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ESTIMATION OF THE SGP4 DRAG TERM FROM TWO OSCULATING ORBIT STATES†

Byoung-Sun Lee‡ and Jae-Woo Park
Communications Satellite Development Center, ETRI, Daejeon 305-350, Korea
E-mail: lbs@etri.re.kr

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ABSTRACT

A method for estimating the NORAD SGP4 atmospheric drag term from minimum osculating orbit states, i.e., two osculating orbits, is developed. The first osculating orbit state is converted into the NORAD TLE-type mean orbit state by iterative procedure. Then the converted TLE is propagated to the second orbit state using the SGP4 model with the incremental SGP4 drag term. The iterative orbit propagation procedure is finished when the difference of the two osculating semi-major axes between the propagated orbit and the given second orbit is minimized. In order to minimize the effect of the short-term variations of the osculating semi-major axis, the osculating argument of latitude of the second orbit is propagated to the same argument of latitude of the first orbit. The method is applied to the estimation of the NORAD-type TLE for the KOMPSAT-1 spacecraft. The SGP4 drag terms are estimated from both NORAD SGP4 orbit propagation and the numerical orbit propagation results. Variations of the estimated drag terms are analyzed for the KOMPSAT-1 satellite orbit determination results.

Keywords: NORAD, TLE, SGP4, Atmospheric Drag, Orbit Propagation, KOMPSAT-1

1. INTRODUCTION

The Simplified General Perturbations 4 (SGP4) is the mathematical model for predicting the Low Earth Orbit (LEO) satellite position and velocity from the NORAD TLE(Hoots & Roehrich 1980). The Two-Line-Element (TLE) is the mean orbital elements of the space object generated by the North American Aerospace Defense Command (NORAD). The NORAD TLE has been widely used by the satellite community who don't have the capability to generate their own satellite orbits. So, the NORAD TLE is used as a basic input for the commercialized satellite ground antenna control system and orbit analysis tools.

Satellite mission control center normally generates the osculating orbital elements for the satellite operations by using their own orbit determination system. When the commercialized satellite ground antenna system is used, the interface between the mission control and antenna control can be simplified by using NORAD-type TLE. The osculating orbital elements can be converted into the NORAD TLE-type mean orbital elements by the iterative approximation procedure (Ernandes 1994,

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‡corresponding author

Lee 2002). However the atmospheric drag effect cannot be estimated in the orbital element conversion. So, the orbit propagation errors are accumulated in the SGP4 model. The atmospheric drag effect of the LEO satellite in the SGP4 model is considered as the SGP4 drag term (B^*). The SGP4 drag term can be estimated by processing of the large number of the tracking data in the conventional orbit determination system (Jochim et al. 1996).

In this paper, a method for estimating the SGP4 B^* drag term from minimum osculating orbit states, i.e., two osculating orbits, is developed. Two osculating orbit state should be timely separated for estimating the SGP4 B^* drag term. The first osculating orbit state is converted into the NORAD TLE-type mean orbit state by iterative procedure. Then the converted TLE is propagated to the second orbit state using the SGP4 model with the incremental B^* drag term. The iterative orbit propagation procedure is finished when the difference of the two osculating semi-major axes between the propagated orbit and the given second orbit is minimized. In order to minimize the effect of the short-term variations of the osculating semi-major axis, the osculating argument of latitude of the second orbit is propagated to the same argument of latitude of the first orbit. The method is applied to the estimation of the NORAD-type TLE for the KOMPSAT-1 spacecraft. The B^* drag terms are estimated from both NORAD SGP4 orbit propagation and the numerical orbit propagation results. Variations of the estimated B^* drag terms are also analyzed for the KOMPSAT-1 satellite orbit determination results.

2. FORMULATION OF THE PROBLEM

2.1 SGP4 Model

The SGP4 model was developed in 1970 and used for the orbit propagation of the near-Earth (period less then 225 minutes) space objects with the TLE from NORAD. The SGP4 model is related with the satellite state vector as follows:

$$\bar{y}(t) = f(\bar{x}_0, B^*, t) \tag{1}$$

where $\bar{y}(t)$ is the state vector(position and velocity) at time t, function f represents the SGP4 model, $\bar{x}_0$ is the mean orbital elements in the TLE at epoch t_0, and B^* is the SGP4 drag term in the TLE.

The SGP4 specific mean orbital elements $\bar{x}_0$ in TLE are

$$\bar{x}_0 = (\bar{e}_0, \bar{i}_0, \bar{\Omega}_0, \bar{\omega}_0, \bar{M}_0, \bar{n}_0) \tag{2}$$

where $\bar{e}_0$ is eccentricity, $\bar{i}_0$ is inclination, $\bar{\Omega}_0$ is right ascension of ascending node, $\bar{\omega}_0$ is argument of perigee, $\bar{M}_0$ is mean anomaly, and $\bar{n}_0$ is anomalistic mean motion. Traditionally the anomalistic mean motion $\bar{n}_0$ is used in SGP4 model instead of the semi-major axis $\bar{a}_0$.

The NORAD SGP4 drag term B^* is related with the ballistic coefficient B by the formula as:

$$B^* = \frac{1}{2} B \rho_0 \tag{3}$$

where ρ_0 is the atmospheric reference density (Montenbruck 2000).

2.2 Conversion into the NORAD TLE

The conversion of the six osculating orbit elements into the six NORAD-type mean elements is the iterative approximation procedures as:

$$x_i^{(a+1)} = x_i^{(a)} + \sum_{j=1}^{6} M^{-1}(y_j - y_j^{(a)}), \qquad i = 1, ..., 6. \tag{4}$$

where x_i represents the NORAD-type mean elements, y_i represents the osculating elements, (a) is the iterative approximation counts, and M^{-1} is the inverse matrix of the partial derivatives of the SPG4 model f respect to the NORAD-type mean elements (Lee et al. 2002).

The matrix M is expressed as:

$$M = \begin{bmatrix} \frac{\partial f_1}{\partial x_1^{(a)}} & \frac{\partial f_1}{\partial x_2^{(a)}} & \cdots & \frac{\partial f_1}{\partial x_6^{(a)}} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_6}{\partial x_1^{(a)}} & \frac{\partial f_6}{\partial x_2^{(a)}} & \cdots & \frac{\partial f_6}{\partial x_6^{(a)}} \end{bmatrix}. \tag{5}$$

The first one of the partial derivatives in Eq. (5) is denoted as:

$$\frac{\partial f_1}{\partial x_1^{(a)}} = \frac{f\left(x_1^{(a)} + \Delta x_1^{(a)}, x_2^{(a)}, ..., x_6^{(a)}\right) - f\left(x_1^{(a)}, x_2^{(a)}, ..., x_6^{(a)}\right)}{\Delta x_1^{(a)}} \tag{6}$$

The iterative procedure in Eq. (4) can be finished when the difference between the given osculating elements and the estimated elements, i.e., $(y_j - y_j^{(a)})$, is small enough for convergence.

While the Eq. (4) provides a useful point-to-point conversion from osculating to mean state vectors, the result is only approximate due to the inevitable modeling deficiencies in the SGP4 theory. Considering the neglect of higher order perturbations as well as sectorial and tesseral gravity field components, the SGP4 model gives rise to position errors of 2 km as a rule of thumb (Montenbruck 2000).

2.3 Estimation of the SGP4 Drag Term

The SGP4 drag term B^* can be estimated as one of the solve-for state parameters when the measurement data such as the GPS navigation solutions are enough for applying the weighted least square procedure (Cho et al. 2002). The orbit determination system is considered over-determined when the number of the GPS navigation solutions(position and velocity vector) are more than seven in the assumption of the seven NORAD-type TLE solve-for state parameters as in Eq. (1).

Otherwise, if only two osculating orbit states are known, the orbit determination system is under-determined for estimating the NORAD-type mean orbital elements and drag term B^*. So, the normal least square procedure could not be applicable. However, the osculating orbit state can be converted into the NORAD-type TLE by applying the iterative approximation procedure as in Eq. (4). Then the SGP4 drag term B^* can be estimated using the timely separated another osculating orbit state. The procedure is depicted in Figure 1.

The osculating argument of latitude (AOL) are used in the procedure 2, 5, and 7 in Figure 1 as:

$$AOL = \bar{\omega} = \omega + M. \tag{7}$$

In procedure 7 in Figure 1, the SGP4 orbit propagation of the second orbit to the same argument of latitude of the first orbit is performed with $B^* = 0$ in the assumption that the drag term affects little within one orbit.

The iterative procedures 8 and 9 can be finished when the difference between the propagated osculating semi-major axis and the given second osculating semi-major axis is minimized. A search for minimized difference starts from the semi-major axis difference in procedure 9 is within solution criteria otherwise no solution is considered. The solution criteria used in this paper is smaller than 1.0×10^{-4} km. So, the solution is not always guaranteed because the system is under-determined.

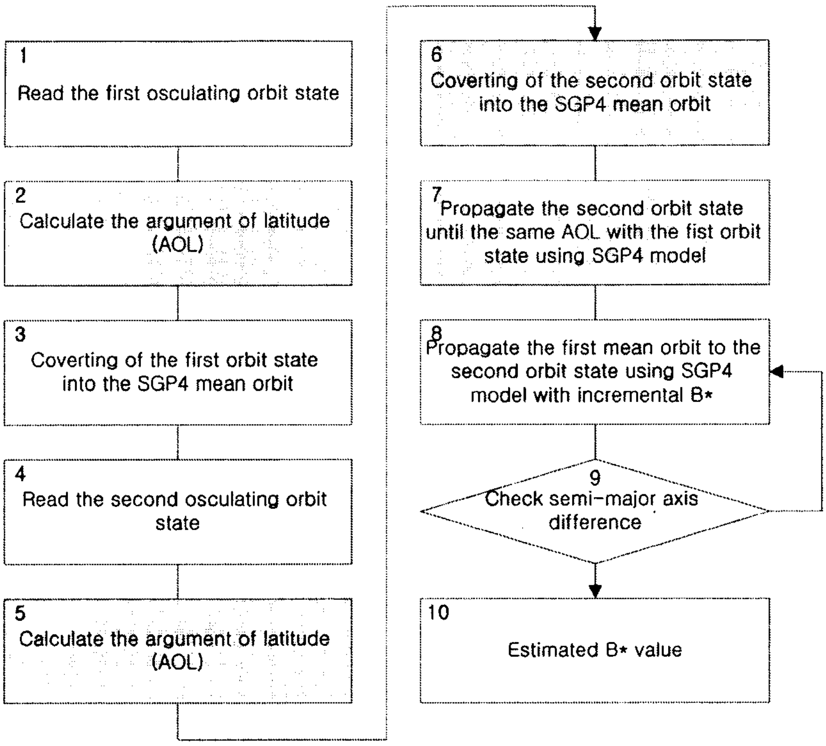

Figure 1. Estimation procedure of the SGP4 drag term B^*.

The shadowed procedure such as 2, 5, 6 and 7 can be skipped when the equalization of the argument of latitude is not considered.

3. APPLICATION TO THE KOMPSAT-1 ORBIT

3.1 Conversion of the KOMPSAT-1 Osculating Orbit

The osculating orbit propagation results of the KOMPSAT-1 with 30 seconds interval and 7 days duration are converted into the SGP4 NORAD-type TLE. The orbit prediction program in the KOMPSAT-1 Mission Analysis and Planning System (MAPS)(Won et al. 1999, Lee et al. 2002) was used for the osculating orbit propagation. Table 1 shows the initial osculating orbit elements and the related parameters.

Figure 2 shows the variation of the osculating semi-major axis and the converted SGP4 mean semi-major axis for 24 hours. The osculating semi-major axis shows two upper peaks and two lower peaks in one orbital revolution with amplitude of about 18 km by the Earth zonal effect. The converted SGP4 mean semi-major axis also shows some variations however the variations could not be characterized in this figure.

Figure 3 represents the variation of the converted SGP4 mean semi-major axis. The amplitude of the variation is about 0.75 km and shows no periodicity unlike the osculating semi-major axis. This shows the modeling errors in the osculating-into-mean orbit conversion due to the inevitable

Table 1. Osculating orbital elements of the KOMPSAT-1 (Epoch : 2001/02/13 00:00:29.0 UTC).

parameter	value	parameter	value
a (km)	7058.30630	Ω(deg)	305.7296
e	0.0023845	ω (deg)	66.6917
i (deg)	98.1532	M (deg)	166.7931
S/C Area (m^2)	8.5	S/C Mass (kg)	448.0
C_d	2.2	C_R	1.5

C_d: Atmospheric Drag Coefficient, C_R: Solar Radiation Pressure Coefficient

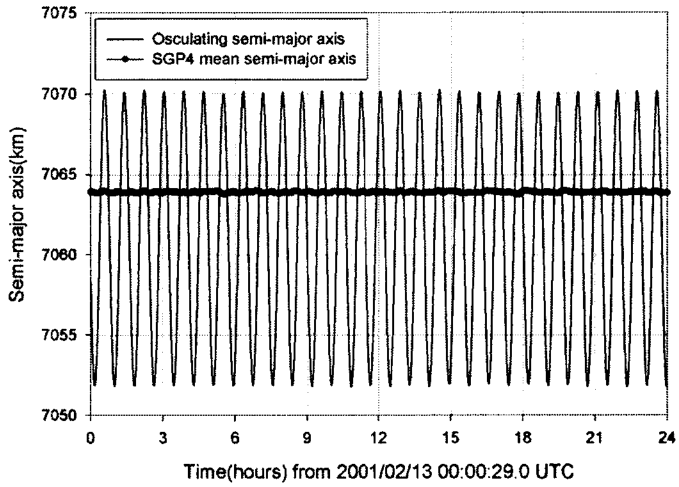

Figure 2. Variation of the osculating semi-major axis and SGP4 mean semi-major axis.

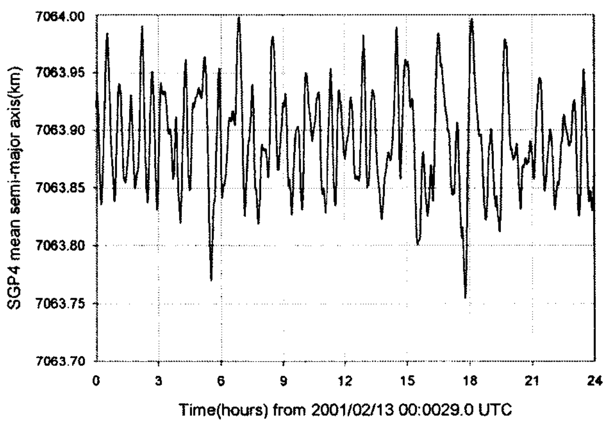

Figure 3. Variation of the converted SGP4 mean semi-major axis for 1 day.

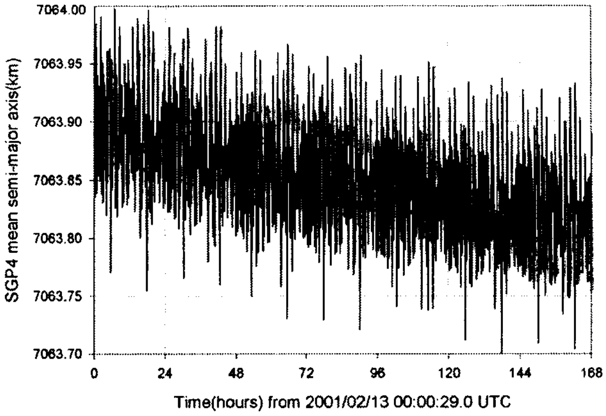

Figure 4. Variation of the converted SGP4 mean semi-major axis for 7 days.

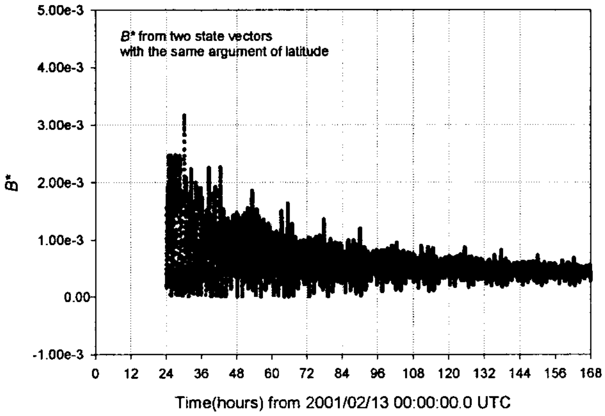

Figure 5. Estimated B^* values from SGP4 orbit state without considering the argument of latitude.

modeling deficiencies in the SGP4 theory. If the conversion into the mean orbit is perfect, the mean semi-major axis should be a straight line with some inclination due to the orbital decay by the atmospheric drag.

Figure 4 shows the variation of the SGP4 mean semi-major axis for the 7 days term. The figure shows the orbital decay of the KOMPSAT-1 during one-week period. The decay rate is calculated as about 11.0 m/day.

3.2 Estimation of the KOMPSAT-1 B^* Value

According to the proposed method in this paper, two osculating orbit states are required for estimating the SGP4 drag term B^* value. Here, the proposed method is applied to the simulated situation and the real situation. Firstly, the orbit propagation results using SGP4 and MAPS are used as two simulated situations. Secondly, the real orbit determination results are applied as a real situation. In order to estimating B^* values, a series of SGP4 orbit propagation with the incremental B^* values for minimizing the difference between the propagated semi-major axis and the given semi-major axis should be performed. So, in estimating the B^* values, the same orbit model is applied

Table 2. Estimated B^* values in 2001/02/18.

No.	Time	Delta SMA(km)	B^* value
1	15:00:59	0.309537×10^{-7}	0.35235×10^{-3}
2	15:01:29	0.184218×10^{-7}	0.35817×10^{-3}
3	15:01:59	0.237560×10^{-7}	0.40238×10^{-3}
4	15:02:29	0.420914×10^{-7}	0.33241×10^{-3}
5	15:02:59	0.230466×10^{-7}	0.31381×10^{-3}

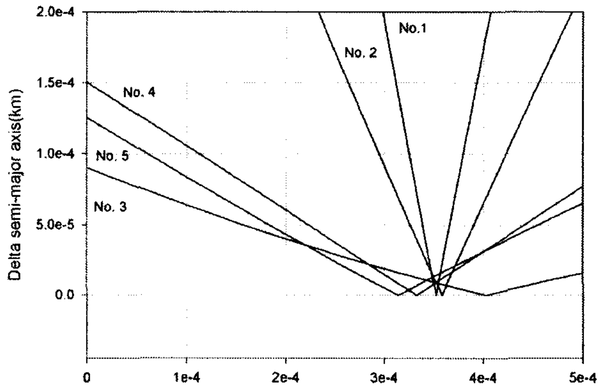

Figure 6. Variation of the semi-major axis difference according to the B^* values for the five consecutive states.

for the SGP4 propagation case and the different orbit model is used for the MAPS case.

3.2.1 Simulated situation using SGP4 orbit propagation results

The SGP4 orbit propagation was performed for 7 days. The TLE and the B^* value of 3.4851×10^{-4} from NORAD in Lee (2002) and the step size of 30 seconds were applied for the propagation. Two kinds of the methods were used for estimating B^* values. One was the B^* estimation without considering the argument of latitude in the procedure of Figure 1. So, the shadowed boxes in Figure 1 were skipped. The other followed the same procedures in Figure 1.

Figure 5 shows the estimated B^* values from SGP4 orbit state without considering the argument of latitude. There are no B^* values within one day because the estimation starts from the one day after the orbit epoch. The figure shows very good results up to 5.5 days. However, estimated B^* values show scattering after 5.5 days. Even many points could not be displayed in Figure 5.

In order to analyze the scattering of the estimated B^* values after 5.5 days, five consecutive states in 2002/02/18 were selected. Table 2 shows the time of the orbit state, the minimum semi-major axis difference in procedure 9 of Figure 1, and the estimated B^* values. Although the semi-major axis differences between the propagated state and the given state are smaller than 1.0×10^{-7} for all cases, the B^* values are different enough to show the scattering in Figure 5.

Figure 6 presents the variation of the semi-major axis difference according to the B^* values. The different characteristics of the variation of the semi-major axis differences are shown.

Figure 7 shows the estimated B^* values from SGP4 orbit state with considering the argument

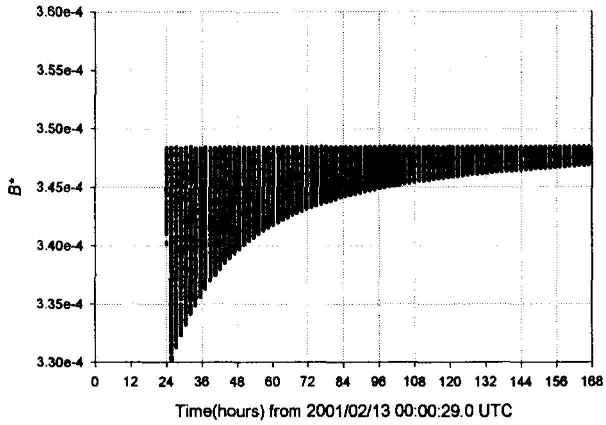

Figure 7. Estimated B^* values from SGP4 orbit state with concerning argument of latitude.

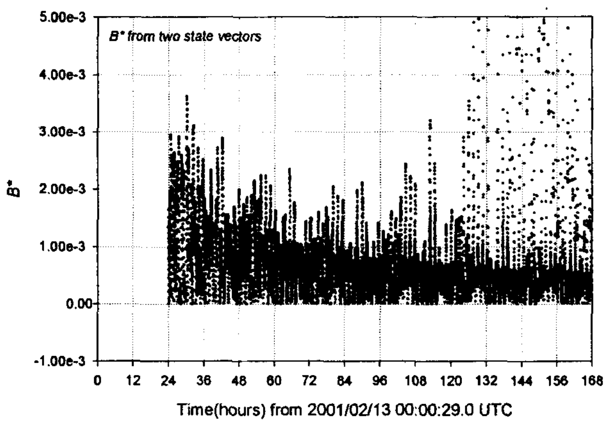

Figure 8. Estimated B^* values from MAPS orbit state without considering the argument of latitude.

of latitude. Unlike the characteristics in Figure 5, the estimated B^* values start from variations with some amplitude and then converged.

3.2.2 Simulated situation using MAPS orbit propagation results

The second orbit state in the previous cases comes from the SGP4 orbit propagation that is the same model for estimating B^* values. So, this kind of situation is just for analysis not for the real satellite operation.

In order to make the situation real, the orbit propagation using the operational KOMPSAT-1 MAPS was performed for 7 days with the orbital elements in Table 1. The orbit propagation results with 30 seconds step was used as the second orbit state for estimating B^* values.

Figure 8 shows the estimated B^* values from MAPS orbit state without considering the argument of latitude. The amplitude of variation of the estimated B^* is very big compared to the SGP 4 propagation case in Figure 5. This is due to the different modeling between the SGP4 and MAPS. The scattering of the estimated B^* is even worse with the time passes.

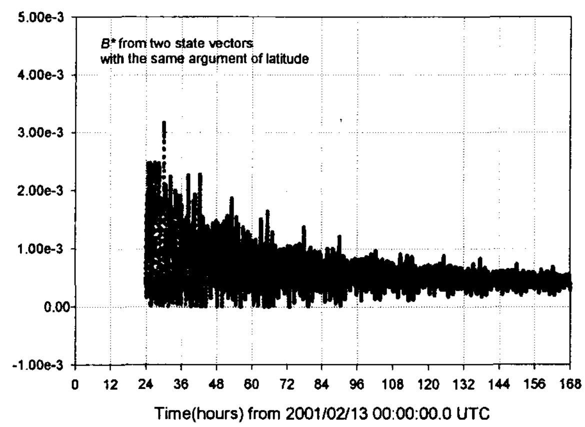

Figure 9. Estimated B^* values from MAPS orbit state with considering the argument of latitude.

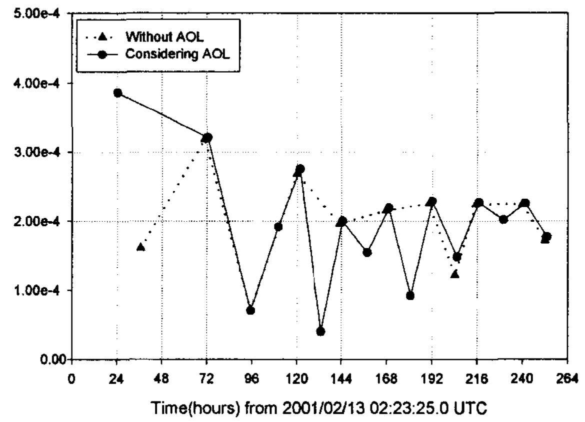

Figure 10. Estimated B^* values from MAPS orbit determination results.

Figure 9 represents the estimated B^* values from MAPS orbit state with considering the argument of latitude. Compared to the SGP 4 propagation case in Figure 7, the amplitude of variation of the estimated B^* is also very big. However, the trend of the estimated B^* is converged with a longer term. Figure 9 looks very similar to the Figure 8 without the scattering of the longer terms.

The above case is somewhat realistic because there are some possibilities for estimating the B^* value from numerically propagated orbit such as MAPS. In this case, the possibility to estimate a good B^* value is increased with the time separation between the two orbit states.

3.2.3 Real situation using MAPS orbit determination results

The orbit determination for the KOMPSAT-1 is normally performed twice a day with the GPS navigation solutions from down linked spacecraft telemetry. Here, the real orbit determination states are used for the estimation of the B^* values. This is a realistic operational application for using SGP4 propagation without NORAD values.

Figure 10 shows the estimated B^* values from orbit states with and without considering the

Table 3. Number of estimated B^* from MAPS orbit determination (Total 19 orbit states).

Cases	B^* Estimated	No solution
Without considering AOL	11	8
Considering AOL	16	3

equalization of the argument of latitude. The amplitude of the variation of the estimated B^* values shows some convergence with the longer term. And the amplitude is comparable to that of the estimated B^* values from NORAD in the same period (Cho et al. 2002).

Table 3 summarizes the number of estimated B^* values from 19 orbit states with and without considering the argument of latitude.

4. CONCLUSIONS

A method for estimating the SGP4 drag term from two osculating orbit states has been developed and analyzed. The iterative SGP4 orbit propagation with the incremental drag term was performed until the difference of the two osculating semi-major axes between the propagated first orbit and the given second orbit was minimized. The osculating argument of latitude is used for minimizing the effect of the short-term variations of the osculating semi-major axis.

The method can be directly applicable when the NORAD type TLE is required to the spacecraft operations such as the tracking antenna control and SGP4 orbit propagation. Implementation of the method is relatively simple compare to the full scale NORAD type TLE estimation. Because the system model is originally under-determined, estimation of the SGP4 drag term by the method is not always guaranteed. However, it can be resolved when a series of the osculating state is applied for estimating the SGP4 drag term.

The simulation and analysis show that the time separation between the two osculating orbit states is a factor for estimating a good SGP4 drag term. Also, consideration of the argument of latitude is confirmed to be favorable for estimating the SGP4 drag term. The comparison of the estimated SGP4 drag term by the method described in this paper and the full scale NORAD TLE orbit determination system will be performed later as a further study.

REFERENCES

Cho, C.-H., Lee, B.-S., Lee, J.-S., Kim, J.-H., & Choi, K.-H. 2002, JA&SS, 19, 197

Ernandes, K. 1994, AMSAT Journal, 17, 26

Hoots, F. R., & Roehrich, R. L. 1980, Models for Propagation of NORAD Element Sets, Project Space Track Report No. 3, Aerospace Defense Command, Peterson AFB, CO., 2nd ed. By T. S. Kelso, December 1988

Jochim, E. F., Gill, E., Montenbruck, O., & Kirschner, M. 1996, Acta Astronautica, 39, 912

Lee, B.-S. 2002, JA&SS, 19, 395

Lee, B.-S., Lee, J.-S., & Kim, J.-H. 2002, JA&SS, 19, 89

Montenbruck, O. 2000, Aerospace Science Technology, 4, 277

Won, C.-H., Lee, J.-S., Lee, B.-S., & Eun, J.-W. 1999, ETRI Journal, 21, 29