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College Students' Conceptions of Mathematics: A Comparison of Korean Students and American Students

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I. Introduction

1. Needs and Aims of This Study

Mathematical activity does not occur entirely within the cognitive realm, it is important to consider what beliefs students hold about mathematics and mathematical activity and how those beliefs influence mathematical experiences. There is a growing body of research supports the views that a person's conceptions of what is important in mathematics influences how he or she approaches mathematical situations(Walker, 1999).

Mathematics is a changing discipline. New branches of mathematics are being created (e.g., chaos theory, fractals), new tools are being developed for studying the discipline (e.g., Mathematica software, graphic calculators), and even what constitutes a mathematical proof is being called into question (e.g., Are computer based proofs of theorems, such as recent proof of the Four Color Problem, legitimate?). Along with these major changes has come a renewed interest in philosophical questions such as "What is mathematics?" and "What does it mean to know mathematics?" Responses to these questions have important implications for the teaching and learning mathematics(Grouws, Howald, and Colangelo, 1996).

Walker(1999) examined students' concepts about learning, knowing, and doing mathematics after studying four years of a Standards based reform high school mathematics curriculum developed by the Core Plus Mathematics Project (CPMP). Upon graduation from high school the students believed that mathematical concepts, principles, and generalizations were slightly more important than facts, formulas, and algorithms. They believed that mathematics was a coherent and dynamic field, that learning mathematics was more about constructing understanding than memorizing, that doing mathematics was more about making sense out of situations than just solving problems, and that mathematics was useful.

By the study "A World of Differences - An International Assessment of Mathematics and Science" held by ETS (1988), Korean 13 year olds achieved the highest score in mathematics proficiency, and Americans achieved the lowest score. For the mathematics attitude "I like mathematics a little or a lot", Korean 13 year

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olds and Americans responded the same level of 7th among the 12 populations. By TIMSS (The Third International Mathematics and Science Study), Korean students' achievement scores were very high (top 2nd among the 45 populations), but their responses to the attitude toward mathematics were pretty low. The results of TIMSS gave much implications to reform current Korean mathematics curriculum (Korean 7th Curriculum).

But, there are few studies on the Korean college level students' conceptions of mathematics and no comparative studies with other countries. It is known well that American college students do well mathematical problem solving and they study mathematics with understanding and creative thinking. So, this study aims to find some implications to reform Korean college level mathematics education by analyzing Korean college students' conceptions of mathematics and by comparing them with that of American college students.

2. Students' Conceptions of Mathematics

Grouws et al. (1996) developed a framework of dimensions for examining student conceptions of mathematics. The framework consisted with seven dimensions such as composition of mathematical knowledge, structure of mathematical knowledge, status of mathematics knowledge, doing mathematics, validating ideas in mathematics, learning mathematics, and useful mathematics. The seven dimensions and their associated poles provided in Table 1 are described below in detail. Each dimension has two poles of positive and negative and is considered as a continuum.

<Table 1> Dimensions and Poles of the Conceptions of Mathematics Inventory

Composition of Mathematical Knowledge		
Knowledge as concepts, principles, and generalizations	vs.	Knowledge as facts, formulas and algorithms
Structure of Mathematical Knowledge		
Mathematics as a coherent system	vs.	Mathematics as a collection of isolated pieces
Status of Mathematical Knowledge		
Mathematics as a dynamic field	vs.	Mathematics as a static entity
Doing Mathematics		
Mathematics as sense making	vs.	Mathematics as results
Validating Ideas in Mathematics		
Logical thought	vs.	Outside authority
Learning Mathematics		
Learning as constructing and understanding	vs.	Learning as memorizing intact knowledge
Usefulness of Mathematics		
Mathematics as a useful endeavor	vs.	Mathematics as a school subject with little value in everyday life or future work

A. Composition of Mathematics Knowledge

The poles of this dimension are the knowledge as concepts, principles and generalizations and the knowledge as facts, formulas, and algorithms. The conception of mathematical knowledge as concepts, principles, and generalizations reflects the belief that mathematics consists of important ideas and relationships among them. These ideas and relationships should guide mathematical activity. Further, the notion used in mathematics also reflects important concepts. The conception of mathematical knowledge as facts, formulas, and algorithms reflects the belief that mathematics is made up of important rules and procedures that allow students to accomplish the necessary mathematics. Mathematical activity is guided by finding the appropriate rule or formula to use in a given situation.

B. Structure of Mathematical Knowledge

This dimension assesses whether students believe that mathematics is a coherent system or a collection of isolated pieces. The conception that mathematics is a coherent system is a belief that there are meaningful connections among the concepts, principles, and skills students learn in mathematics classes. When students learn new mathematics the students having this conception expect that prior knowledge and problem solving skills are relevant and helpful and act accordingly. In contrast, holding the belief that mathematics is a collection of isolated pieces discourages one from looking for connections between mathematical topics or among the processes used to solve mathematical problems. Not only the strands such as geometry and algebra are considered separate, the roles of proof, construction, and problem solving are independent as well.

C. Status of Mathematical Knowledge

The poles of this dimension are mathematics as a dynamic field and mathematics as a statistic entity. If students believe that mathematics is a dynamic field, they believe that mathematics is growing and changing and that this growth impacts the discipline as a whole. On the other hand, if students believe that mathematics is a static field, they believe that mathematics is a compilation of information that remains fixed once it is developed.

D. Doing Mathematics

The poles of this dimension are mathematics as sense making and mathematics as results. Mathematics as sense making reflects a belief that the process of doing mathematics depends on valuing, exploring, comprehending, and expanding the concepts and principles underlying mathematics. In contrast, mathematics as results reflects the belief that the process of doing mathematics is implementing procedures and finding results. At most it is acknowledged that mathematicians created such procedures for specific uses and that they understand the underlying principles involved. Doing mathematics involves remember- ing and carefully following step by step procedures.

E. Validating Ideas in Mathematics

The poles of this dimension are validation through logical thought and validation established by an outside authority. Logical thought represents the belief that the validity of mathematical knowledge is established through personal reflection and individual thought and reasoning. In contrast, outside authority represents the belief that one receives valid mathematical knowledge from an authority; a textbook, a knowledgeable peer, a teacher, or a mathematician.

F. Leaning Mathematics

The poles of this dimension are learning as constructing and understanding and learning as memorizing. Learning as constructing and understanding represents the belief that one creates new knowledge by fitting things with past experiences. The learner feels a need to be involved actively in experiences and resolves any conflicts with previous knowledge that arises. In contrast, learning as memorizing represents the belief that learning mathematics is a process of mentally storing what one has been thought; that is, the learner is a passive receiver who records existing knowledge. Learning well means to be able to quickly and accurately recall needed information.

G. Usefulness of Mathematics

The poles of this dimension are mathematics as a useful endeavor and mathematics as a school subject with little value in everyday life. The statement regarding mathematics as useful reflects the belief that students use mathematics later in life as opposed to only in other mathematics classes.

II. A Study of College Students' Conceptions of Mathematics

1. Subjects

Data were collected from two samples of students. The first sample was comprised of 60 Korean college students participating in a calculus course as liberal arts in the university the researcher belonged at the end of the 2nd semester in 2001. The second sample was 30 American college students, the researcher lectured in Michigan State, participated in a pre-calculus course at the end of spring semester in 2000. Among the 60 Korean students, 40 were belonged in the department of mathematics education and others are from those departments of liberal arts or social sciences.

2. Instrument

This study uses the Conceptions of Mathematics Inventory(CMI) developed by Grouws et. al. (1996). Grouws et. al. (1996) developed three major themes of student conceptions of mathematics: The nature of mathematical knowledge, the character of mathematical activity, and the essence of learning mathematics. And then developed the CMI to measure the students' conceptions of mathematics. The Inventory is consisted with seven dimensions: Three dimensions of composition of mathematical knowledge, structure of mathematical knowledge, and status of mathematical knowledge developed from the theme of the nature of mathematical knowledge, two dimensions of doing mathematics and validating ideas in mathematics developed from the theme of the character of mathematical activity, one dimension of learning mathematics from the theme of the essence of learning mathematics, and one dimension of usefulness of mathematics.

The CMI is a 56 items, 6 point Likert scale instrument, with response choices of strongly disagree, disagree, slightly disagree, slightly agree, agree, and strongly agree to the statements. The conceptions in each dimension are assessed using eight statements. Of the eight statements, four are written to reflect one pole and the other four to reflect the other pole. The pole in the first column of Table 1 will be reflected to as the positive pole. The positively worded and the negatively worded statements were scored differently. The scoring used in the analysis is provided in Table 2.

<Table 2> Scoring of the Conceptions of Mathematics Inventory

Response	Positive Item	Negative Item
Strongly Agree	6	1
Agree	5	2
Slightly Agree	4	3
Slightly Disagree	3	4
Disagree	2	5
Strongly Disagree	1	6

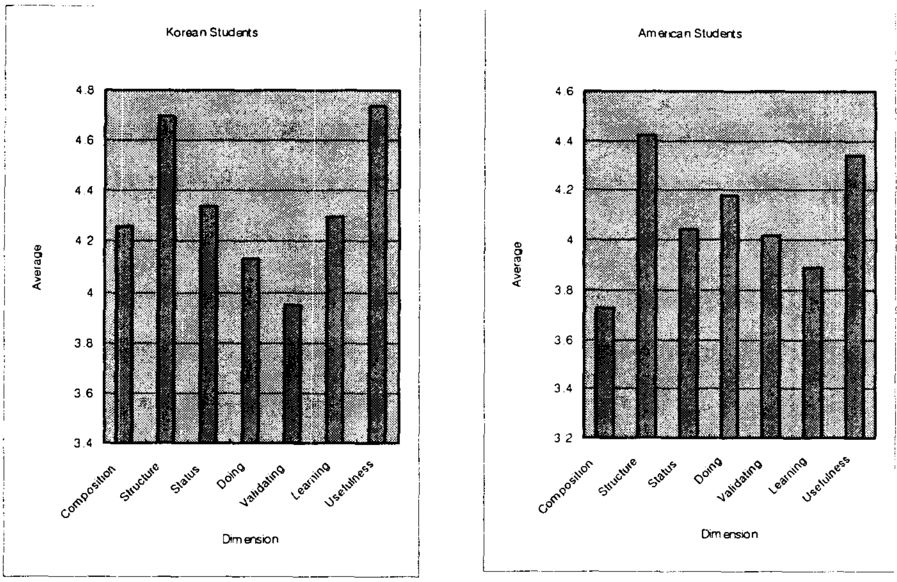
III. Results and Discussion

1. Overview of Student Conceptions of Mathematics

Before looking at each of the dimensions in detail, it will be helpful to look at the overall patterns of the two samples. Figure 1 displays a bar graph of the overall average ratings for the two samples. The average rating for each dimension of two samples are above the midpoint 3.5. This indicates that the conceptions held by the students are positive in general.

For the Korean sample, the strongest conception is in the dimension of usefulness of mathematics. The two poles of this dimension are mathematics as a useful endeavor, and mathematics as a school subject with little value in everyday life or future work. Korean sample studying calculus as free liberal art seems to understand well that mathematics is a useful subject in their everyday life. The weakest conception is in the dimension of validating mathematics. The two poles are logical thought and outside authority. Most of Korean secondary school students study supplementary or in depth materials at private academies after their school hours. They have little time to study by themselves.

For the American sample, the strongest conception is in the dimension of structure of mathematics knowledge. The two poles of this dimension are mathematics as a coherent system, and mathematics as a collection of isolated pieces.



[Figure 1] Dimension Averages on Korean and American Students

The American students seem to understand well that mathematical ideas should be connected and built upon each other. The weakest conception is in the dimension of composition knowledge. The two poles of this dimension are knowledge as conceptions, principles, and generalizations; and knowledge as facts, formulas and algorithms. The American sample took pre-calculus course, and could solve most of the exercises or problems by using the formulas or the algorithms they learned in the same chapters. Korean sample shows more positive responses in five dimensions such as (1) composition of mathematical knowledge, (2) structure of mathematical knowledge, (5) validating ideas in mathematics, (6) learning as constructing, and (7) mathematics as a useful endeavor, and American sample is more positive in two dimensions such as (3) status of mathematical knowledge, and (4) doing mathematics. For the dimensions of (1), (2), (3), (6), and (7), the differences are statistically significant (see Table 3).

In the next sections, we will consider the students' conceptions of mathematics in each of these dimensions by comparing the average rating of the two samples.

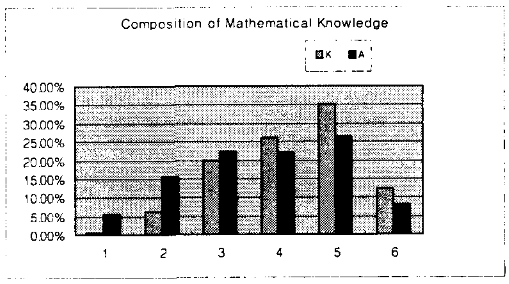
<Table 3> Group K and group A Statistics by Dimensions

Dimension	Group K		Group A		p - value
	Mean	S.D.	Mean	S.D.	
1	4.2591	1.1359	3.7250	1.3633	0.0000
2	4.7000	0.9061	4.4200	1.1644	0.0026
3	4.3386	1.0040	4.0400	1.2515	0.0030
4	4.1295	1.3428	4.1750	1.4404	1.2945 *
5	3.9523	1.2860	4.0200	1.3890	0.5586 *
6	4.3000	1.2863	3.8900	1.4863	0.0008
7	4.7364	1.0118	4.3350	1.3756	0.0002

2. Composition of mathematical knowledge

One positive item in this dimension is the statement "Mathematical knowledge consists mainly of ideas and concepts and the connections among them." One negative item is the statement "There is always a rule to follow when solving a mathematical problem."

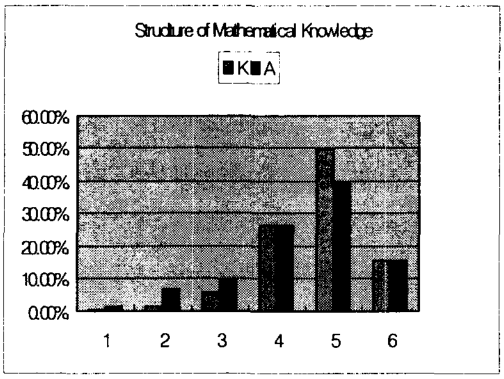
The mean score of group K (Korean college students) is 4.2591 and the mean score of group A (American college students) is 3.7250. A significant difference (p=0.0000) was found in the responses on this dimension. On this dimension, 21.0% of American students disagreed and only 6.82% of Korean students disagreed (see Figure 2). Korean students showed more positive responses in the compositions of mathematics knowledge.



[Figure 2] Composition of Mathematics Knowledge Survey Results

3. Structure of mathematical knowledge

One positive item in this dimension is the statement "Mathematics is mostly thinking about relationships among things such as numbers, points, and lines. One negative item is the statement "Mathematics consists of many unrelated topics."



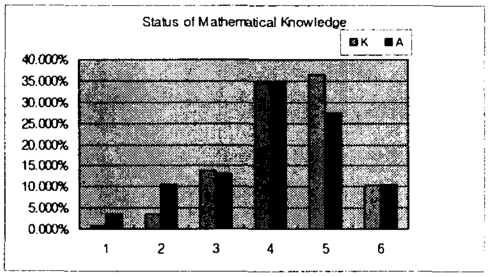
[Figure 3] Structure of Mathematical Knowledge Survey Results

The mean of group K is 4.7000 and the mean of group A is 4.4200. A significant difference (p = 0.0026) was found in the responses on this dimension. On this dimension, 91.8% of Korean students were positive and 81.5% of American students were positive (see Figure 3). The Korean students showed significantly positive responses in the conceptions of structure of mathematical knowledge than the American students.

4. Status of Mathematical Knowledge

One positive item in this dimension is the statement "The field of mathematics is always growing and changing." One negative item is the statement "New discoveries are seldom made in mathematics."

The mean of group K is 4.3386 and the mean of group A is 4.0400. A significant difference (p = 0.0030) was found in the responses on this dimension. The Korean students showed more positive responses in the conceptions of status of mathematical knowledge. On this dimension, the same percents (10%) of the each group showed strongly agreed, but 4.3%of Korean students disagreed or strongly disagreed and 14.0% of American students disagreed or strongly disagreed (see Figure 4).



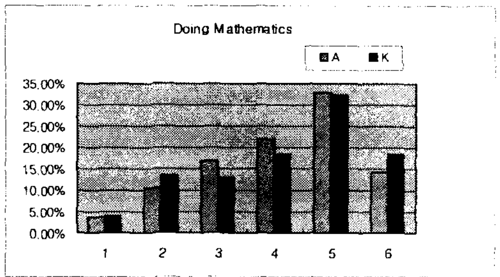
[Figure 4] Status of Mathematical Knowledge Survey Results

5. Doing Mathematics

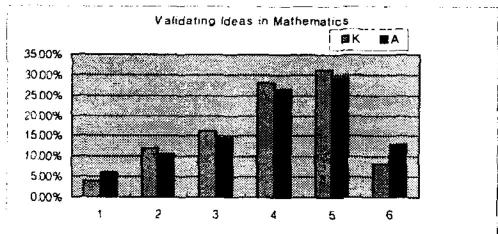
One positive item in this dimension is the statement "When working mathematics problems, it is important that what you are doing makes sense to you." One negative item is the statement "If you knew every possible formula, then you could easily solve any mathematical problem."

The mean of group K is 4.1295 and the mean of group A is 4.1750. American students' responses are a little more positive than Korean students. But the difference is not statistically significant (p=1.2945).

On this dimension, 14.3% of Korean students strongly agreed and 18.5% of American students strongly agreed (see Figure 5). This result seems to be affected by the instructional styles of the two countries. Korean schools more emphasize algorithmic knowledge or computational skills to achieve high scores for college entrance examinations, and American schools emphasize cooperative learning focused on investigations and problem solving.



[Figure 5] Doing Mathematics Survey Results



[Figure 6] Validating Ideas in Mathematics Survey Results

6. Validating Ideas in Mathematics

One positive item in this dimension is the statement "When your method of solving a problem is different from your teacher's method, your method can be as correct as your teacher's." One negative item is the statement "You know something is true in mathematics when it is in a book or a teacher tells you."

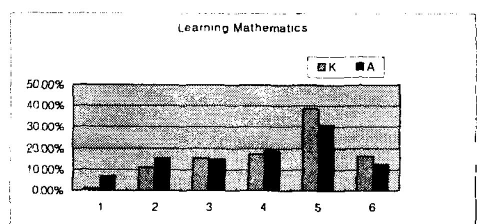
The mean of group K is 3.9523 and the mean of group A is 4.0200. American students' responses are a little more positive than Korean students. But the difference is not statistically significant (p=0.5586). On this dimension, 8.2% of Korean students strongly agreed and 13.0% of American students strongly agreed (see Figure 6).

A lot of Korean students try to memorize mathematical facts, formulas, or algorithms to solve more items of any examinations they have to take during their school lives. They believe that learning method focused on understanding concepts or the relationships among concepts and reasoning mathematical principles or generalizations is not so effective to prepare college entrance examinations.

7. Learning Mathematics

One positive item in this dimension is the statement "Learning mathematics involves more thinking than remembering information." One negative item is the statement "Learning to do mathematics problems is mostly a matter of memorizing the steps to follow."

The mean of group K is 4.3000 and the mean of group A is 3.8900. Korean students' responses are more positive than American students. The difference is statistically significant (p=0.0008). This result seems to be affected by the students' responses showed strongly disagreed on this dimension. 1.14% of Korean students strongly disagreed and 7% of American students strongly disagreed (see Figure 7).

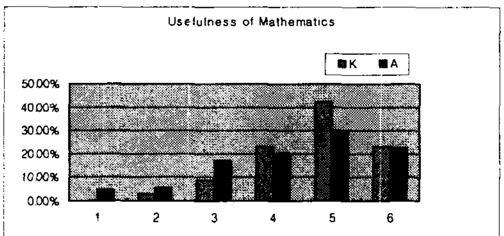


[Figure 7] Learning Mathematics Survey Results

8. Usefulness of Mathematics

One positive item in this dimension is the statement "I will use mathematics in many ways as an adult." One negative item is the statement "I expect to have little use for mathematics when I get out of school." The mean of group K is 4.7364 and the mean of group A is 4.3350. Korean students' responses are more positive than American students. The difference is statistically significant (p=0.0002).

On this dimension, 65.2% of Korean students agreed or strongly agreed, but 52.5% of American students agreed or strongly agreed (see Figure 8). 10%of American students disagreed or strongly disagreed to this dimension. This result seems to be affected by the fact that the calculus Korean sample took is pretty advanced mathematics and very useful branch of mathematics than the pre calculus American sample took.



[Figure 8] Usefulness of Mathematics Survey Results

IV. Results

Students' conceptions of mathematics affect their successful studying mathematics. By the Third International Mathematics and Science Study (TIMSS) and the Second International Mathematics and Science Study, Korean elementary and secondary school students showed higher level of achievement scores than other countries. Really Korean 13 year old students were 2nd in the TIMSS achievement test. But, Korean students' attitudes toward mathematics were lower than middle levels. There were no comparative studies on the college level students' conceptions of mathematics between Korea and other countries before this study.

This study tested Korean and American college level students' conceptions of mathematics with CMI instrument and compared the characteristics of the two samples. The first sample was comprised of 60 Korean college students participated in a calculus course as liberal arts in the university the researcher belonged at the end of the 2nd semester in 2001. The second sample was comprised of 30 American college students, the researcher lectured in Michigan State, participated in a pre-calculus course at the end of spring semester in 2000. Among the 60 Korean students, 40 were belonged in the department of mathematics education and others are from those departments of liberal arts or social sciences.

Korean sample showed more positive responses in five dimensions such as (1) composition of mathematical knowledge, (2) structure of mathematical knowledge, (5) validating ideas in mathematics, (6) learning as constructing, and (7) mathematics as a useful endeavor, and American sample was more positive in two dimensions such as (3) status of mathematical knowledge, and (4) doing mathematics. For the dimensions of (1), (2), (3), (6), and (7), the differences are statistically significant.

By this study, we can conclude that Korean college students understand the current new trends of mathematics education, but they are relatively weak in doing mathematics according to the trends.

The sample size of this study is pretty small to declare the differences of the two countries' college students' conceptions of mathematics. Next study, to find the causes to lead to the differences found in this study, with nationwide samples of the two countries is needed in near future.

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대학생의 수학 개념: 한국 학생과 미국 학생의 비교

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이 논문은 수학적 개념의 뜻과 그 중요성을 살펴본 다음, 연구자가 소속되어 있는 한국의 대학생과 연구자가 연구년 동안 강의한 바 있는 미국의 대학생이 갖고 있는 수학적 개념의 수준에 대하여 조사하여 보고, 그 차이점을 비교하여 수학교육의 개선을 위한 시사점을 찾아보고자 하였다.

본 연구는 수학적 개념을 수학적 지식의 구성, 수학적 지식의 구조, 수학적 지식의 현상, 수학을 행하기, 수학적 아이디어의 가치 인식, 구성으로서의 학습, 유용한 노력으로서의 수학으로 분류하고 각 개념에 대한 양국 학생들의 인식 정도를 설문조사 방식으로 조사하였다.
본 연구에서 한국 학생들은 수학적 개념에 대한 7개의 영역 중에서 '수학적 지식의 현상', '수학을 행하기'를 제외한 5개의 영역에서 더 높은 수준을 보였다. 앞으로 한국의 수학교육은 수학을 실제로 행하는 활동을 더욱 강조하여야 할 것이다.

key words: college students(대학생), conceptions of mathematics(수학의 개념),
Korean students(한국 학생), American students(미국 학생),
comparison study(비교 연구)