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A Nash Bargaining Solution of Electric Power Transactions Embedding Transmission Pricing in the Competitive Electricity Market

Dong-Joo Kang*, Balho H. Kim, Koo-Hyung Chung** and Young-Hwan Moon***

Abstract - The economic operation of a utility in a deregulated environment brings about optimization problems different from those in vertically integrated one [1]. While each utility operates its own generation capacity to maximize profit, the market operator (or system operator) manages and allocates all the system resources and facilities to achieve the maximum social welfare. This paper presents a sequential application of non-cooperative and cooperative game theories in analyzing the entire power transaction process.

Keywords: deregulation, optimization, maximum social welfare

1. Introduction

Competitive electricity markets can be analyzed with game theory. The energy bidding game in a power pool can be considered a non-cooperative game in that each participant competes to win more profit than the others. On the contrary, it is also regarded as cooperative game in that the participants need to cooperate to apply the result of the bidding game or to accomplish a common profit in the physically interconnected power system. The meaning of "cooperative" in the previous sentence is not collusion but an adjusting process by market operators (MOs) or system operators (SOs) or the bilateral contract between two market participants besides the scheduled dispatch by bidding only when allowed. The models in a competitive electricity market can be categorized as pool, bilateral, and a hybrid of the two. This paper chooses the hybrid model to realize an open market situation.

2. Non-Cooperative Energy Bidding Game

Competition in the restructured electricity market requires the market players to build a strategy to maximize their own profit, while cost minimization was the ultimate goal in vertically integrated utilities [2,3]. To analyze the behavior of a profit maximizing utility, we define the production cost function and the profit function as follows:

$$C_i(P_i) = a_i + b_i P_i + c_i P_i^2 \tag{1}$$

$$PF_i(P_i^{allocated}) = \rho_{MCP} P_i^{allocated} - C_i(P_i^{allocated}) \tag{2}$$

where ρ_i is the bid price, C_i is the cost function of the ith generator, PF_i is the profit function of the ith generator, P_i is the generation quantity of the ith generator, $P_i^{allocated}$ is the allocated quantity in a bidding, and ρ_{MCP} is the market clearing price. The steps of solution process are shown in Fig. 1.

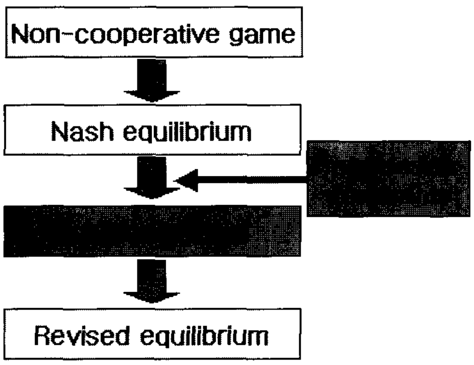

Fig. 1 Overall solution process

In addition, the electricity demand responding to the spot price is assumed to be

$$D_t = D_0 - S\rho_{MCP}\,. \tag{3}$$

Here, D is the initial letter of demand; Index t indicates the t-th bidding stage in periodically repeated bidding process; D_0 is the initial demand; and S is the elasticity of demand depending on Market Clearing Price (MCP). The dimension of demand is [MW] and the dimension of the MCP is [won/MW].

* Electricity Market Technology Research Group, ETRL, KERI, Korea. (djkang@keri.re.kr, yhmoon@keri.re.kr)
** Dept. of Electrical and Electronic Engineering, Hongik Univerity, Korea. (bhkim@wow.hongik.ac.kr, berlinetta@hanmail.net)
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For mathematical simplification, only two generators are assumed to participate in the market. Then we expect the following three cases.

i). $\rho_A > \rho_B$	ii). $\rho_A < \rho_B$	iii). $\rho_A = \rho_B$

ρ_A is the bid price offered by Generator A and ρ_B is the bid price offered by Generator B.

i) $\rho_A > \rho_B$ or ii) $\rho_A < \rho_B$

If $\rho_A > \rho_B$, then the market price, ρ_{MCP}, is determined as ρ_A, and the market demand at ρ_A is

$$D_t = D_0 - S\rho_{MCP}. \quad (4)$$

Generator B comes to have the optimal generation quantity, P_B^*, by the condition $\partial PF_B / \partial P_B = 0$ maximizing its own profit. The Allocated volume for utility A in the market is the rest volume after B's dispatch.

Consequently, the optimal bidding strategies of the two utilities, A and B, are given as follows.

A's strategy : (ρ_A*, $P_A = D_t - P_B$*)
B's strategy : (ρ_B, P_B*)

iii) $\rho_A = \rho_B$

In this case, there exists no equilibrium point because one utility can make more benefit by modifying its own bidding strategy.However, on rare occasions, the bid prices submitted by generators are equal. So it is possible that we neglect that rare case.

3. Nash Bargaining Problem

In this section, we apply the Nash bargaining solution in adjusting the Nash equilibrium point, obtained by solving the non-cooperative game in Section 2, to reflect system constraints like transmission capacity. The main features of the Nash Bargaining Game formulation are as follows [4,5].

1) The outcome of the cooperative game satisfying the Nash bargaining game's axioms could be characterized by the following mathematical maximization problem.

Maximize $(x-a)^h (y-b)^k$ subject to $y = f(x)$

2) The objective function is the polynomial in which each term is a product of individual profit functions subject to a common contract.

3) In formulating the profit function, the generation quantity in formulating the Nash bargaining game has two kinds of components. One is the preliminary quantity calculated from the previous section, and the other is the adjustment, to be added to or subtracted from the initial solution, to satisfy system constraints.

4) The willingness-to-pay price for purchasing transaction power from another generator is considered another variable in addition to generation power quantity and transaction power quantity of the objective function.

5) The final result should be Pareto Optimal so that no generator would change or withdraw from the equilibrium point for better profit.

The Nash bargaining problem between i and j is generally formulated as

$$\text{Max} \sum_{k \in K} \prod_{i,j} R_{ij}^k (p_{ij}, T_{ij}) \quad (5)$$

Where $R_{ij}^k(p_{ij}, T_{ij}) = p_{ij}T_{ij} - C_i(P_{gi} + T_{ij}) + C_i(P_{gi})$ and with the following definitions:

R_{ij}^k is the profit function of contract k,
K is the number of contracts,
i, j are generators under contract K,
p_{ij} is the transaction price per unit of transaction, T_{ij},
T_{ij} is the transaction power generated by generator i and purchased by j,
P_{gi} is the power generated by generator i, and
C_i is the generation cost of generator i.

T_{12} and p_{12} indicate the transaction quantity and price, respectively. The Nash bargaining problem between the two players is denoted by $R_{12}(p_{12}, T_{12})$. According to Eq. (5), the two-generator bargaining problem is described as

$$L = \max (R_1 \cdot R_2) \quad (6)$$

Where

$$R_1 = p_T T_{12} - C_1(P_{g1} + T_{12}) + C_1(P_{g1})$$
$$R_2 = -p_T T_{12} - C_1(P_{g2} - T_{12}) + C_2(P_{g2})$$

The first term in R_1 and R_2 is the payment for a given transaction, and the second term is the change in the area operation cost owing to the transaction. Each generator has a constrained generation capacity, and profits are constrained by the condition $R_1 > 0$ and $R_2 > 0$, indicating that negative profit is unacceptable. When there are no transactions, it means no adjustments are made to the initial solution determined by the bidding game in Section 2.

4. Case Study

In this section, we analyze the game situation described in Sections 1 and 2. The two-bus and two-generator system is illustrated in Fig. 2. And the data of the two generators are given in Table 1.

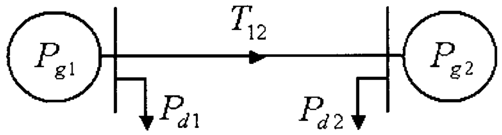

Fig. 2 Two-bus and two-generator system

Table 1 The parameters of the two generators

Player	Cost Coefficient			Gen. Limit	
	a_i	b_i	c_i	Min	Max
Gen. A	0	6.0	0.22	10	250
Gen. B	0	2.0	0.42	20	200
$D_0 = 450$, $S = 1.5$					

In Table 1, D_0 and S indicate the initial system demand and demand elasticity, respectively. If the market clearing price, ρ_{MCP}, is determined as ρ_B, the optimal bidding price of B, ρ_B^*, is calculated from $\partial PF_B / \rho_B = 0$.

$$\rho_B^* = \frac{2c_A^2(D_0 + Sb_B + 2D_0Sc_B) + c_A\{b_B + b_A + 2c_B(Sb_A + D_0)\} + c_Bb_A}{4c_A^2(S + S^2c_B) + 2c_A(1 + 2Sc_B) + c_B} \quad (7)$$

From Eq. (7), we obtain $\rho_B^* = 98.03$, the optimal bidding price of generator B. Because generator A offered a lower bidding price than B, A was allocated optimal quantity $P_A^* = 94$ from $P_B^* = (\rho_B - b_A)/2c_B$. Finally, ρ_B^* should satisfy the upper boundary as follows.

$$D_t - P_B^* \geq 0$$

$$\rho_A \leq \frac{2D_0c_B + b_B}{2Sc_B + 1}$$

This is calculated to $\rho_B < 116$, so we come to know $\rho_B^* = 98.03$ satisfies this condition. This calculated market clearing price determines market demand. The profit function of generator A at this time is

$$PF_A = \frac{1}{4c_B}(\rho_B - b_A)^2 - a_A \quad (8)$$

and the final results are arranged in Table 2.

Table 2 The equilibrium point from the strategies of the two generators

	A's best strategy	B's best strategy
ρ_A	· ·	98.03
(P_A, P_B)	(209, ·)	(· , 94)
Dt	303	
(PF_A, PF_B)	(9624.45, 5315.7)	

This result must be applied in the power system subject to physical and economic constraints. Therefore, we need a re-optimization procedure for the Nash bargaining solution to obtain the final adjusted solution considering those constraints. In this paper, we only reflect the usage cost based transmission pricing proportional to the transmission capacity used. So the transmission price is assumed to be specified as \$100/MWh. Fig. 3 describes the results of the non-cooperative Nash game solved in Section 2.

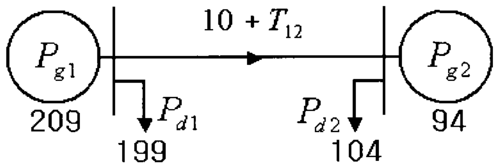

Fig. 3 Two-bus and two-generator system

The Nash bargaining game is formulated in accordance with Eqs. (5) and (6).

$$L = \max\,(R_1 \cdot R_2) \quad (9)$$

subject to

$$R_1 = p_T T_{12} - C_1(P_{g1} + T_{12}) + C_1(P_{g1}) - 1000$$
$$R_2 = -p_T T_{12} - C_1(P_{g2} - T_{12}) + C_2(P_{g2}) - 100T_{12}$$

R_1, R_2 is expanded as follows:

$$R_1 = p_T T_{12} - 6(P_{g1} + T_{12}) - 0.22(P_{g1} + T_{12})2 + 6P_{g1} + 0.22P_{g12} - 1000$$
$$= p_T T_{12} - 6T_{12} - 0.22(199 + T_{12})^2 + 0.22(199)^2 - 1000$$
$$= p_T T_{12} - 81.56\, T_{12} - 0.22\, T_{12}^2 - 1000 \quad (10)$$

$$R_2 = -p_T T_{12} - C_2(P_{g2} - T_{12}) + C_2(P_{g2}) - 100T_{12}$$
$$= -p_T T_{12} + T_{12} - 0.42(94 - T_{12})^2 + 0.42 \cdot 94^2 - 100T_{12}$$
$$= -p_T T_{12} - 20.04T_{12} - 0.42T_{12}^2 \quad (11)$$

$$L = \max(p_T T_{12} - 81.56T_{12} - 0.22T_{12}^2 - 1000)(-p_T T_{12} - 20.04T_{12} - 0.42T_{12}^2)$$
$$= \max(1000p_T T_{12} + 61.52p_T T_{12}^2 - 0.20p_T T_{12}^3 - p_T^2 T_{12}^2 + 2054.46T_{12}^2 + 38.67T_{12}^3 + 0.0924T_{12}^4)$$

To maximize L, we use

$$\frac{\partial L}{\partial p_T} = 1000T_{12} + 61.52T_{12}^2 - 0.20T_{12}^3 - 2p_T T_{12}^2 = 0$$

$$\frac{\partial L}{\partial T_{12}} = 1000T_{12} + 61.56T_{12}^2 - 0.20T_{12}^3 - 2p_T^2 T_{12} + 20.04$$

$$+ 4108.82T_{12} + 116.01T_{12}^2 + 0.3696T_{12}^3 = 0.$$

Eq. (9) is a non-linear problem with a non-linear objective and constraints, and multi-extreme points exist in the optimization problem. These results are shown in Table 3.

Table 3 Results of Nash bargaining game

		Gen [MW]	Marginal price [$/MWh]	Savings [$/h]
Before Trans.	G1	209	41.14	-
	G2	94	98.03	-
After Trans.	G1	233	81.33	377.98
	G2	70	81.33	377.98
Transaction Power T_{12}			34.00 [MW]	
Price			62.33[$/MWh]	
Payment			2109.10[$]	

5. Conclusions

This paper presents a sequential application of the non-cooperative and cooperative game theories in analyzing the process of power transaction in a competitive generation market. We first introduced a competitive generation pool model to determine the Nash equilibrium and secondly applied the Nash bargaining solution to the results of the non-cooperative game to reflect the impact of transmission cost. This resulting model is expected to reduce the dead-weight loss of social welfare (which means the lost part of social welfare due to non-optimal distribution of entire social resource) in deregulated markets in which the resources are allocated not by system-wide economic dispatch but by dispatch offers and market participants' dispatch bids. For more general applicability, this study will be extended to the market model for *n* players including *n* generators, *n* purchasers, and a transmission operator.

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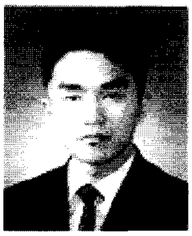
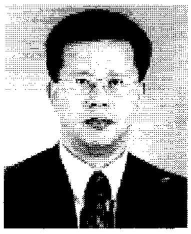
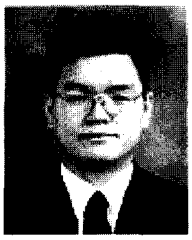


Dong-Joo Kang
He received the B.S. and M.S. degrees in Electrical Engineering from Hong-ik University, Korea, in 1999 and 2001, respectively. He has been with KERI (Korea Electro-technology Research Institute) since 2001. His interests include analysis of power industry restructuring, application of game theory analysis to power transactions, and development of electricity market simulators. He is a member of KIEE.



Balho H. Kim
He received the B.S. degree from Seoul National University, Korea, in 1984, and M.S. and Ph.D. degrees from the University of Texas at Austin in 1992 and 1996, respectively. He was with KEPCO (Korea Electric Power Corporation) from 1984 to 1990 and joined Hongik University in 1997. His research fields include distributed optimal power flow, public pricing, B/C analysis, and power system planning and operation.

Koo-Hyung Chung
He received the B.S. and M.S. degrees from Hong-Ik University in 2001 and 2003, respectively. He is currently working toward the Ph.D. degree at Hong-Ik University. He has been working for P&P (Power & People) since 2001. His research interests are market analysis and power system operation.



Young-Hwan Moon
He received the B.S. and M.S. degrees from Seoul National University, Korea, in 1979 and 1981, respectively, and the Ph.D. degree from the University of Texas at Arlington in 1990, all in Electrical Engineering. He has been with KERI (Korea Electro-technology Research Institute) since 1981 and is now a principal research engineer. His interests include modeling of power systems, application of power system stabilizer, and control of HVDC transmission. He is a member of IEEE and Tau Beta Pi.