Statistical Study of Ductility Factors for Elastic Perfectly Plastic SDOF Systems

탄소성 단자유도 구조물에 대한 연성계수의 통계적 분석

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국문요약

반응수정계수의 핵심구성요소인 연성계수에 대하여 통계적 분석을 수행하였다. 연성계수의 체계적인 산정을 위하여 총 1,860개의 지진기록을 수집하였다. 수집된 지진기록을 지반 전단파의 평균속도에 따라 4가지로 분류하고, 탄소성 이력거동을 가지는 단자유도 구조물에 대하여 비탄성 스펙트럼을 작성하였다. 작성된 비탄성 스펙트럼으로부터 연성계수를 구하고, 변위연성비, 토질조건, 규모 및 진앙거리가 연성계수에 미치는 영향을 분석하였다. 토질 조건별로 평균연성계수를 구하고, 산정된 연성계수의 산포도를 검토하기 위하여 변동계수를 산정하였다.

주요어 : 반응수정계수, 연성계수, 변위연성비, 토질조건

ABSTRACT

This paper present a summary of the results of statistical study of the ductility factor which is key component of response modification factor(R). To compute the ductility factor, a group of 1,860 ground motions recorded from various earthquake was considered. Based on the local site conditions at the recording station, ground motions were classified into four groups according to average shear wave velocity. Inleastic spectrum were computed for elastic perfectly plastic SDOF systems undergoing different level of inelastic deformation and period. Ductility factors were calculated by deviding elastic response spectrum by inelastic response spectrum. The influence of displacement ductility ratio, site condition, magnitude and epicentral distance on ductility factors were studied. The coefficient of variation was computed to evaluated the dispersion of ductility factors as the defined ratio of the standard deviation to the mean.

Key words : response modification factor, ductility factor, displacement ductility ratio, site condition

1. Introduction

Due to economic reasons, present design philosophy allows building structures to undergo inelastic deformations in the event of earthquake ground motions. As a results of this design philosophy, the design base shear is calculated by dividing the base shear for elastic response by the response modification factor(R). The concept of a response modification factor was proposed based on the premise that well-detailed seismic framing systems could sustain large inelastic deformations without collapse and develop lateral strengths in excess of their design strength. The response modification factor was assumed to represent the ratio of the forces that the framing system were to behave entirely elastically to the prescribed design forces at the strength level. However, studies by researchers(e.g., Bertero, 1986 & 1991) and design professionals, including Project ATC-34, have identified major shortcommings in the value and formulation of the response modification factors used in seismic codes. Therefore, it is necessary to re-evaluate the response modification factors in seismic design.

In this study, statistical analysis of ductility factors which is a key component of response modification factor were accomplished. Ductility factors were computed for elastic perfectly plastic SDOF systems undergoing different level of inelastic deformation and period when subjected to a large number of recorded earthquake ground motions. The influence of displacement ductility ratio, site condition, magnitude and epicentral distance on ductility factors were studied.

2. Backgruond of ductility factor

2.1 Components of response modification factor

In the mid-1980s, data from experimental research at the University of California at Berkeley were used to develop base shear-roof displacement relationships for steel braced frames and a draft formulation for the response modification factor.

Using these data, the Berkeley researchers proposed splitting *R* into three factors that account for contributions from ductility, reserve strength and viscous damping, as follows(1) :

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본 논문에 대한 토의를 2003년 6월 30일까지 학회로 보내 주시면 그 결과를 게재하겠습니다. (논문접수일 : 2003. 1. 27 / 심사종료일 : 2003. 2. 21)

$$R = R_\mu R_s R_\xi \tag{1}$$

In this expression R_μ is the ductility factor, R_s is the strength factor, and R_ξ is the damping factor.

Recent studies, including those in the companion Project ATC-34, support a new formulation for R; that is, a formulation in which R is expressed as the product of three factors(1) :

$$R = R_\mu R_s R_R \tag{2}$$

where R_μ is the period-dependent ductility factor, R_s is the period-dependent strength factor, and R_R is the redundancy factor.

2.2 Evaluation procedure of ductility factor

The level of inelastic deformation experienced by the system under a given ground motion is typically given by the displacement ductility ratio, μ, which is defined as the ratio of maximum absolute relative displacement to its yield displacement.

$$\mu = \frac{\max.|u(t)|}{u_y} \tag{3}$$

The time history of the response of a nonlinear single-degree-of-freedom(SDOF) system to earthquake ground motion is given by the solution of the following equation

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + F(t) = -m\,\ddot{u}_g(t) \tag{4}$$

where m, c and $F(t)$ are the mass, damping coefficient, and restoring force of the system, respectively; $u(t)$ is the relative displacement; $\ddot{u}_g(t)$ is the ground acceleration; and the dot over a quantity represents its derivative with respect to time. The initial period of the system is given by,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m\,u_y}{F_y}} \tag{5}$$

where k is the initial stiffness of the system; and F_y is the system's yield strength, respectively.

The ductility factor(i.e, the reduction in strength demand), R_μ, is defined as the ratio of the elastic strength demand to the inelastic strength demand,

$$R_\mu = \frac{F_y(\mu = 1)}{F_y(\mu = \mu_i)} \tag{6}$$

where $F_y(\mu = 1)$ is the lateral strength required to avoid yielding in the system under a given motion and $F_y(\mu = \mu_i)$ is the lateral strength required to maintain the displacement ductility ratio demand, μ, less than or equal to a pre-determined target ductility ratio, μ_i, where subject to the same ground motion.

For a given ground motion and a maximum tolerable displacement ductility demand μ_i, the problem is to compute the minimum lateral strength capacity $F_y(\mu = \mu_i)$ that has to be supplied to the structure in order to avoid ductility ratio demands larger than μ_i. Eq. (6) can be rewritten as

$$R_\mu = \frac{C_y(\mu = 1)}{C_y(\mu = \mu_i)} \tag{7}$$

where $C_y(\mu = 1)$ is seismic coefficient(yielding strength divided by the weight of the structure) required to avoid yielding; and $C_y(\mu = \mu_i)$ is minimum seismic coefficient required to control the displacement ductility demand to μ_i. As shown in Fig. 1, $C_y(\mu = 1)$ and $C_y(\mu = \mu_i)$ correspond to ordinates of a elastic response spectrum and a constant displacement ductility inelastic response spectrum, respectively.

For a given ground motion, computation of $F_y(\mu = \mu_i)$ involves iteration, for each period and for each target ductility, of the lateral yield strength F_y using Eq. (4) until the computed ductility demand μ is the same as the target ductility μ_i, within a certain tolerance. For a given ground acceleration time history, $\ddot{u}_g(t)$, a R_μ spectrum can be constructed by plotting the ductility factors computed with Eq. (7) of a family of SDOF systems with different periods of vibration undergoing different levels of inelastic deformation, μ_i, when subjected to $\ddot{u}_g(t)$.

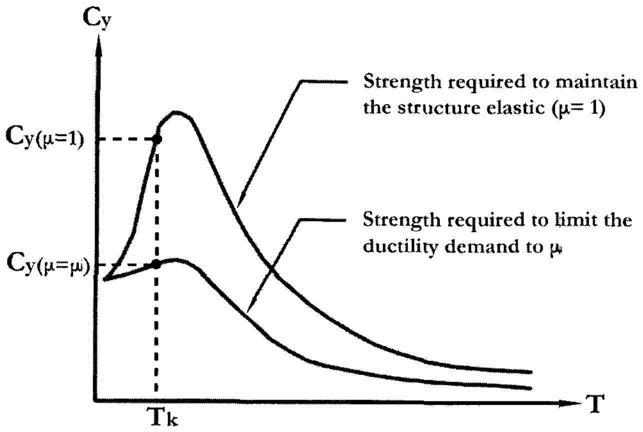


Fig. 1 Linear and constant ductility nonlinear response spectra(12)

3. Earthquake ground motions

There is a general consensus that one of the largest sources of uncertainty in the estimation of the response of inelastic structures during earthquakes is the prediction of the intensity and characteristics of future earthquake ground motions at a given site. In this study, an effort was made to consider a relatively large number of recorded ground motion to study the effects of the variability of the characteristics of ground motions on ductility factors.

To study the influence of local site condition, magnitude and epicentral distance on ductility factors, a group of 1,860 ground motions recorded on a wide range of soil conditions during 47 different earthquake was considered. A particulary large number of earthquake ground motions was selected in order to assess the dispersion of the ductility factors. Some of the ground motions used in this study were listed in Table 1. The others were omitted becasue of limited space. All of the selected records represent free-field conditions, basement and ground level. Based on the local site conditions at the recording station, ground motions were classified into four groups according to average shear wave velocity, $\overline{v_s}$, as follows.

(a) Site AB(Rock Site) : $\overline{v_s} \geq 750\ m/s$
(b) Site C(Dense Soil) : $360\ m/s \leq \overline{v_s} < 750\ m/s$
(c) Site D(Stiff Soil) : $180\ m/s \leq \overline{v_s} < 360\ m/s$
(d) Site E(Soft Soil) : $\overline{v_s} < 180\ m/s$

This site classification is a similar to that used in present building codes, especially in UBC(1997) and NEHRP(1997).

In Fig. 2, the distribution of earthquake ground motions comprising the data-set with regard to magnitude, epicentral distance and site classification are shown. The figures demonstrate that the data except site E(soft soil), is well-distributed with respect to all three parameters, hence results of analysis will not have significant bias.

4. Method of analysis

Nonlinear time history analysis were carried out on SDOF systems. BISPEC[4] was used to perform the dynamic analysis. For a given period of vibration and a given target displacement ductility ratio, constant displacement ductility inelastic response spectrum was computed by iteration on the system's non-dimensional yielding strength $C_y(\mu = \mu_i)$ until the displacement ductility demand computed with Eq. (3) is, within a certain tolerance, the same as the target ductility. The tolerance is chosen such that $C_y(\mu = \mu_i)$ is considered satisfactory if the computed ductility is within 1% of target ductility.

The following values of target ductilities are selected for this study : 1(elastic), 2, 3, 4, 5 and 6. For each earthquake record and each target ductility, the inelastic response spectrum are computed for a set of 60 discrete periods ranging from 0.05 to 3.0 seconds. Considering the large number of records, ductilities, and periods of vibration and the large computational effort involved in calculating constant displacement ductility inelastic spectrum through iteration, this study is limited to SDOF systems that have a elastic perfectly plastic behavior and constant damping coefficient corresponding to a damping ratio ξ of 5% based on elastic properties. Fig. 3 show the constant displacement ductility inelastic response spectra and ductility factors for some of the earthquake ground motions.

5. Statistical study

5.1 Influence of earthquake parameters on ductility factor

The influence of displacement ductility ratio(μ) and period(T) on ductility factor were studied. As mentioned before, the effects of earthquake ground motion parameters on ductility factor were studied. Earthquake ground motion parameters were considered the site condition, magnitude (M) and epicentral distance.

5.1.1 Displacement ductility ratio (μ)

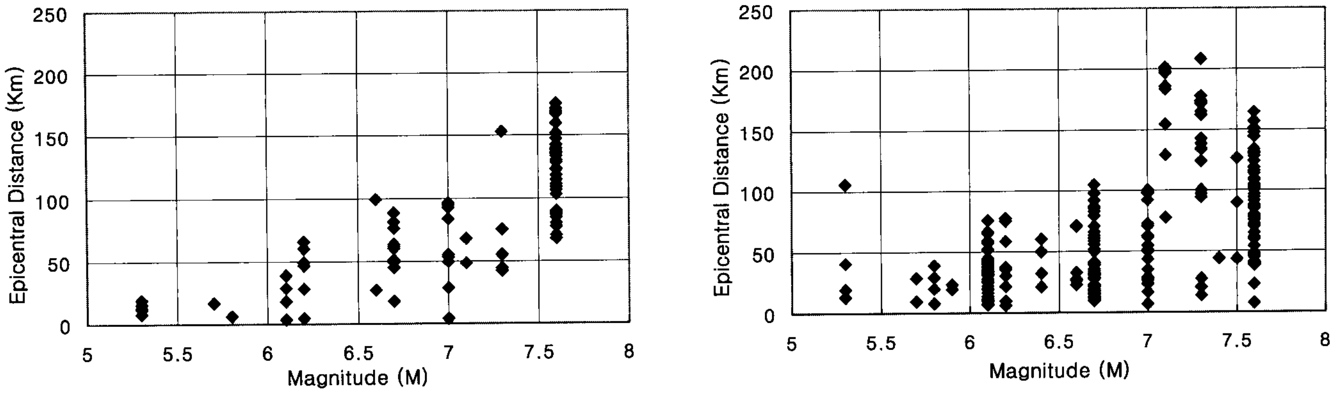
The influence of displacement ductility ratio(μ) on ductility factors is studied by computing the mean ductility factor spectra for ground motions recorded on earthquake with various levels of magnitude and epicentral distance for each site condition. Fig. 4 show the some of the mean ductilily factor spectra with various levels of magnitude and epicentral distance for each site conditions. The figure shows the displacement ductility ratio(μ) have a great influence on ductility factors regardless of the site condition and the level of target ductility. As shown in this figure, the ductility factors are characterized by the following features ; first, the ductility factor increase with increasing target ductility, with the rate of increase being period dependent ; and second, for a given target ductility, the ductility factor exhibit an important variation with changes in period, particularly in the short-period range.

5.1.2 Site condition

The effect of site condition on ductility factors is studied by computing the mean ductility factor spectra for earthquake ground motions with various levels of magnitude, epicentral distance and displacement ductility ratio. Some of the mean ductility factor spectra with various levels of magnitude, epicentral distance and displacement ductility ratio are shown in Fig. 5. This figure displays that the

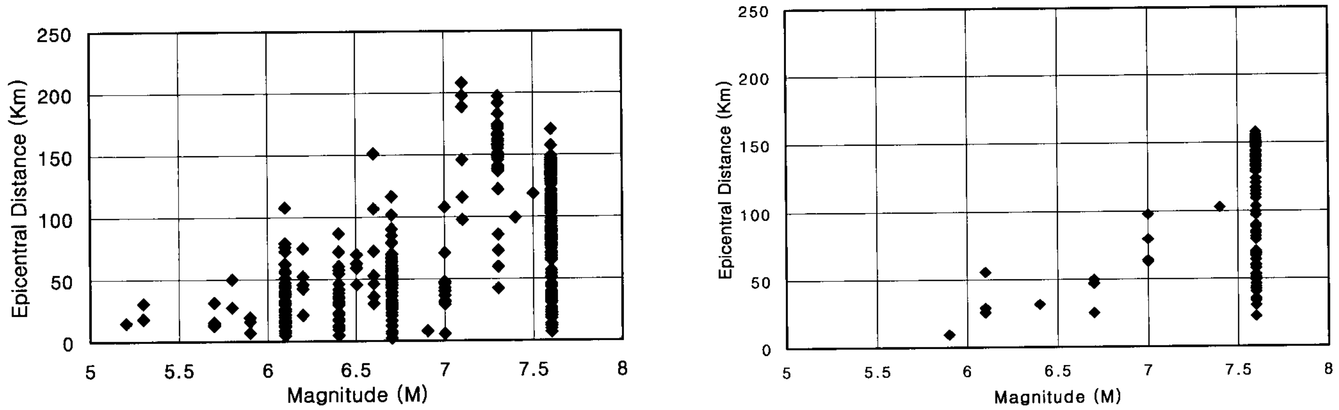
Table 1 Some of the earthquake ground motions

No	Earthquake	M	Station	Site geology	Dist.(km)	Component	PGA(a/g)
1	Anza, CA	5.3	Anza Array-fire station	Alluvium	15.9	315	0.0715
2	Big Bear, CA	6.5	San Fernando E & Hospitality	Deep Alluvium	45.4	180	0.1009
3	Borrego valley, CA	6.6	El Centro Array Station 9	Deep Alluvium	46.3	0	0.0597
4	Cape Mendocino, CA	7.0	Cape Mendocino Petrolla	Cretaceous Rock	3.8	90	1.0396
5	Chi Chi Taiwan	7.6	CWB station TCU129	S_D (1997 UBC)	11.9	0	0.6336
6	Chi Chi Taiwan	7.6	CWB station TCU120	S_C (1997 UBC)	23.2	90	0.2249
7	Chi Chi Taiwan	7.6	CWB station CHY002	S_E (1997 UBC)	42.4	270	0.1169
8	Chi Chi Taiwan	7.6	CWB station HWA056	S_B (1997 UBC)	80.3	0	0.1070
9	Chi Chi Taiwan	7.6	CWB station TCU096	S_D (1997 UBC)	104.8	0	0.1072
10	Coalinga, CA	6.5	Parkfield-cholame 5w	Alluvium, Sandstone	59.3	270	0.1470
11	Coyote, CA	5.7	Gilroy Array Station 1	Rock	16.7	230	0.1029
12	Duzec, Turkey	7.1	Lamont 1061	-	-	270	0.1339
13	Erzincan, Turkey	6.9	95 Erzincan	-	-	90	0.4957
14	Hector Mine, CA	7.1	13123 Riverside Airport	Alluvium	59.6	270	0.0500
15	Hollister	5.2	1028 Hollister City Hall	-	-	181	0.0889
16	Imperial Valley, CA	6.4	5053 Calexico Fire Station	Alluvium	17.4	225	0.2748
17	Imperial Valley, CA	6.4	6604 Cerro Prieto	Rock	21.5	147	0.1689
18	Kern County, CA	7.5	80053 Pasadena	Alluvium, Granite	126.2	270	0.0529
19	Kobe, Japan	6.9	Kakogawa	-	-	0	0.2509
20	Kocaeli, Turkey	7.4	Cekmece	Stiff Soil	99.7	0	0.1789
21	Landers, CA	7.3	24 Lucerne Valley	Decomposed Granite	42.0	275	0.7206
22	Landers, CA	7.3	32075 Baker Fire Station	Alluvium	123.9	50	0.1079
23	Livemore, CA	5.8	57134 San Ramon Fire Station	-	-	70	0.0579
24	Loma Prieta, CA	7.0	47379 Gilroy Array #1	Rock	28.4	0	0.4108
25	Loma Prieta, CA	7.0	58373 Apeel 10 - Skyline	Sandstone	62.6	0	0.1029
26	Loma Prieta, CA	7.0	58163 Yerba Buena Island	Franciscan	95.4	90	0.0679
27	Lytle Creek, CA	5.3	Cedar Springs, Allen Ranch	Granitic	19.3	95	0.0710
28	Morgan Hill, CA	6.1	57383 Gilroy Array #6	Rock	35.9	0	0.2219
29	Mt. Lewis, CA	6.2	57191 Halls Valley	Alluvium	-	90	0.1589
30	North Palm Springs, CA	6.2	12206 Silent Valley	Weathered Granite	27.9	0	0.1389
31	Northridge, CA	6.7	90006 Sun Valley	Alluvium	11.1	0	0.3028
32	Northridge, CA	6.7	90017 LA-Wonderland Ave	Granite Rocks	18.2	185	0.1719
33	Northridge, CA	6.7	14403 LA-116th St School	Terrace Deposits	40.5	90	0.2078
34	Northridge, CA	6.7	25340 Ventura	Alluvium	69.7	360	0.0752
35	Northridge, CA	6.7	23598 Rancho Cucamonga	Granitic Rock	88.7	90	0.0710
36	Parkfield, CA	6.1	1014 Cholame #5	Alluvium, Sandstone	36.8	85	0.4418
37	Point Mugu, CA	5.3	272 Port Hueneme	Alluvium 300M	17.9	180	0.1120
38	San Fernando, CA	6.6	127 Lake Hughes #9	Gneiss	26.7	21	0.1569
39	San Fernando, CA	6.6	262 Palmdale Fire Station	Alluvium	32.6	210	0.1509
40	San Fernando, CA	6.6	111 Cedar Springs	Granitic	99.4	95	0.0200
41	San Francisco, CA	5.3	1117 Golden Gate Park	Chert	11.5	100	0.1120
42	Santa BarBara, CA	5.8	283 Santa Barbara Courthouse	Boulder Alluvium	8.1	132	0.1019
43	Superstition Hills, CA	6.6	5060 Brawley	Alluvium	22.1	225	0.1559
44	Trinidad, CA	7.2	1498 Rio Dell Overpass	-	-	0	0.1629
45	Victoria, Mexico	6.4	6604 Cerro Prieto	Rock	32.0	315	0.5867
46	Westmorland, CA	5.9	5060 Brawley Airport	Alluvium	16.0	225	0.1689
47	Whitter Narrows, CA	6.1	90093 Arcadia - Campus Dr	Alluvium	8.6	9	0.2998
48	Whitter Narrows, CA	6.1	90034 LA - Fletcher Dr	Nonmarine Deposit	16.4	144	0.1709
49	Whitter Narrows, CA	6.1	14196 Inglewood - Union Oil	Terrace Deposit	25.4	0	0.2988
50	Whitter Narrows, CA	6.1	24278 Castaic	Sandstone	75.9	0	0.0710



(a) Site AB(Rock site)

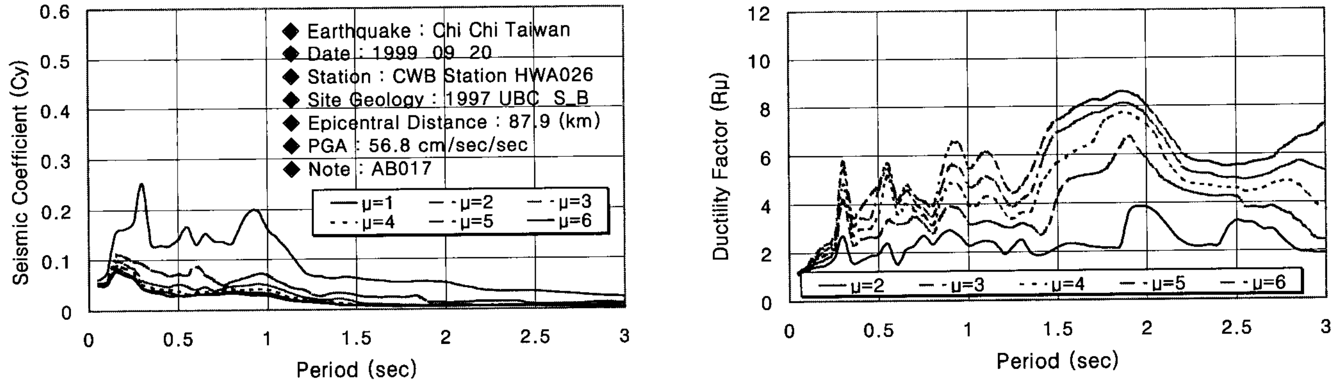
(b) Site C(Dense soil)



(c) Site D(Stiff soil)

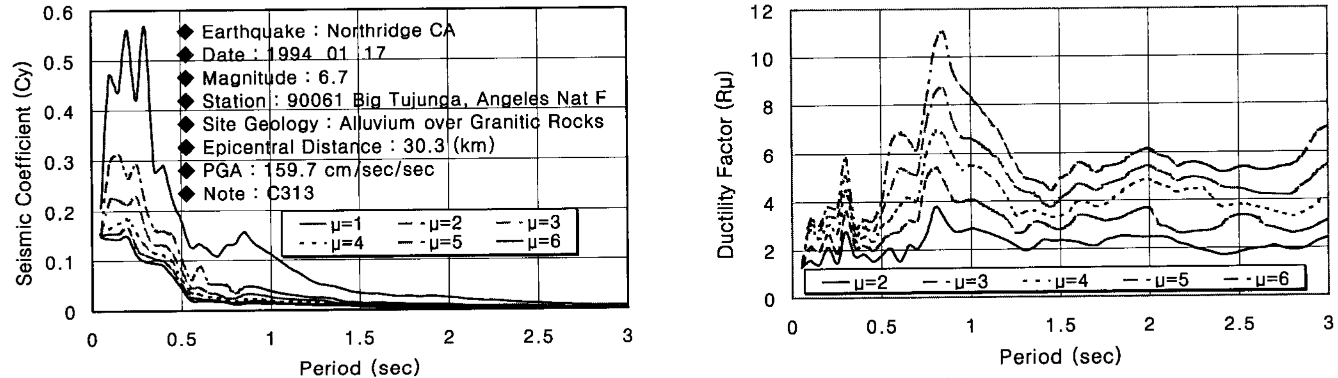
(d) Site E(Soft soil)

Fig. 2 Distribution of earthquake ground motions



(a) Inelastic response spectra

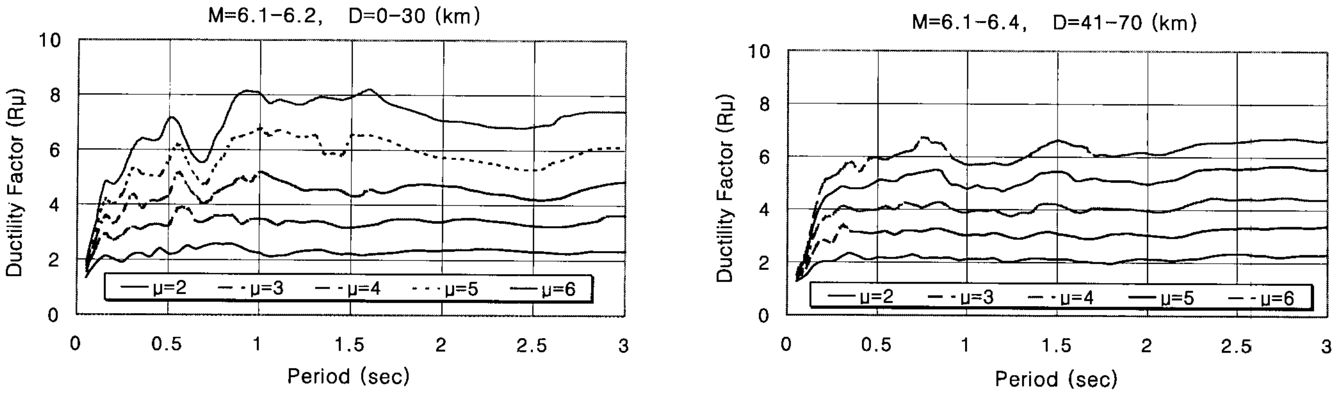
(b) Ductility factor



(c) Inelastic response spectra

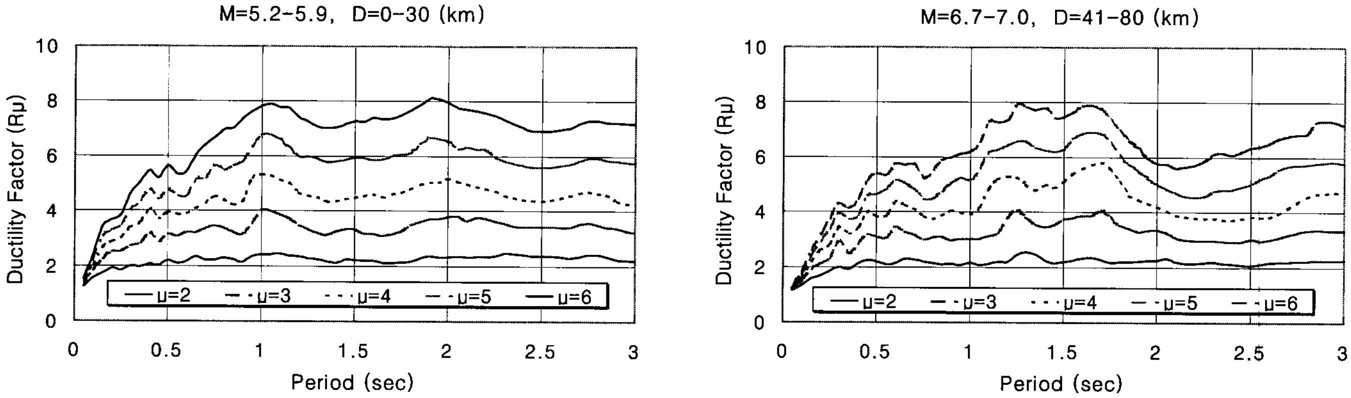
(d) Ductility factor

Fig. 3 Example of inelastic response spectra and ductility factor



(a) Site AB(Rock site)

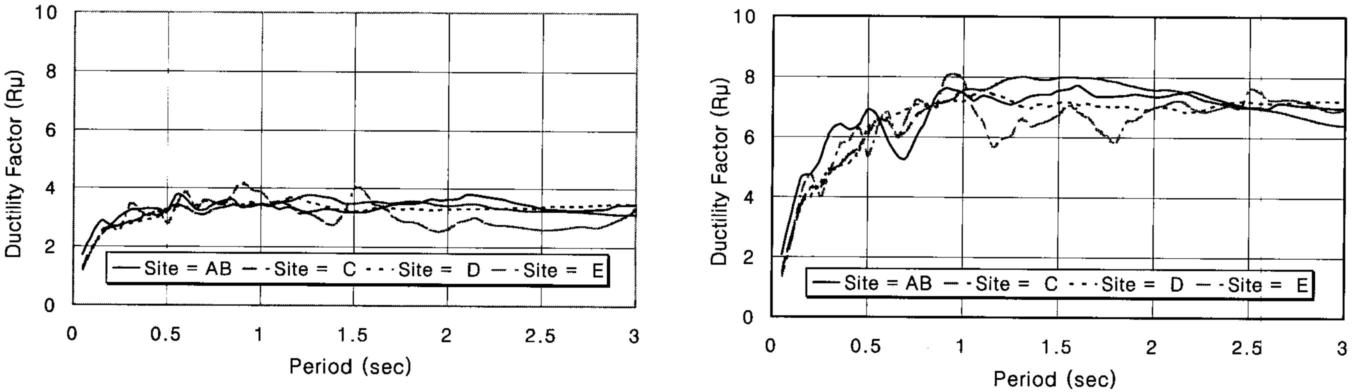
(b) Site C(Dense soil)



(c) Site D(Stiff soil)

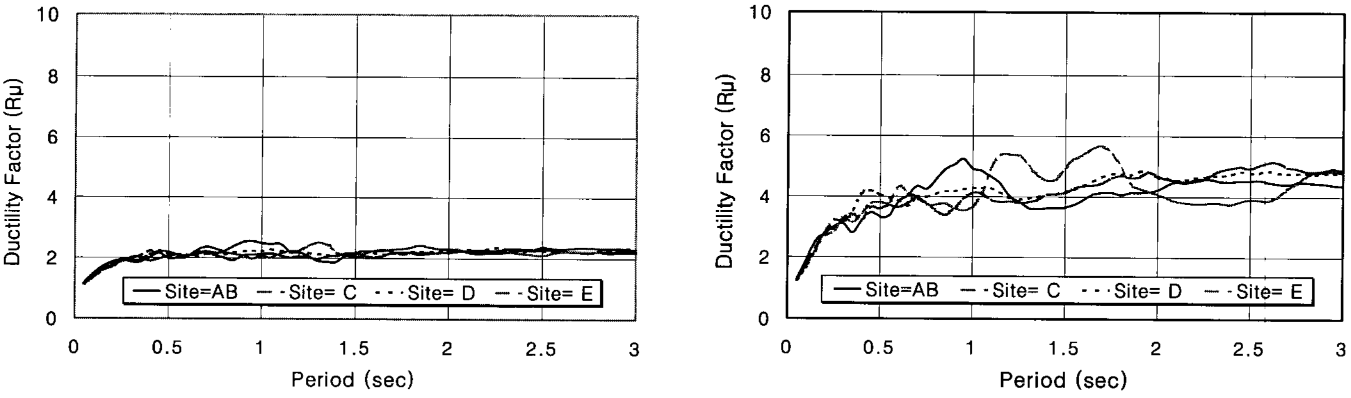
(d) Site E(Soft soil)

Fig. 4 Influence of displacement ductility ratio on ductility factors



(a) M=6.1~6.2, D=0~40(km), μ=3

(b) M=6.1~6.2, D=0~40(km), μ=6



(c) M=6.7~7.0, D=41~70(km), μ=2

(d) M=6.7~7.0, D=41~70(km), μ=4

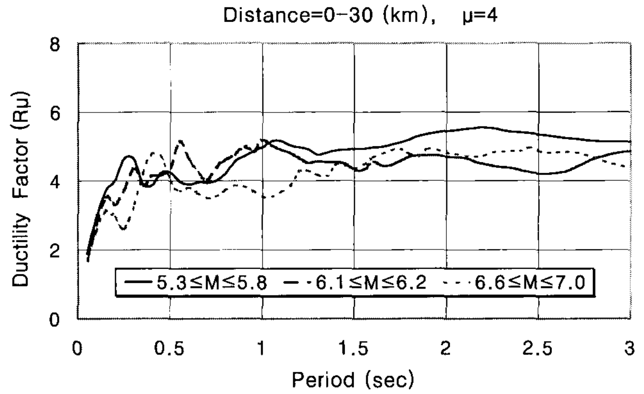
Fig. 5 Influence of site condition on ductility factors influcnce of site condition on ductility factor, with the exception of site E(soft soil), is negligible. The mean ductility factor spectra for site E(soft soil) have a great variation with changes in period compared with other site conditions.

5.1.3 Earthquake magnitude(M)

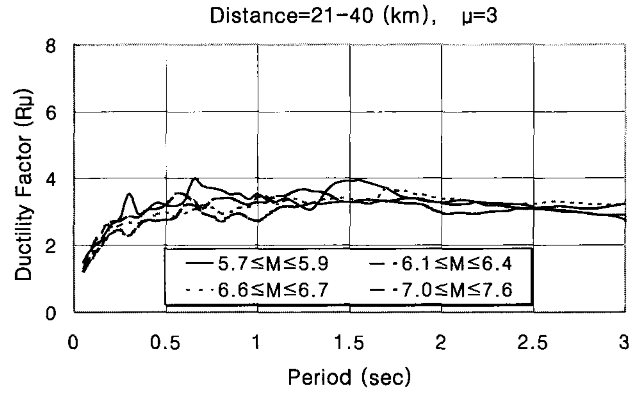
The effect of earthquake magnitude on ductility factors is studied by calculating the mean ductility factor spectra for ground motions recorded on earthquake with various levels of epicentral distance and displacement ductility ratio for each site condition. Here, the analysis for site E(soft soil) was excluded because of uneven distribution of earthquake ground motions. The influence of earthquake magnitude on ductility factors are shown in Fig. 6. It can be seen that regardless of the site condition, level of epicentral distance and ductility, the influence of earthquake magnitude on ductility factors is negligible.

5.1.4 Epicentral distance

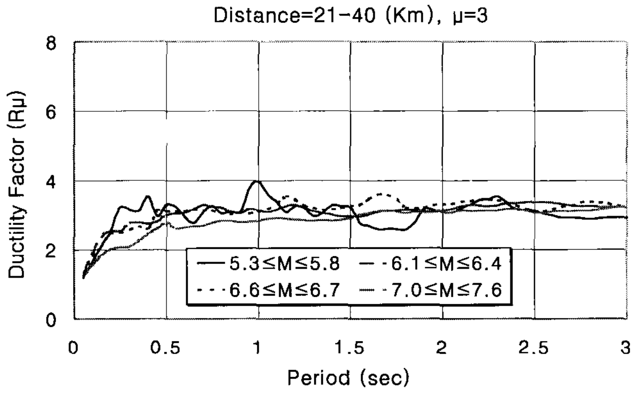
Fig. 7 presents the effect of epicentral distance on ductility factors. The influence of epicentral distance on ductility factors is studied by computing mean ductility factor spectra for earthquake ground motions with different level of earthquake magnitude and displacement ductility ratio for each site condition. Similarly, the analysis for site E(soft soil) was excluded. It can be seen that mean ductility factors are



(a) Site AB(Rock site)

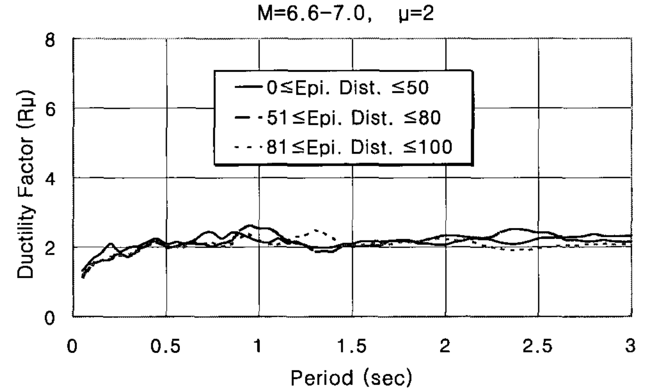


(b) Site C(Dense soil)

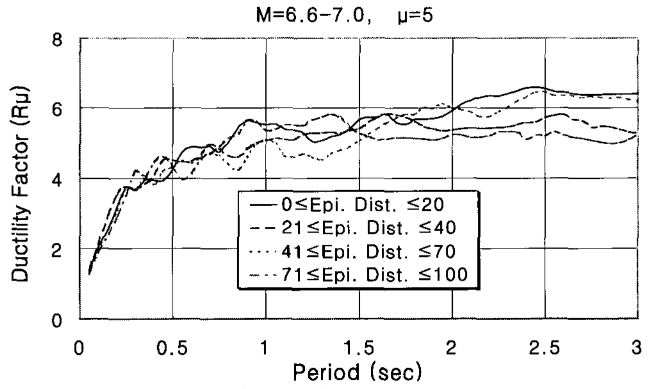


(c) Site D(Stiff soil)

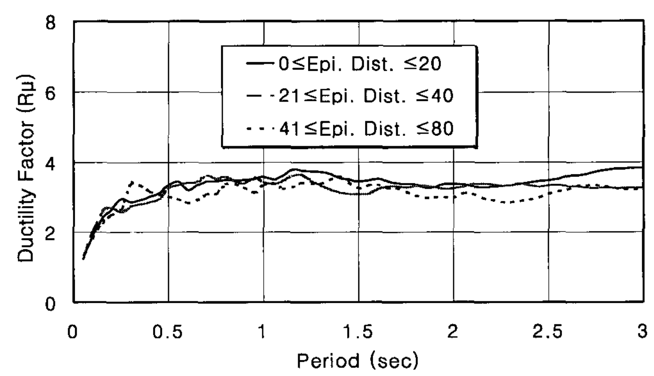
Fig. 6 Influence of earthquake magnitude on ductility factors



(a) Site AB(Rock site)



(b) Site C(Dense soil)



(c) Site D(Stiff site)

Fig. 7 Influence of epicentral distance on ductility factors practically the same for all groups of epicentral distances. Thus, epicentral distances have a negligible effect on ductility factors.

5.2 Mean ductility factor

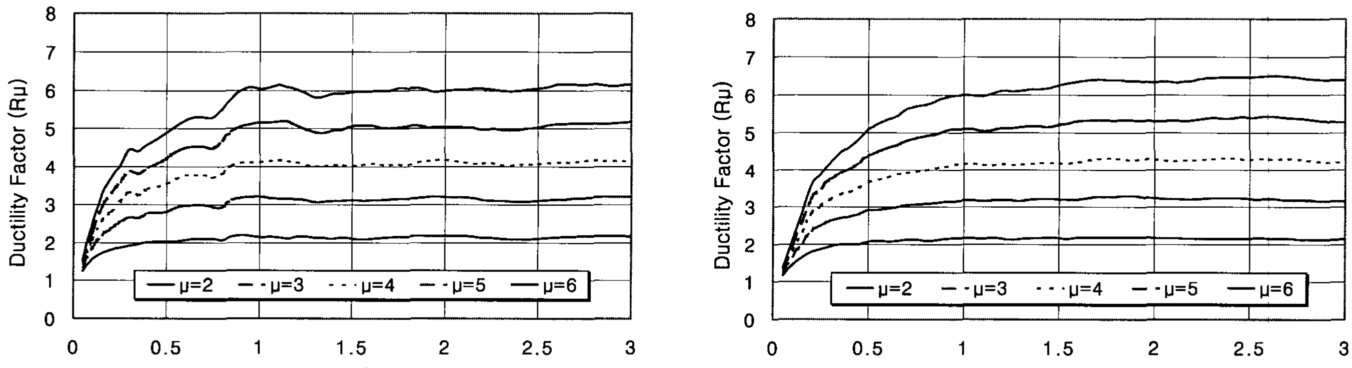
The mean of ductility factor spectra of all ground motions for elasto-plastic systems with target displacement ductility ratios $\mu=2$, 3, 4, 5 and 6, for each site conditions, are shown in Fig. 8. As shown in this figure, the ductility factors(R_μ)-target displacement ductility ratios(μ)-period (T) relationship are characterized by the following features ;

- The ductility factors approach $R_\mu=1$ as the period tend to zero.
- The ductility factors increase with increasing target ductility where the rate of increase depends on periods.
- For a given target displacement ductility, the ductility factor exhibit an important variation with changes in period, particularly in the short-period range.
- For a long-period range, mean ductility factors are approximately constant and approach the target displacement ductility ratio.

5.3 Coefficient of variation

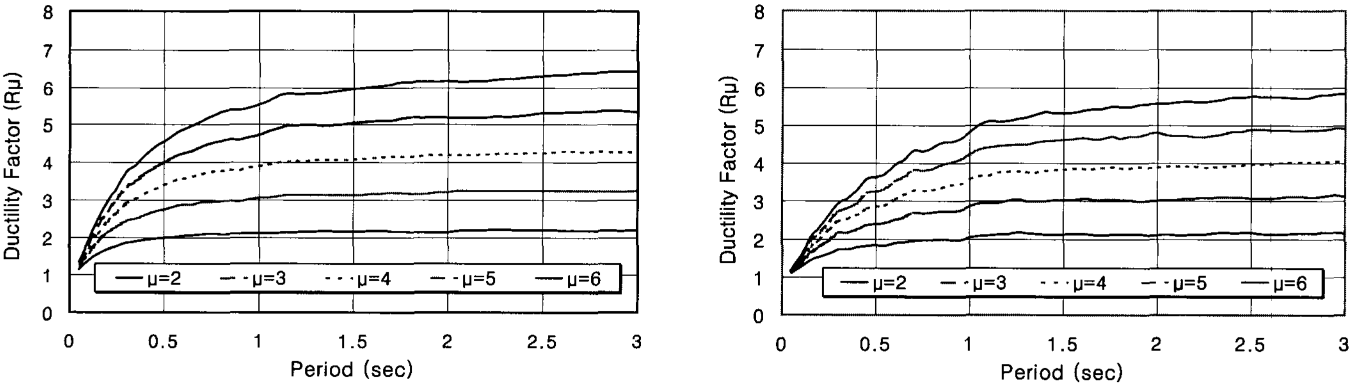
As mentioned before, ductility factors increase with increasing ductility demands. For a given system with period of vibration T and a given target displacement ductility ratio μ, the ductility factor will typically vary within a certain range when subjected to a family of ground motions. Thus, it is important to study not only the influence of the displacement ductility ratio on mean ductility factors but also on the dispersion of these ductility factors. One way of evaluating the dispersion of ductility factors is by computing the coefficient of variation (COV), which is defined as the ratio of the standard deviation to the mean.

Coefficient of variation of ductility factors for systems subjected to ground motions recorded on each site conditions are shown in Fig. 9. As illustrated by this figure, with the exception of systems with very short periods ($T < 0.2$ s), coefficient of variation of ductility factors exhibit only small variations with changes in the period of vibration. Regardless of the site condition at the recording station, the dispersion in ductility factors increases with increasing displacement ductility ratio. As presented in Table 2, a similar results are founded in technical literature.(10),(11),(13)



(a) Site AB(Rock site)

(b) Site C(Dense soil)



(c) Site D(Stiff soil)

(d) Site E(Soft soil)

Fig. 8 Mean ductility factors

Table 2 Mean value of COV

	Miranda [11,13]	KNHC [10,13]	This study			
			AB	C	D	E
μ=2	0.25	0.23	0.22	0.23	0.22	0.22
μ=4	0.35	0.33	0.32	0.34	0.34	0.34
μ=6	0.40	0.39	0.39	0.40	0.39	0.40

6. Conclusions

The primary purpose of this study was to estimate the reduction in lateral strength demands produced by allowing hysteretic behavior to take place in structures in the event of earthquake ground motions. For this purpose, a statistical study of ductility factors was accomplished. The statistical study comprised ductility factors computed for SDOF systems undergoing different levels of displacement demand when subjected to a relatively large number of earthquake ground motions recorded on different site conditions. The following conclusions can be drawn from the results of this study.

The ductility factor is primarily affected by the period of the system and the displacement ductility ratio. However, the effect of earthquake magnitude and epicentral distance on ductility factor is negligible. The site condition, except site E(soft soil), showed a small effect on ductility factors.

For a given displacement ductility ratio, indenpendent of site conditions, ductility factors exhibit important variations with changes in period, particularly in short-period range. For a long-period range, mean ductility factors are approximately constant and approach the target displacement ductility ratio.

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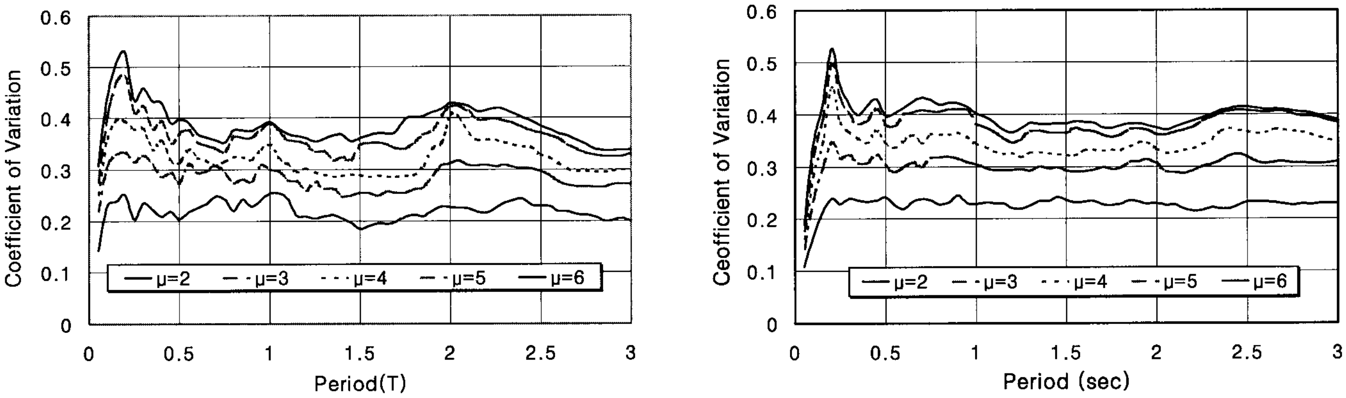
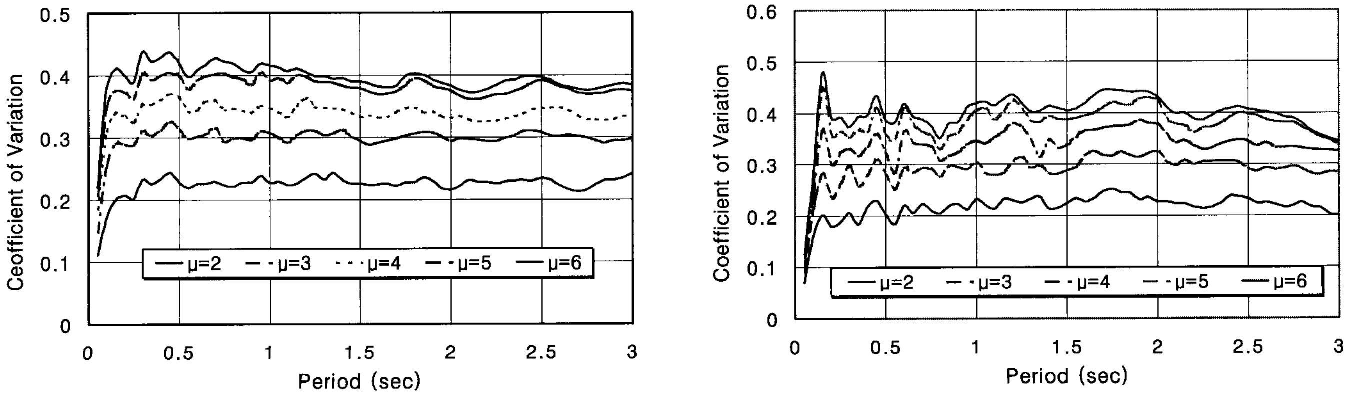


(a) Site AB(Rock site)

(b) Site C(Dense soil)



(c) Site D(Stiff soil)

(d) Site E(Soft soil)

Fig. 9 Coefficient of variation

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