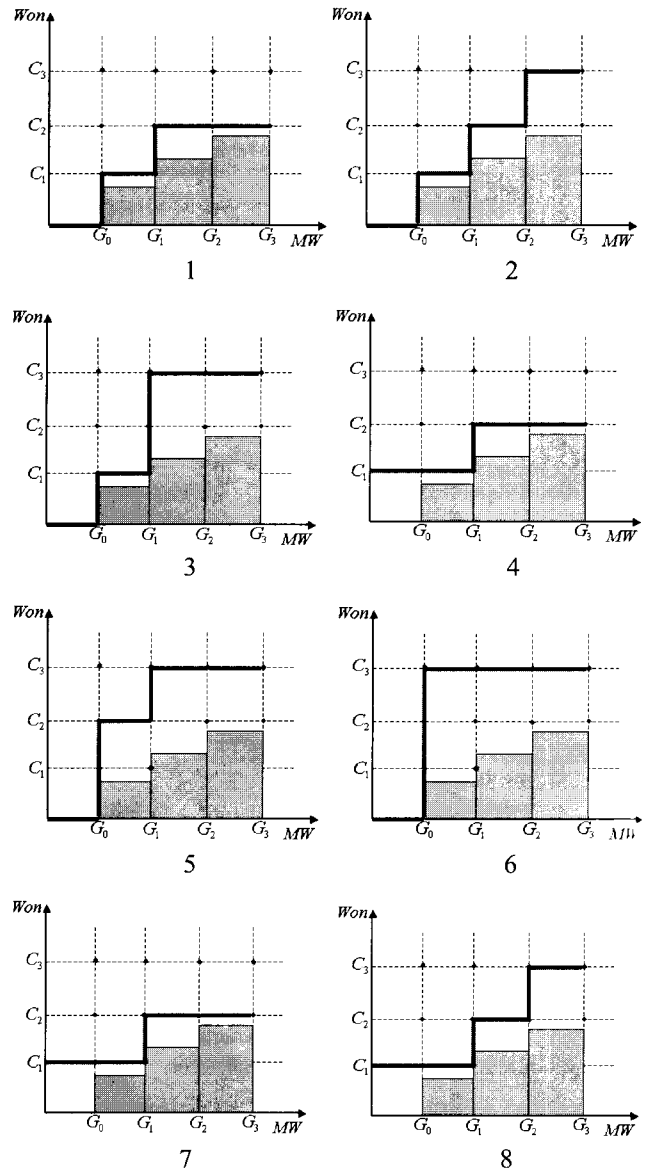
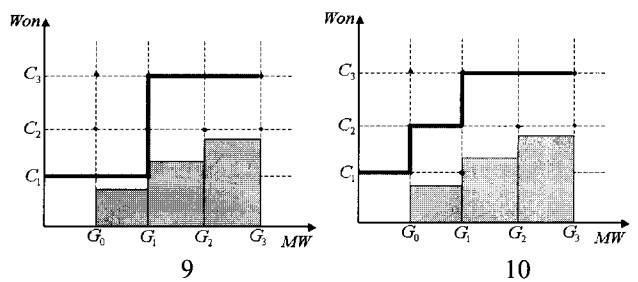


Fig. 4 The strategic supply curves of A



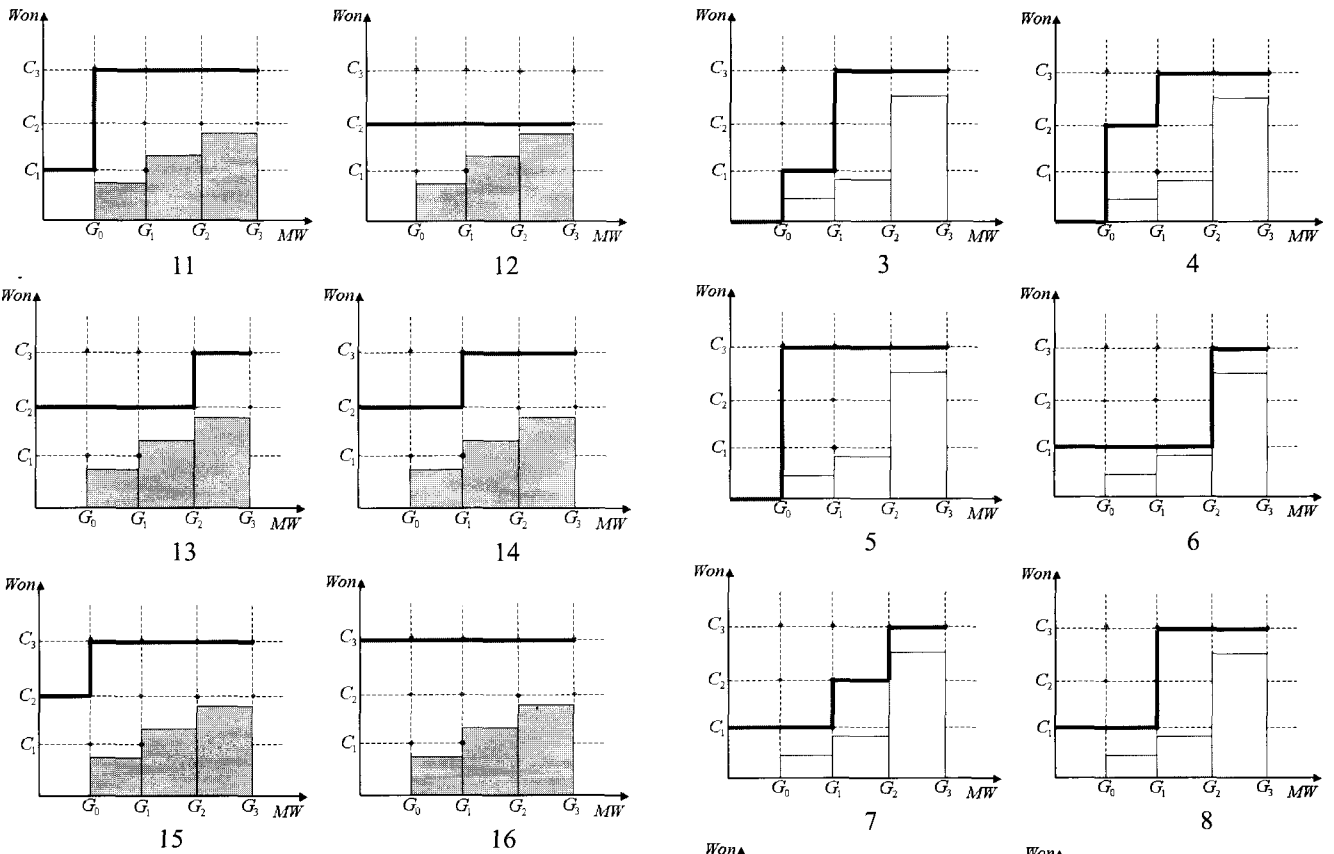


Fig. 4 The strategic supply curves of A (continued)

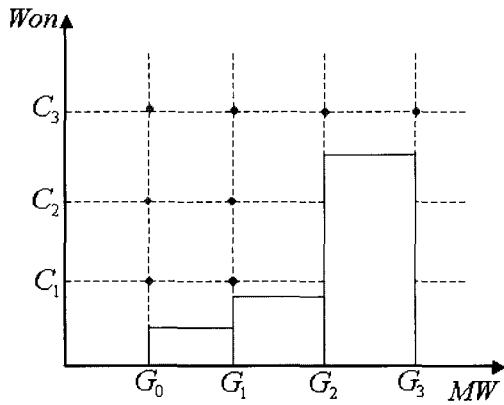


Fig. 5 The cost function of Generator B

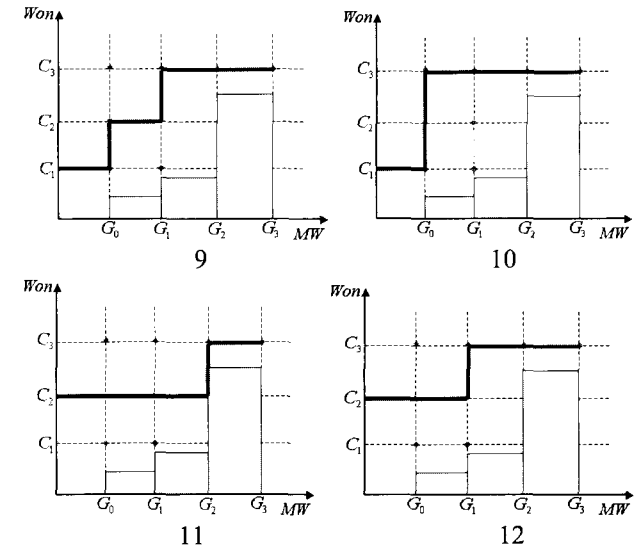


Fig. 6 The strategic supply curves of B

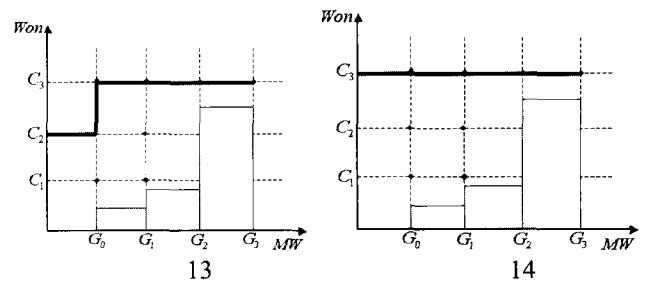
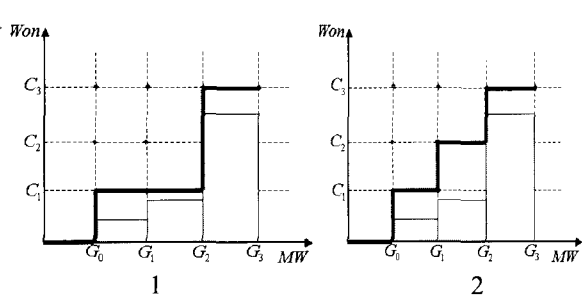


Fig. 6 The strategic supply curves of B (continued)

Let us assume that  $G_0 = 100$ ,  $G_1 = 200$ ,  $G_2 = 300$ ,  $G_3 = 400$ , and  $C_1 = 100$ ,  $C_2 = 200$ ,  $C_3 = 300$ . The market demand is fixed at  $D = 600$ [MW] in this case study. Next, we calculated the payoffs when A and B participate in a bidding game with their strategies. For example, if generator A bids its energy with strategic supply curve 2 in Fig. 4 and B bids its energy with strategic supply curve 6 in Fig. 6, the two generators' payoffs are calculated as shown in Fig. 7, where the solid line is the supply curve selected by generator A and the dotted line is the supply curve selected by generator B. So the generation quantity allocated to each participant for the dispatch is determined as in Fig. 8.

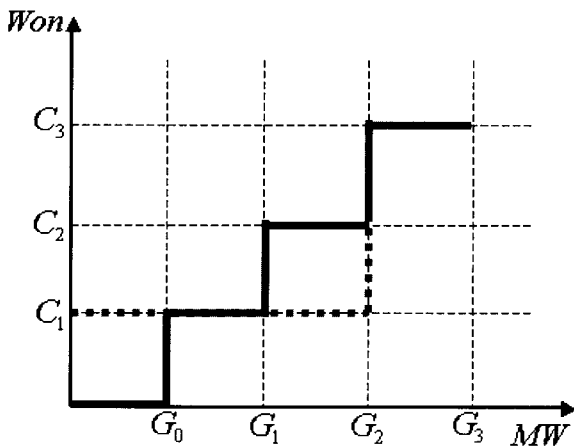


Fig. 7 The strategic choices of A and B

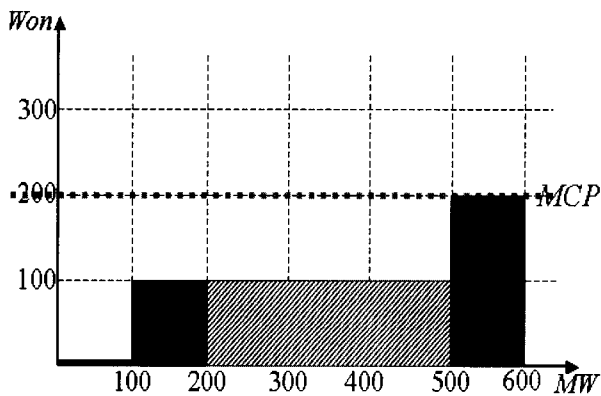


Fig. 8 The supply function

The MCP is determined as 200[won/MWh], and generator A supplies 300[MWh] and generator B supplies 300[MWh] respectively. Finally we can solve the profit of A and B.

As illustrated in Figure 9, the shaded bar graphs are A's generation cost and the area of oblique lines show A's profit. So the profit is calculated as

$$PF_A = 200 \times 300 - (100 \times 80 + 100 \times 120) = 40000[\text{Won}]$$

Similarly, in Figure 10, B's profit is the area of oblique lines and is calculated as

$$PF_B = 200 \times 300 - (100 \times 40 + 100 \times 85) = 47500[\text{Won}]$$

So the profit vector of the two generators is (40000,47500), and all the profit vectors are calculated as shown in Table 1.

Here, we can find out that the strategy in row 9 and the strategy in column 8 are the dominant strategies of A and B, respectively.

In Table 2, the strategy vector of (SA,SB) = (9,8) gives us the most profitable outcome. Therefore, (9,8) is the strategy expected to bring us the maximum profit in this game.

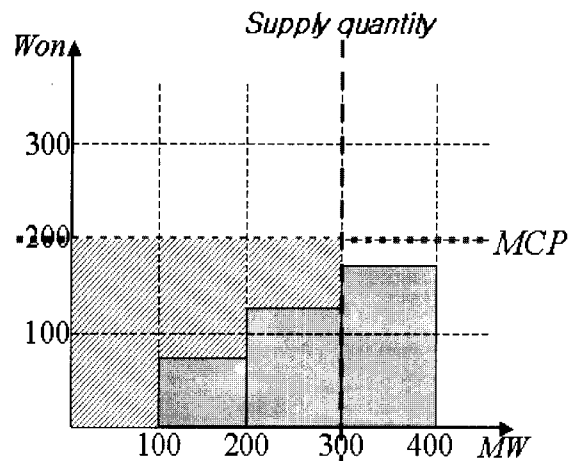


Fig. 9 The cost function and profit of A

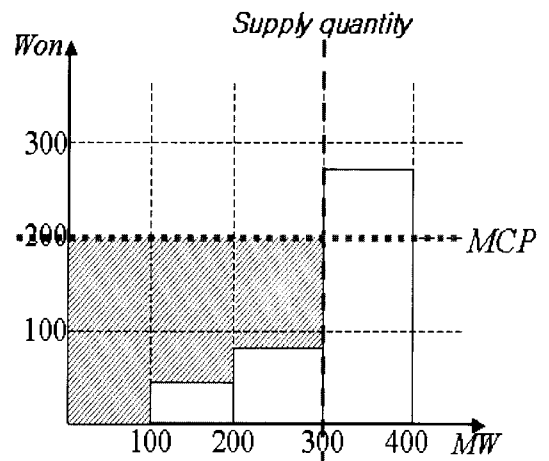


Fig. 10 The cost function & profit of B

**Table 1** The payoff vector of A and B

A \ B	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	300,350	250,375	275,400	300,350	250,375	275,400	300,350	450,500	450,275	450,300	300,350	450,200	450,275	450,300
2	400,450	350,500	350,500	360,370	420,325	400,475	450,320	360,550	250,375	275,400	300,350	250,375	275,400	300,350
3	360,350	250,375	275,400	300,350	250,375	275,400	300,350	450,500	450,275	450,300	300,350	450,200	450,275	450,300
4	350,350	250,375	275,400	360,350	250,375	275,400	300,350	450,500	450,275	450,300	300,350	450,200	450,275	450,300
5	340,350	270,375	275,450	360,370	250,375	275,450	370,350	470,550	450,275	450,300	360,350	450,200	450,275	450,300
6	300,350	270,375	275,450	360,370	250,375	275,450	370,350	470,550	450,275	450,300	360,350	450,200	470,275	470,350
7	300,350	270,375	275,400	360,350	250,375	255,450	370,350	470,550	450,275	450,300	360,370	450,250	470,275	475,350
8	300,350	250,375	275,400	360,350	250,375	255,450	370,350	550,550	450,275	450,300	360,350	450,250	470,300	450,365
9	300,350	220,375	275,480	320,550	250,500	255,525	370,550	575,625	450,550	450,550	360,550	480,550	430,500	450,450
10	270,350	220,375	275,450	300,370	250,375	275,450	360,350	550,550	450,275	425,350	360,350	475,300	430,275	470,390
11	250,350	220,375	275,400	300,350	250,375	275,450	360,350	480,450	450,275	425,350	320,350	450,350	430,275	470,390
12	250,350	220,375	275,400	360,350	250,375	275,450	360,350	480,400	450,275	425,375	320,350	450,460	430,275	470,380
13	250,350	220,375	275,450	360,370	250,375	275,400	360,350	450,420	450,275	425,375	320,350	450,480	420,275	485,370
14	250,350	220,375	275,400	360,350	250,375	275,400	300,350	450,200	450,275	425,375	300,350	450,200	420,275	496,370
15	250,350	220,375	275,400	300,350	250,375	275,400	300,350	450,200	450,275	425,375	300,350	450,200	420,275	488,370
16	250,350	220,375	275,400	300,350	250,375	275,400	300,350	450,200	450,275	400,375	300,350	450,200	420,275	450,350

**Table 2** The Nash equilibrium of A and B

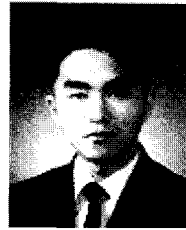
A \ B	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	300,350	250,375	275,400	300,350	250,375	275,400	300,350	450,500	450,275	450,300	300,350	450,200	450,275	450,300
2	400,450	350,500	350,500	360,370	420,325	400,475	450,320	360,550	250,375	275,400	300,350	250,375	275,400	300,350
3	360,350	250,375	275,400	300,350	250,375	275,400	300,350	450,500	450,275	450,300	300,350	450,200	450,275	450,300
4	350,350	250,375	275,400	360,350	250,375	275,400	300,350	450,500	450,275	450,300	300,350	450,200	450,275	450,300
5	340,350	270,375	275,450	360,370	250,375	275,450	370,350	470,550	450,275	450,300	360,350	450,200	450,275	450,300
6	300,350	270,375	275,450	360,370	250,375	275,450	370,350	470,550	450,275	450,300	360,350	450,200	470,275	470,350
7	300,350	270,375	275,400	360,350	250,375	255,450	370,350	470,550	450,275	450,300	360,370	450,250	470,275	475,350
8	300,350	250,375	275,400	360,350	250,375	255,450	370,350	550,550	450,275	450,300	360,350	450,250	470,300	450,365
9	300,350	220,375	275,480	320,550	250,500	255,525	370,550	575,625	450,550	450,550	360,550	480,550	430,500	450,450
10	270,350	220,375	275,450	300,370	250,375	275,450	360,350	550,550	450,275	425,350	360,350	475,300	430,275	470,390
11	250,350	220,375	275,400	300,350	250,375	275,450	360,350	480,450	450,275	425,350	320,350	450,350	430,275	470,390
12	250,350	220,375	275,400	360,350	250,375	275,450	360,350	480,400	450,275	425,375	320,350	450,460	430,275	470,380
13	250,350	220,375	275,450	360,370	250,375	275,400	360,350	450,420	450,275	425,375	320,350	450,480	420,275	485,370
14	250,350	220,375	275,400	360,350	250,375	275,400	300,350	450,200	450,275	425,375	300,350	450,200	420,275	496,370
15	250,350	220,375	275,400	300,350	250,375	275,400	300,350	450,200	450,275	425,375	300,350	450,200	420,275	488,370
16	250,350	220,375	275,400	300,350	250,375	275,400	300,350	450,200	450,275	400,375	300,350	450,200	420,275	450,350

## 5. Conclusions

This paper proposed a process determining the optimal bidding strategy in a price-based competitive generation market. In the proposed process, we first define the points whose components contain the generation quantity and bidding price as strategy vectors, and the defined strategy vectors collected and connected to make a dispatch offer are named the strategic supply curve in this paper. Next, we calculate the payoffs of the generators and finally reach the Nash equilibrium. This resulting model is expected to assist the generator participating in the bidding process, without knowing the price information of others, to build a bidding strategy for ensuring a "stable and high" payoff. We use the expression "stable and high" because the Nash equilibrium doesn't always ensure maximum profit to any participant. Because the Nash equilibrium basically assumes a player to be a *risk-averse* person based on statistical research in behavioral science, the strategy bearing risk is avoided even if it has the possibility of maximum payoff simultaneously. Future extensions of this study are planned and will include player type definitions like *risk-averse*, *risk-neutral*, and *risk-taker*. In addition, the price-quantity band will be expanded to 10 and the number of generators participating in bidding process will be increased to be applicable to the Korean TWBP(Two-Way Bidding Pool) market.

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