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Series Design of Compressors for Two-Stage Centrifugal Chiller

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A preliminary series design of compressors for a two-stage centrifugal chiller is suggested. Six groups of hydrodynamically similar compressors, ranging from 233RT to 1,200RT, are introduced. Flow rates, impeller diameters, and wheel speeds for each group are determined from hydrodynamic similarity to share impellers of adjacent groups. It is expected that these compressors can have the same performance and efficiency from the smallest model to the largest one.

Key Words : Two-Stage Centrifugal Chiller, Series Design, Hydrodynamic Similarity

Nomenclature

a : Speed of sound, m/s
COP : Coefficient of Performance in Eq. (9), dimensionless
D : Diameter of an impeller, m
D_{1st} : Diameter of the first-stage impeller, m
D_{2nd} : Diameter of the second-stage impeller, m
D_s : Specific diameter in Eq. (13), dimensionless
g : Gravitational acceleration, m/s^2
h : Enthalpy, J/kg
H : Polytropic head in Eq. (16), m
m : Mass flow rate, kg/s
Ma : Wheel Mach number in Eq. (14), dimensionless
N : Rotational speed, rps
N_s : Specific speed in Eq. (12), dimensionless
p : Pressure, Pa
Q : Volume flow rate, m^3/s
Q_{1st} : Inlet volume flow rate of 1st-stage compressor, m^3/s
Q_{2nd} : Inlet volume flow rate of 2nd-stage compressor, m^3/s
Re : Reynolds number in Eq. (15), dimensionless
s : Entropy, J/kg ℃
T : Temperature, ℃
U : Wheel speed, m/s
x : Quality, dimensionless
γ : The ratio of maximum to minimum chiller capacity within a group
γ_D : The ratio of maximum to minimum specific diameter within a group
γ_N : The ratio of maximum to minimum specific speed within a group
ϕ : Flow coefficient in Eq. (10), dimensionless
ν : Dynamic viscosity, m^2/s
ψ : Head coefficient in Eq. (11), dimensionless

Subscripts

1st : First-stage
2nd : Second-stage
1～6 : State as defined in Fig. 2
c : Condenser
e : Evaporator
eco : Economizer
s : Specific parameter
sat : Saturated state
sub : Subcooled state

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1. Introduction

During the last two decades, there have been great improvements in the centrifugal chiller industry : enhancement of COP and development of new chillers for environmental protection. In particular, because of environmental issues, serious efforts have been made during the last decade on the development of compressor using R134a with zero ODP (Ozone Depletion Potential). R134a has higher pressure at the same saturation temperature compared to R11 and R123 which had been commonly used in centrifugal chillers, and thus it has higher density. However, the speed of sound is almost the same as that of previous refrigerants. Therefore, compressors using R134a require smaller volumetric size and almost the same circumferential velocity of impeller, and thus designers usually employ high-speed gears to allow for this situation. Up to now, it has been a practice to use single-stage centrifugal compressors because they are simple to design and inexpensive. Recently, as the competition on chiller COP becomes severer, new two-stage centrifugal chillers using R134a have been announced (Hitachi, 2001 ; Mitsubishi, 2001). It is reported that they have 10% higher COP than single-stage chillers do. Although the current two-stage centrifugal chillers are too expensive to play major roles in the marketplace, it is expected that it will occupy major share of the market in the foreseeing future because of serious efficiency competition. Thus, we have investigated the economical and technological aspects associated with the development of two-stage compressors.

The usual range of chiller capacity in the commercial chiller market for standard temperature condition (i.e. Korean Standard KSB6270 or ARI Standard 550/590) is quite broad and amounts to 150~1500RT. The chiller capacity is proportional to the compressor capacity, and thus compressor capacity can be used as the index of the chiller capacity. In this paper we will discuss the design parameters of compressor with molded impellers that are usually used in the field of centrifugal chiller industry. To cover this broad range of chiller capacity for the required polytropic head, it is required to divide the compressor range into several groups (compressor frames). In a given group, designers usually fix (1) the design of high-speed gear, (2) rpm, (3) compressor casing (duct diameter of compressor inlet and outlet, inlet guide vane, volute), (4) lubrication system including bearing, and (5) the design of impeller mold. The design change to cover a capacity range in a group is to trim blades of the impeller and diffuser vanes only (i.e. so called "Flow Cut"). In the series design, it is necessary to consider the hydrodynamic similarity of impeller and diffuser to save the R&D resources and production cost.

The number of compressor groups should be determined from a compromise between economy and efficiency. If the capacity range is too high, the compressor performance would not meet the required design goals in compressor efficiency and performance characteristics. If the capacity range is too small, it will require too much R&D resources and the expensive impeller molds.

The purpose of this study is to propose a number of compressor series for two stage chillers from around 200RT to 1200RT considering the market inquiries. The representative compressor in each group should have the hydrodynamic similarity, i.e. the same values of tip Mach number, flow coefficient, polytropic head coefficient, specific speed, and specific diameter except Reynolds number that is high enough to neglect its effect.

The similar design would guarantee us to have basically the same performance and efficiency in all groups. Thus, if the hydrodynamic performance and efficiency are tested thoroughly, we can confirm the adequacy of hydrodynamic design of the compressor in whole series.

2. Analysis of Two-Stage Centrifugal Chiller

2.1 Refrigeration cycle analysis

Configuration and cycle diagrams of a two-stage centrifugal chiller are shown in Figs. 1 and 2. Unlike the single-stage centrifugal chiller, this

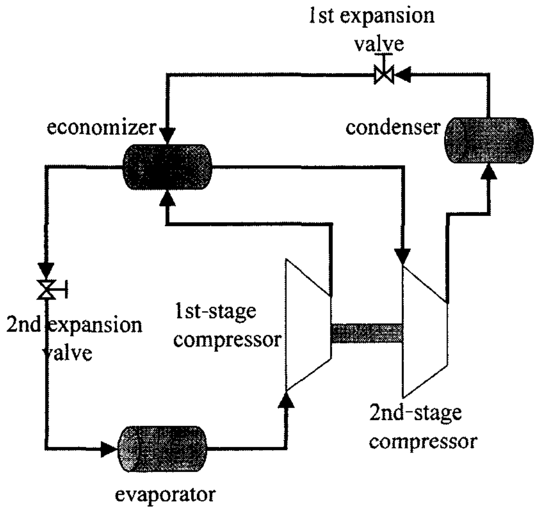

Fig. 1 Component diagram of a two-stage centrifugal chiller

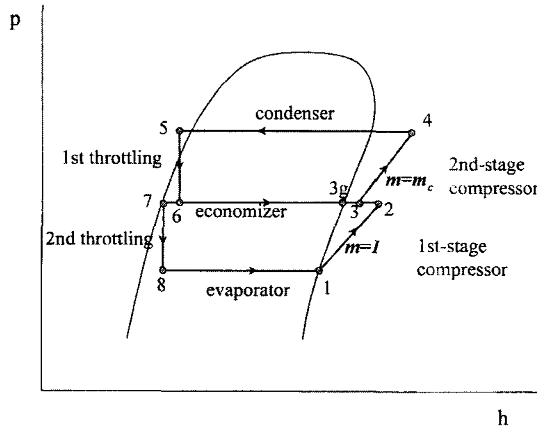

Fig. 2 Cycle diagram of a two-stage centrifugal chiller

cycle has an economizer. The purposes of using the economizer are three-fold ; (1) it reduces inlet temperature of second-stage compressor, so that polytropic head required in the second compressor decreases (2) the density of compressor inlet gas increases, so that the size of second stage compressor can decrease (3) it reduces the quality of refrigerant flowing into the evaporator by reverting flash gas into the second compressor so that the chiller efficiency increases.

The cycle analysis of a two-stage chiller is a little more complex than that of a single-stage chiller. Here, several assumptions are made to simplify the calculation. We neglect the pressure drop and heat exchange through the duct. We did not consider the characteristics of evaporator, condenser, expansion valve and compressor with respect to the variation of refrigerant mass flow rate. Thus, the system matching between these components is not considered because this study is not meant to examine off-design performances. Thus, the input variables to define the state of the system are as follows.

(1) evaporation pressure, P_e
(2) condensing pressure, P_c
(3) economizer pressure, P_{eco}
(4) isentropic efficiency of the first-stage compressor, η_{1st}
(5) isentropic efficiency of the second-stage compressor, η_{2nd}
(6) the amount of subcooling in the condenser, T_{sub}

The thermodynamic states in Fig. 2 are computed straightforwardly as follows. Here we have assumed that mass flow rates along the first-stage compressor and the second-stage compressor are "1" and "m_c" respectively.

a. Obtain entropy and enthalpy at state 1 as a function of P_e.

$$\begin{aligned} s_1 &= s_{sat}(P_e) \\ h_1 &= h_{sat}(P_e) \end{aligned} \tag{1}$$

b. Determine properties at state 2.

$$\begin{aligned} h_2 - h_1 &= \frac{h(P_{eco}, s_1) - h_1}{\eta_{1st}} \\ T_2 &= T(P_{eco}, h_2) \end{aligned} \tag{2}$$

c. Determine properties at state 5.

$$\begin{aligned} T_5 &= T_{sat}(P_c) - T_{sub} \\ h_5 &= h(P_c, T_5) \end{aligned} \tag{3}$$

d. Determine the state 6.

$$\begin{aligned} h_6 &= h_5 \\ T_6 &= T_{sat}(P_{eco}) \\ x_6 &= \frac{h_6 - h_7}{h_{3g} - h_7} \end{aligned} \tag{4}$$

e. Determine the second-stage mass flow rate.

$$m_c = 1 + \frac{x_6}{(1 - x_6)} \tag{5}$$

f. Determine the state 3.

$$h_s = \frac{h_2 + (m_c - 1)\, h_{3g}}{m_c} \qquad T_3 = T(P_{eco}, h_3) \tag{6}$$

g. Determine the state 4.

$$h_4 - h_3 = \frac{h(P_c, s_3) - h_3}{\eta_{2nd}} \qquad T_4 = T(P_c, h_4) \tag{7}$$

h. Determine the state 8.

$$h_8 = h_7 \tag{8}$$

The above analysis does not have any iteration process but allows us to compute the cycle straightforwardly. From the cycle analysis, it is easy to define COP as follows.

$$\text{COP} = \frac{h_1 - h_8}{h_2 - h_1 + m_c(h_4 - h_3)} \tag{9}$$

R134a is used as the refrigerant and thermodynamic properties are obtained from REFPROP Ver. 6.01 (NIST, 2000).

2.2 Nondimensional analysis of a compressor

There might be many ways to devide the chiller capacity range of each group. The best choice of the capacity range of the group may come from the importance of the group in the market. If the market does not prefer the particular chiller capacity, it would be better to have the chiller capacity range divided by an equal ratio. Here, we want to devide chiller series ranging from around 200 to 1,200RT into six separate groups according to its capacity. In this case, the ratio of maximum to minimum capacity within a group appears to be 1.35 $(=(1200/200)^{1/6})$. The exact initial capacity of the chiller series will be determined from the nondimensional analysis at the end of the paper.

We introduce nondimensional parameters of compressors as follows.

Flow coefficient :

$$\phi = \frac{Q}{ND^3} = \frac{2\pi}{N_s D_s^3} \tag{10}$$

Polytropic head coefficient :

$$\psi = \frac{gH}{U^2} = \frac{4}{N_s^2 D_s^2} \tag{11}$$

Specific speed :

$$N_s = \frac{2\pi N \sqrt{Q}}{(gH)^{3/4}} \tag{12}$$

Specific diameter :

$$D_s = \frac{D(gH)^{1/4}}{\sqrt{Q}} \tag{13}$$

Mach number :

$$Ma = \frac{U}{a_1} = \frac{\pi ND}{a_1(P_e, T_{sat}(P_e))} \tag{14}$$

Reynolds number :

$$\text{Re} = \frac{UD}{\nu} \tag{15}$$

Here, we have used the definition of Mallen-Saville (Aungier, 2000) as in Eq. (16) to evaluate the compressor's polytropic head.

$$gH_{1st} = h_2 - h_1 - \frac{(s_2 - s_1)(T_2 - T_1)}{\ln(T_2/T_1)} \qquad gH_{2nd} = h_4 - h_3 - \frac{(s_4 - s_3)(T_4 - T_3)}{\ln(T_4/T_3)} \tag{16}$$

The usual polytropic head is defined for a polytropic process for any given initial and final state. However, the isentropic process is not defined by any polytropic process in general. Thus, the conventional polytropic head is not a good way to approximate the compressor work $(\int v dP)$ near the isentropic path.

3. The Result of Cycle Analysis

3.1 Optimal P_{eco}

Usual LTD (Leaving Temperature Difference) values of evaporator and condenser in the centrifugal chiller industry are about 1℃. Since leaving water temperatures of evaporator and condenser at the design capacity is 7℃ and 37℃ from Korean Standard (KSB6270, 1985), design saturation temperatures at evaporator and con-

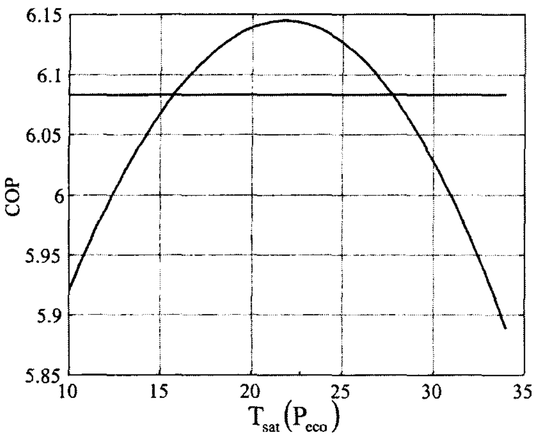

Fig. 3 Cycle COP variation with $T_{sat}(P_{eco})$

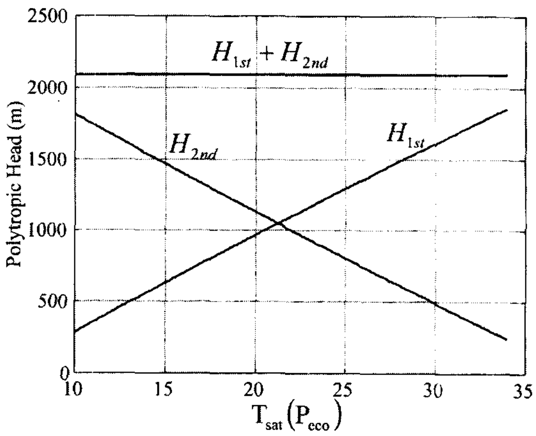

Fig. 4 Polytropic heads of the first-and the second-stage compressor

denser for the centrifugal chiller are determined as follows.

$$T_{sat}(P_e)=6℃,\ T_{sat}(P_c)=38℃$$

However, there is no guide from the standards for the decision of saturation temperature at economizer, and it is up to designer's choice. Therefore, it is meaningful to see the variation of cycle COP with respect to the variation of economizer's saturation temperature. Here, we assumed that the isentropic efficiencies are 78% for both compressors and the amount of subcooling at the condenser is 1℃.

In Fig. 3, we show the variation of cycle efficiency with respect to the economizer temperature T_{sat} (P_{eco}). Here COP is the ratio of refrigeration capacity to the sum of each compressor's works as defined in Eq. (9).

As shown in Fig. 3, the maximum COP occurs at $T_{sat}(P_{eco})=21.7$℃ : however, the value of COP is insensitive to the change of the saturation temperature $T_{sat}(P_{eco})$. For example, the range of $T_{sat}(P_{eco})$ corresponding to the range of COP>0.99 COP_{max} is 16℃~28℃ (which is the interval defined by the intersections between the horizontal line and the parabolic one in Fig. 3). This large range of $T_{sat}(P_{eco})$ lets designers choose the same values of nondimensional parameters for the first and second compressors in a given chiller.

The variation of polytropic head of compressors with respect to $T_{sat}(P_{eco})$ is shown in Fig. 4.

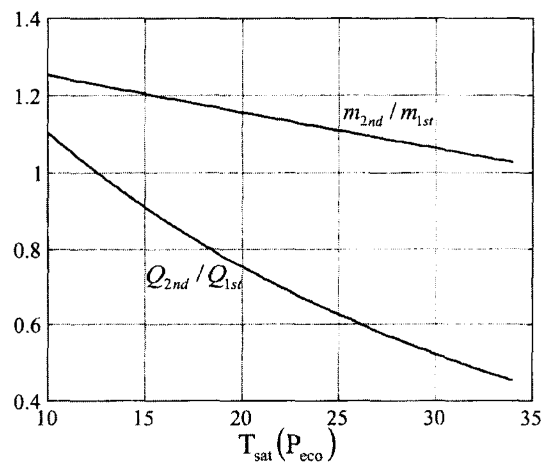

Fig. 5 The ratios of mass and volumetric flow rates between the second-and the first-stage compressor

As $T_{sat}(P_{eco})$ increases, H_{1st} and H_{2nd} increase and decrease almost linearly respectively. Thus, the sum remains almost constant. polytropic heads of the first- and second-stage compressors have the same values at $T_{sat}(P_{eco})=21.2$℃ ; this value is near the optimal $T_{sat}(P_{eco})$ obtained from the cycle analysis and it is consistent with the fact that multi-stage air compressor has the maximum efficiency when the polytropic head is distributed evenly.

The ratios of both the mass flow rate and the inlet volumetric flow rate between the first- and the second-stage compressors are shown in Fig. 5.

Of course, the ratio of mass flow rate m_{2nd}/m_{1st} is greater than unity as shown in Fig. 5. However, the ratio of the volumetric flow rate Q_{2nd}/Q_{1st} is less than "1" for almost all temperature range because the refrigerant density at the second-stage compressor's inlet is much higher than that at the first-stage compressor's inlet. The ratio of volumetric flow rate Q_{2nd}/Q_{1st} at the optimal $T_{sat}(P_{eco})$ amounts to only 74%. It is noticeable that the ratio of volumetric flow rate varies from 0.9 to 0.55 within the range COP$>$0.99COP$_{max}$ (i.e. 16℃$< T_{sat}(P_{eco}) <$28℃). Thus, even if the volumetric flow rate of the second-stage compressor varies significantly by some reasons, the cycle efficiency does not change significantly as long as the compressor's isentropic efficiency remains unchanged.

3.2 Determination of nondimensional parameters of compressors

To determine nondimensional parameters of compressors, we consider the recommended range of the nondimensional parameters for good performance and efficiency of the compressors. The number of independent parameters among parameters defined in section 2.2 are found to be three. However, the Reynolds number in our application is too large to consider its dependency, and the Reynolds number will not be considered further anymore. Therefore, the number of independent nondimensional compressor parameters becomes two. The nondimensional parameters generally used for efficiency issue are N_s and ϕ. The recommended ranges of N_s and ϕ are known to be N_s=0.6~0.85 and ϕ=0.11~0.21 respectively (ASHRAE, 1996).

The ratio of recommended minimum N_s to maximum N_s is 0.706(=0.6/0.85). If we assume the same compressor head for the first- and the second-stage compressors, the ratio of volumetric flow rate Q_{2nd}/Q_{1st} can be determined to be 0.720 from the cycle analysis. Thus, $N_{s,2nd}=\sqrt{0.720}\,N_{s,1st}$. To locate the design specific speeds at both stages within the recommended range, 0.6~0.85, $N_{s,1st}$ must be greater than $0.6/\sqrt{(0.720)}=0.707$. So, the N_s range for first-stage compressor is 0.707~0.85, and the corresponding N_s for the second-stage compressor lies between 0.6 and 0.707. If we define γ as the ratio of maximum to minimum volumetric flow rate in one compressor group, this leads us to have $\gamma=(0.85/0.707)^2=1.446$. Since this value is larger than the ratio 1.35 of chiller capacity within a group mentioned in section 2.2, it is possible to achieve the original target, i.e. to divide the range of chiller capacity 200~1200RT into 6 groups. This choice allows us to design hydrodynamically similar compressor for different groups. However, the designs of the first- and the second-stage compressor must be different because of different nondimensional parameter N_s.

Thus, we consider the other choice, i.e. imposing the same N_s values for both compressors. In this case, the impeller design can be completely the same for both first- and second-stage compressors as well as for the different groups except scaling parameter, and there exist only one design of impeller in the whole series design. Since $N_{s,1st}=N_{s,2nd}$,

$$\sqrt{Q_{1st}/Q_{2nd}}=(H_{1st}/H_{2nd})^{3/4} \tag{17}$$

Since we know that both Q_{2nd}/Q_{1st} and H_{2nd}/H_{1st} are dependent on $T_{sat}(P_{eco})$ from the cycle analysis, we can solve Eq. (17) and obtain $T_{sat}(P_{eco})$=23.4℃. From this, H_{1st} and H_{2nd} can be computed and found to be H_1=1,193 m and H_2=908 m. This value of $T_{sat}(P_{eco})$ is only 1.7℃ larger than the optimal $T_{sat}(P_{eco})$, so that the effect of this choice on the cycle efficiency is considered to be negligible. Since the first- and the second-stage compressor can use the same recommended range 0.6~0.85 of N_s, the value of γ can be as large as $\gamma=(0.85/0.6)^2$.

If we define γ_N to be the ratio of maximum N_s to the minimum N_s in a given compressor group and γ_D to be the ratio of maximum D_s to the minimum D_s in a given compressor group, we obtain the following equations.

$$\frac{N_{s,\max}}{N_{s,\min}}=\gamma_N=\sqrt{\frac{Q_{\max}}{Q_{\min}}}=\gamma^{1/2} \tag{18}$$

$$\frac{N_{s,\max}}{N_{s,\min}}=\gamma_D=\sqrt{\frac{Q_{\max}}{Q_{\min}}}=\gamma^{1/2} \tag{19}$$

The use of the same N_s for the first- and second-stage compressor determines the values of polytropic head in the first- and second-stage compressors. In a similar way, use of the same D_s for the first- and second-stage compressors leads to a particular choice of impeller diameters, i.e.

$$\frac{D_{1st}(gH_{1st})^{1/4}}{\sqrt{Q_{1st}}}=\frac{D_{2nd}(gH_{2nd})^{1/4}}{\sqrt{Q_{2nd}}} \tag{20}$$

Thus, D_{1st}/D_{2nd} is determined as follows.

$$\frac{D_{1st}}{D_{2nd}}=\sqrt{\frac{Q_{1st}}{Q_{2nd}}}\left(\frac{H_{2nd}}{H_{1st}}\right)^{1/4}=\sqrt{\frac{H_{1st}}{H_{2nd}}}=1.146 \tag{21}$$

Blade trimming of open impellers is a common and economical method to change the flow rate for the compressor using cast impeller. Rodgers (2001) showed that by trimming impeller blades the flow rate can change from 20% to 100% with efficiency sacrifice of only 6% and can change from 50% to 100% with efficiency sacrifice of only 2%. However, his study is for air-compressor and there is no such study in the case of refrigerant compressors. Thus, we decide to be more conservative and choose narrower range of N_s than recommended by ASHRAE (1996) and Rodgers (2001). We will determine the γ value from the consideration of sharing impeller molds belonging to adjacent compressor groups. Since we consider six two-stage compressor groups, we need to have 12 impeller molds. However, if we require the diameter of the second-stage impeller in a given group to be the same as the diameter of the first-stage impeller in the group with one-grade smaller capacity, we need only 7 different impeller molds instead of 12 impeller molds. This means that about half of the cost for impeller molds can be saved. When we define the group identification number to be "i", we can obtain the following relation for the impeller diameter, i.e.

$$\frac{D_{i+1,1st}}{D_{i+1,2nd}}=\frac{D_{i+1,1st}}{D_{i,1st}}=1.146 \tag{22}$$

In addition, $D_{i,1st}/D_{i+1,1st}$ can be arranged as follows.

$$\begin{aligned}\frac{D_{i+1,1st}}{D_{i,1st}}&=\frac{D_s\sqrt{Q_{i+1}}/(gH_{1st})^{1/4}}{D_s\sqrt{Q_i}/(gH_{1st})^{1/4}}\\&=\sqrt{\frac{Q_{i+1}}{Q_i}}=\sqrt{\gamma}\end{aligned} \tag{23}$$

From Eqs (22) and (23),

$$\gamma_N=\gamma_D=1.146,\ \gamma=\gamma_D^2=1.314 \tag{24}$$

This result allows us determine the series design parameters. This value of γ_N is much less than the recommended value of ASHRAE, and thus it is quite possible to have a good design of impeller without sacrificing efficiency when trimming impeller blade. With this value of $\gamma=1.314$, and the six groups between 200RT and 1200RT, we must change the starting RT from 200RT to 233.4RT to have the largest capacity of 1200RT. The detailed values of series design parameter are shown in Table 1.

Table 1 The series-designed specifications of compressors

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Capacity range (RT)	233~307	307~403	403~529	529~695	695~913	913~1,200
max. Q_{1st} (m^3/s)	0.359	0.471	0.619	0.814	1.069	1.405
max. Q_{2nd} (m^3/s)	0.238	0.313	0.411	0.540	0.710	0.932
rpm	13,711	11,962	10,435	9,104	7,942	6,929
D_{1st} (mm)	208	239	273	313	359	412
D_{2nd} (mm)	182	208	239	273	313	359
ϕ	0.133~0.174					
N_s	0.667~0.765					
D_s	4.14~3.61					
ψ	0.524					

4. Conclusion

We have performed cycle analysis of two-stage chiller and nondimensional analysis of two-stage compressor in order to impose hydrodynamic similarity of compressor as much as possible. We could determine all nondimensional parameters except Reynolds number whose effect is negligible, and obtain the following conclusion.

(1) The heads of the first- and second-stage compressors are determined from the requirement of the same N_s values of both the first- and second-stage compressors. This requirement also determines the design saturation temperature of the economizer, $T_{sat}(P_{eco})=23.4$℃. The polytropic heads for the first- and the second-stage compressors are determined to be $H_1=1{,}193$m and $H_2=908$ m respectively.

(2) The requirement of the same D_s values for both the first- and the second-stage compressors leads to the result that the ratio of the first-stage impeller's diameter to the second-stage impeller's diameter in a given group becomes 1.146.

(3) The requirement, that the diameter of second-stage impeller in a given group should be the same as the diameter of first-stage impeller in the group with one-grade smaller capacity, leads to the complete determination of series design except the starting chiller capacity.

(4) The series design parameters such as rated capacities, impeller diameters, rpm, and volume flow rates are determined for six different groups. The hydrodynamic similar compressor design would guarantee us to have basically the same performance and efficiency in all groups.

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