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## 고이득 관측기를 이용한 슬라이딩 모드 제어기 설계

## (Output Feedback Sliding Mode Control with High-gain Observer)

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## 요 약

불확실성이 있는 표준형으로 표현된 단일 입력 단일 출력 비선형 시스템을 고려하였다. 모델링 불확실성을 극복할 수 있고 상태 변수를 예측할 수 있는 고이득 관측기를 이용하였다. 크기가 제한된 출력만을 이용한 슬라이딩 모드 제어기를 이용하여 폐회로를 안정화하였다. 크기가 제한된 출력만을 이용한 제어기를 사용하여 상태변수에 임펄스와 같은 현상이 나타나는 것을 제거하였다. 제안된 방법은 크기가 제한된 제어기 설계를 쉽게 하였다.

## Abstract

We consider a single-input-single-output nonlinear system which is represented in a normal form. The model contains the uncertainty. A high-gain observer is used to estimate the states variables to reject a modeling uncertainty. We design the globally bounded output feedback controller using sliding mode control to stabilize the closed-loop system. The globally bounded output feedback controller reduce the peaking in the states variables. The proposed method give a more design freedom in the design of the globally bounded controller than that of the previous work.

**Key Words** : the separation principle do not hold for

## I. Introduction

The output feedback control scheme can be classified two schemes. One scheme is static output feedback which does not use an observer to estimate the states of system. But the scheme is limited to relative degree one system. The other scheme is the use of observe-based one which could be used for relative degree higher than one system. Since

nonlinear system in the presence of imperfect feedback cancellation and modeling uncertainty, a high-gain observer has been used to reject disturbance due to imperfect feedback cancellation and modeling uncertainty<sup>[1]</sup>. However, the use of a high-gain observer results in impulse-like behavior, so called peaking, of state variables which are usually physical variables. For example, the works<sup>[2,3]</sup> exhibit such a peaking in the state variables. Therefore it is desired to remove the peaking phenomenon in state variables. The globally bounded control was introduced to remove the peaking in state variables<sup>[4]</sup>. Since the idea of the globally bounded control was introduced, the idea has been used for the design of continuous output feedback control<sup>[5-8]</sup> which is satisfied Lipschitz

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condition for the controller. The paper<sup>[9]</sup> worked on the sliding mode control using a high-gain observer and globally bounded control. Since the sliding mode control scheme does not satisfy the Lipschitz condition, the analysis and design scheme are different with the continuous one. The work<sup>[9]</sup> demonstrated that the peaking of states variables was reduced with a sliding mode control. There are two controller design elements involved in the controller design of the work. One element is the design of globally bounded control and the other one is to design the controller satisfied the sliding mode condition when distance between the states and estimates of the states is small enough. The globally bounded control was obtained via saturation over the some region. However, a relatively large magnitude of control input was required because of the restriction on the saturation region. In this paper, we propose a method to give a more freedom to choose the saturation region. The proposed method can reduce the magnitude of the control effort due to the freedom of choosing the saturation region. The rest of paper is organized as follows. Section II introduces a class of system and problem statement to be considered. Section III states a high-gain observer structure and its property with a globally bounded control. We also describe a globally bounded sliding mode control design method that ensures stabilize the closed-loop system in the section III. Finally, we demonstrate the performance of the proposed method through an example in section IV.

## II. Problem Statement

We consider the following single-input single-output nonlinear system defined over a domain  $D$

$$\begin{aligned}\dot{x} &= Ax + b[f(x) + g(x)u] \\ y &= Cx\end{aligned}\quad (1)$$

where  $x$ ,  $u$ ,  $y$  are state variables, control input, output, respectively, and  $x = [x_1 \ x_2 \ \dots \ x_n]^T$ ,

$$A = \begin{bmatrix} 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix}_{n \times n}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n \times 1},$$

$$c = [1 \ 0 \ \dots \ 0]_{1 \times n}.$$

**Remark 1** The nonlinear system (1), so called a normal form, can be derived from the nonlinear system,  $\dot{w} = F(w) + G(w)u$ ,  $y = h(w)$ , under the uniform relative degree assumption<sup>[10]</sup>.

Let  $f_o(x)$  be a known nominal model of  $f(x)$ . Suppose that  $f_o(x)$  sufficiently smooth,  $f_o(0) = 0$ , and  $g(x)$  is nonsingular for all  $x \in D$ . We also assume that the uncertainty of the equation(1) satisfies the following assumption.

**Assumption 1** For all  $x \in D$ , there is a scalar Lipschitz function  $\rho(x)$  such that

$$|f(x) - f_o(x)| \leq \rho(x) \quad (2)$$

Our goal is the design of output feedback controller to stabilize the nonlinear system given by the equation (1) over domain  $D$ .

## III. Output feedback controller design

1. Robust high-gain observer with a globally bounded control

To design an output feedback controller, we use a following high-gain observer to estimate the state variable  $x$

$$\begin{aligned}\hat{x}_i &= x_{i+1} + \frac{a_i}{\epsilon^i} (y - \hat{x}_1), \quad i = 1, \dots, n-1 \\ \hat{x}_n &= \frac{a_n}{\epsilon^n} (y - \hat{x}_1) + f_0(\hat{x}) + g(\hat{x})u\end{aligned}\quad (3)$$

where  $\hat{x}_i$  is the estimate of the state variables  $x_i$  and  $\epsilon$  is a positive constant to be specified.

The positive constant  $a_i$  are chosen such that the roots of the following equation are in the open left half plane.

$$s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n = 0$$

We rewrite the observer equation (3) into the compact form

$$\dot{\hat{x}} = A\hat{x} + b[f_0(\hat{x}) + g(\hat{x})u] + D(\varepsilon)Lc(x - \hat{x}) \quad (4)$$

where  $L = [a_1 \dots a_n]^T$ , and  $D(\varepsilon) = \text{diag}[1/\varepsilon \ 1/\varepsilon^2 \dots 1/\varepsilon^n]$ .

Let  $e_i = x_i - \hat{x}_i$  be the estimation error, and define the scaled variables  $\zeta_i = (1/\varepsilon^{n-i})e_i$ . The closed-loop equation can be rewritten as

$$\begin{aligned} \dot{x} &= Ax + b[f(x) + g(x)u] \\ \varepsilon \dot{\zeta} &= (A - Lc)\zeta + \varepsilon b[f(x) - f_0(\hat{x}) + \{g(x) - g(\hat{x})\}u] \end{aligned} \quad (5)$$

where  $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_n]^T$ . Note that  $(A - Lc)$  is a Hurwitz matrix. The closed-loop system can be viewed as a singularly perturbed system with  $x$  as the slow variables and  $\zeta$  as the fast one. Define

$$\begin{aligned} \Omega_0 &= \{x \in R^n \mid \|x\| \leq c_o\}, \\ \Omega_{\zeta_0} &= \{\zeta \in R^n \mid \|\zeta\| < c_\zeta / \varepsilon^{n-1}\} \\ \Omega &= \Omega_0 \times \Omega_{\zeta_0} \end{aligned}$$

where  $c_\zeta$  is an arbitrary positive constant. The set  $\Omega$  is defined for the region of allowable initial condition. Note that  $\Omega_0$  could be any compact set which is a subset of  $D$ . We just define it for simplicity. We use a globally bounded control functions as a control input. We will specify the control input  $u$  to make a globally bounded control later on. The following lemma states that the fast variables decays very rapidly during a short time period. The proof of the lemma is the same as the proof of Lemma 1 in [9], hence it is omitted.

**Lemma 1** Consider the closed-loop system (5) and suppose that the control input  $u$  is globally bounded. Then, for all  $(x(0), \zeta(0)) \in \Omega$ , there exist  $\varepsilon_1$  and  $T_1 = T_1(\varepsilon) \leq T_3$  such that for all  $0 < \varepsilon < \varepsilon_1$ ,  $\|\zeta\| < \bar{\kappa}\varepsilon$  for all  $t \in [T_1, T_4)$  where  $T_3$  is a finite time and  $T_4 > T_3$  is the first time  $x(t)$  exits from the any compact set containing the set  $\Omega_0$ .

**Remark 2** The equation (5) can be rewritten as  $\varepsilon \dot{\zeta} = (A - Lc)\zeta + \varepsilon b\delta(x, e)$ , where  $\delta(\cdot) = f(x) - f_0(\hat{x}) + \{g(x) - g(\hat{x})\}u$ . We can treat the term  $\delta$  as a perturbation term. The perturbation term is multiplied by  $\varepsilon$ . Hence its effect diminishes as  $\varepsilon \rightarrow 0$ . The solution of equation (5) has contains terms of the form  $(1/\varepsilon^{n-1})e^{\lambda t/\varepsilon}$ . Therefore  $\hat{x}(t)$  exhibits impulse-like behavior  $\varepsilon \rightarrow 0$ . Since  $x(t)$  is coupled with control input which contains  $\hat{x}$ ,  $x(t)$  can also exhibit impulse-like behavior. However the use of the globally bounded control can reduce peaking in state variables.

## 2. Controller design

Controller design has two elements to be considered. One element is the design of globally bounded control and the other element is the stabilization of the system with the fast variables  $\zeta$  being  $O(\varepsilon)$ . First, we consider the design of the control input which can stabilize the closed-loop system with the fast variables  $\zeta$  being  $O(\varepsilon)$  using sliding mode control, since sliding mode control scheme is a simple and robust control scheme<sup>[11]</sup>. We choose a sliding surface  $S = M\hat{x}$ , where  $M = [m_1, m_2, \dots, m_{n-1}, 1]$  and  $m_i$  are chosen such that  $\mathcal{A}$  is Hurwitz where

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ -m_1 & -m_2 & \dots & -m_{n-2} & -m_{n-1} \end{bmatrix}_{(n-1) \times (n-1)}$$

Consider the following function

$$\begin{aligned} \Psi(\hat{x}) &= (1/g(\hat{x}))[-M(A\hat{x} + bf_0(\hat{x})) \\ &\quad - (\rho(\hat{x}) + a_1 \text{sgn}(S(\hat{x})))] \end{aligned} \quad (6)$$

where  $\alpha_1$  is a positive constant. The function  $\Psi(\hat{x})$  satisfies the sliding mode condition when  $\zeta \leq \bar{k}\epsilon$  as we will show in the proof of Lemma 2.

Note that the function  $\Psi(\hat{x})$  may not satisfy the sliding mode control when the fast variables  $\zeta$  are not  $O(\epsilon)$ . In particular, the sliding mode condition may not be satisfied when  $t \leq T_1$ . The remaining part is the design of a globally bounded control input. We take a control input  $u(\hat{x})$  as  $\Psi(\hat{x})$ , saturated outside any compact  $\Omega_o \subset W$  when  $\hat{x}(t)$  exits  $W$  in the first time. In particular, let  $\phi_1 = -(1/g(\hat{x}))M(A\hat{x} + bf_o(\hat{x}))$ ,  $\phi_2 = -(\rho(\hat{x}) + \alpha_1)$  and  $S_i = \max_{\hat{x} \in W} |\phi_i(\hat{x})|$  and take

$$u(\hat{x}) = S_1 \text{sat}(\phi_1(\hat{x})/S_1) + S_2 \text{sat}(\phi_2(\hat{x})/S_2) \text{sgn}(S(\hat{x})) \quad (7)$$

where  $\text{sat}(\cdot)$  is the saturation function. One can verify that  $u(\hat{x})$  is a globally bounded control input.

**Lemma 2** Consider the closed-loop system (5) with control input  $u(\hat{x})$  defined by (7). Then the sliding mode condition

$$S(\hat{x})S(\hat{x}) \leq -\alpha_2 |S(\hat{x})|$$

is satisfied for  $t \geq T_1$ .

**Proof:** The proof of this lemma have two parts. One part is to prove to satisfy the sliding mode condition as long as  $\zeta$  is  $O(\epsilon)$ . Remaining part is to prove that  $\zeta$  is  $O(\epsilon)$  for  $t \geq T_1$ . Since the first part of proof is similar to the proof of [9], we prove it briefly.

$$\begin{aligned} S(\hat{x})S(\hat{x}) &= S(\hat{x})M[A\hat{x} + b(f_o(\hat{x}) + g(\hat{x})u) \\ &\quad + D(\epsilon)Lc(x - \hat{x})] \\ &= S(\hat{x})M[A\hat{x} + b(f_o(\hat{x}) + g(\hat{x})u \pm b\phi(\hat{x})) \\ &\quad + O(\epsilon)] \\ &\leq -(\rho(\hat{x}) + \alpha_1)|S(\hat{x})| + (\rho(\hat{x}) \\ &\quad + O(\epsilon))|S(\hat{x})| \\ &\leq -\alpha_2 |S(\hat{x})| \end{aligned}$$

where  $\phi(\hat{x}) = f(\hat{x}) - f_o(\hat{x})$  and  $\alpha_2$  is a some positive constant. Note that for

$D(\epsilon)Lc(x - \hat{x}) = \pm b\phi(\hat{x}) + O(\epsilon)$ , we use the result of [9]. To show that  $\|\zeta\| < \bar{k}\epsilon$  for  $t \geq T_1$ , it is sufficient to show that  $x(t) \in W_1$  for all  $t$  where  $W_1 \subset D$  is some compact set to be defined. Consider the set

$$W_1 = \{x \in R^n \mid \|Mx\| \leq c_{s1}, \|z\| \leq c_{z1}\}$$

where  $c_{z1} = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \frac{2\|Pb\|c_{s1}}{\lambda_{\min}(Q)}}$ ,  $P$  and  $Q$  are the positive definite matrices such that  $PA + A^T P = -Q$ , and  $c_{s1}$  is large enough such that  $W \subset W_1$ . Lemma 2 implies that  $\hat{x}$  might be outside set  $W_1$  for  $t < T_1$ , but since  $\hat{x}_i = x_i - \epsilon^{n-i}\zeta_i$ ,  $\hat{x}$  is in the set  $W_1$  for  $t \in [T_1, T_4]$ . Define  $V(\hat{z}) = \hat{z}^T P \hat{z}$  where  $\hat{z} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{n-1}]^T$ . The derivative of  $V(\hat{z})$  along the trajectories of equation(4) is given by

$$\begin{aligned} V(\hat{z}) &= -\hat{z}^T Q \hat{z} + 2\hat{z}^T P b S(\hat{z}) + 2\hat{z}^T P D(\epsilon) \hat{\zeta} \\ &\leq -\lambda_{\min}(Q) \|\hat{z}\|^2 + 2\|Pb\| \|\hat{z}\| |S(\hat{z})| + O(\epsilon) \\ V(\hat{z}) &\leq 0, \quad \text{for } \|\hat{z}\| \geq \delta \end{aligned}$$

where  $\hat{b} = [0 \ 0 \ \dots \ 0 \ 1]^T_{1 \times (n-1)}$ ,

$$D(\epsilon) = \text{diag}[\epsilon^{n-2}, \epsilon^{n-3}, \dots, \epsilon, 1],$$

$$\hat{c} = [1 \ 0 \ \dots \ 0]_{1 \times (n-1)}, \quad \hat{\zeta} = [\zeta_1, \dots, \zeta_{n-1}], \text{ and}$$

$\delta > 2c_{s1}\|Pb\|/\lambda_{\min}(Q)$  is a positive constant.

Define the set  $W_2 = \{V(\hat{z}) \leq \delta_1\}$  where  $\delta_1 = \lambda_{\max}(P)\delta^2$ . Since  $V(\hat{z}) \leq 0$  for  $\|\hat{z}\| \geq \delta$ ,  $\hat{z} \in W_2$  for  $t \geq T_1$ . Since  $S(\hat{x})S(\hat{x}) < 0$  and  $W_2 \subset \{\|\hat{z}\| \leq c_{z1}\}$ ,  $\hat{x} \in W_1$  for  $t \geq T_1$ . This implies that  $\hat{x}$  never leave the set for  $t \geq T_1$ .

Thus  $x \in W_1$  for all  $t$ . Therefore we can conclude that  $\|\zeta\| < \bar{k}\epsilon$  for  $t \geq T_1$ . Note that the work[9] used the similar set with  $W_1$  for the saturation region. But we use the set  $W_1$  to show that  $\hat{x}(t)$  does not leave the set  $W_1$ , not

use for the design of globally bounded control. We use the any compact set containing initial condition for the saturation region. Lemma 2 implies that the trajectories of the equation (4) reach the sliding manifold  $S(\hat{x})=0$  within a finite time and remain in the sliding manifold thereafter. On the sliding manifold, the following equation holds

$$\begin{aligned} \dot{x}_{n-1} &= x_n \\ &= -(m_1 \hat{x}_1 + m_2 \hat{x}_2 + \dots + m_{n-1} \hat{x}_{n-1}) + \zeta_n \\ &= -\{m_1(x_1 - \varepsilon^{n-1} \zeta_1) + \dots + m_{n-1} \\ &\quad (x_{n-1} - \varepsilon \zeta_{n-1})\} + \zeta_n \end{aligned} \quad (8)$$

After substituting the equation (8) into the closed-loop equation (5), we have

$$\begin{aligned} \dot{z} &= \bar{A}z + E\mathcal{D}(\varepsilon)\zeta \quad \text{where } z = [x_1, \dots, x_{n-1}]^T, \\ \mathcal{D}(\varepsilon) &= \text{diag}[\varepsilon^{n-1}, \dots, \varepsilon, 1], \quad \text{and} \\ E &= \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \dots & \dots & 0 & 0 \\ -m_1 & -m_2 & \dots & -m_{n-1} & 1 \end{bmatrix}_{n-1, n-1} \end{aligned}$$

There are positive definite matrices  $Q_1, Q_2, P_1, P_2$  such that  $P_1 \bar{A} + \bar{A}^T P_1 = -Q_1$  and  $P_2(A - Lc) + (A - Lc)^T P_2 = -Q_2$ , since  $\bar{A}$  and  $(A - Lc)$  are Hurwitz matrices. Consider the Lyapunov function  $V_c(z, \zeta) = z^T P_1 z + \zeta^T P_2 \zeta$  for the closed-loop system. Then

$$\begin{aligned} V_c(z, \zeta) &= -z^T Q_1 z - (1/\varepsilon) \zeta^T Q_2 \zeta \\ &\quad + 2z^T P_1 E \mathcal{D}(\varepsilon) \zeta \\ &\quad + 2\zeta^T P_2 b[f(x) - f_0(\hat{x}) + (g(x) - \hat{g}(\hat{x})) \\ &\quad u(\hat{x})] \end{aligned}$$

Since  $f(\cdot), f_0(\cdot), g(\cdot)$  are smooth enough functions, the following inequality holds

$$|f(x) - f_0(\hat{x}) + (g(x) - \hat{g}(\hat{x}))u(\hat{x})| \leq k_1 \|x\| + k_2 \|\zeta\| \quad (9)$$

where  $k_1$  and  $k_2$  are some positive constant. Using (8) and (9), we have

$$\begin{aligned} V_c(z, \zeta) &\leq -\lambda_{\min}(Q_1) \|z\|^2 - (1/\varepsilon) \lambda_{\min}(Q_2) \|\zeta\|^2 \\ &\quad + k_3 \|\zeta\|^2 + 2k_4 \|\zeta\| \|z\| \\ &= - \begin{bmatrix} \|z\| \\ \|\zeta\| \end{bmatrix}^T \begin{bmatrix} \lambda_{\min}(Q_1) & -k_4 \\ -k_4 & (1/\varepsilon) \lambda_{\min}(Q_2) - k_3 \end{bmatrix} \begin{bmatrix} \|z\| \\ \|\zeta\| \end{bmatrix} \end{aligned} \quad (15)$$

for some positive constant  $k_3$  and  $k_4$ . It can be verified that  $\begin{bmatrix} \lambda_{\min}(Q_1) & -k_4 \\ -k_4 & (1/\varepsilon) \lambda_{\min}(Q_2) - k_3 \end{bmatrix}$  is a positive definite matrix for sufficiently small  $\varepsilon$ . Therefore we can conclude that the closed-loop system (5) is asymptotically stable.

#### IV. Example

Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_1 \sin x_1 + u \\ y &= x_1 \end{aligned}$$

where  $a_1$  is an unknown coefficient that satisfies  $|a_1| \leq 0.4$  and domain  $D = \{x \in R^2 \mid \|x\| \leq 10.5\}$ . Suppose that initial states belong to the set  $\Omega_0 = \{x \in R^2 \mid \|x\| \leq 1\}$ . A high gain observer is constructed as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + (1/\varepsilon)(y - \hat{x}_1) \\ \dot{\hat{x}}_2 &= (1/\varepsilon^2)(y - \hat{x}_1) + u \end{aligned}$$

For comparison, we design the controller following the design scheme of [9], and refer to case 1. Choose the sliding surface  $S(\hat{x}) = \hat{x}_1 + \hat{x}_2$ . We use the saturation region given by the design procedure of [9], It is given by

$$W_{\text{case1}} = \{\hat{x} \in R^2 \mid -6 \leq \hat{x}_1 \leq 6, -2.5 \leq \hat{x}_1 + \hat{x}_2 \leq 2.5\}.$$

Let  $\psi(\hat{x}) = -\hat{x}_2 - (0.5\|\hat{x}\| + 0.1)\text{sgn}(s(\hat{x}))$  and the control input is given by

$$u_{\text{case1}}(\hat{x}) = -S_{\text{case11}} \text{sat}(\hat{x}_2 / S_{\text{case11}}) - S_{\text{case12}} \text{sat}\left(\frac{0.5\|\hat{x}\| + 0.1}{S_{\text{case12}}}\right) \text{sgn}(S(\hat{x}))$$

where  $S_{\text{case11}} = \max_{\hat{x} \in W_{\text{case1}}} (|\hat{x}_2|) = 8.5$  and  $S_{\text{case12}} = \max_{\hat{x} \in W_{\text{case1}}} (0.5\|\hat{x}\| + 0.1) = 6.1$ .

Following the design scheme proposed in this paper, we choose  $W = \{\hat{x} \in R^2 \mid \|\hat{x}\| \leq 1.2\}$ . One can verify that  $\Omega_0 \subset W$ . The control input is given by

$$u(\hat{x}) = -S_1 \text{sat}(\hat{x}_2/S_1) - S_2 \text{sat}\left(\frac{0.5\|\hat{x}\| + 0.1}{S_2}\right) \text{sgn}(S(\hat{x}))$$

where  $S_1 = \max_{\hat{x} \in W} (|\hat{x}_2|) = 1.2$  and

$$S_2 = \max_{\hat{x} \in W} (0.5\|\hat{x}\| + 0.1) = 0.9436$$

We simulate the response for  $x(0) = [1 \ 0]^T$ ,  $\hat{x}(0) = [0 \ -1]^T$ ,  $a_1 = -0.4$ , and  $\epsilon = 0.04$ . Fig. 1 and Fig. 2 show that state variables,  $x_1(t)$  and  $x_2(t)$ , does not exhibit peaking phenomenon in the both cases as we expected, since globally bounded controllers are used. One can also observe that the estimates of states,  $\hat{x}_1$  and  $\hat{x}_2$ , exhibit a peaking phenomenon. One can observe that the attractivity to the sliding manifold is achieved after short periods of time from Fig. 3. In other words, the reaching condition to the sliding manifold satisfy after the distance between the states and estimates of states being small enough. Fig. 4 shows that the magnitude of the control input for both cases. The magnitude of control input proposed in this paper is much smaller than that of case 1. This comes from the use of saturation region using the proposed method.

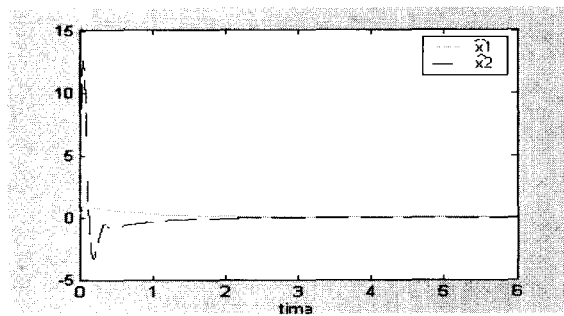
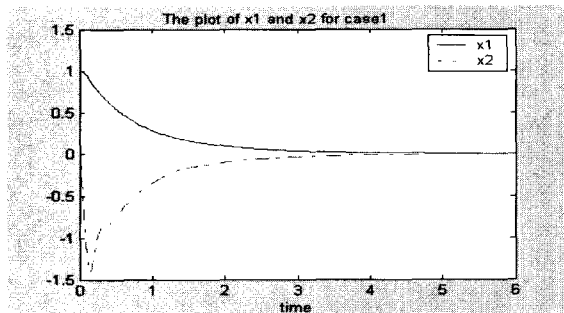


그림 1. 경우 1 에 대한 상태변수 와 상태변수 예측치  
Fig. 1. The plot of states and estimates of states for case 1.

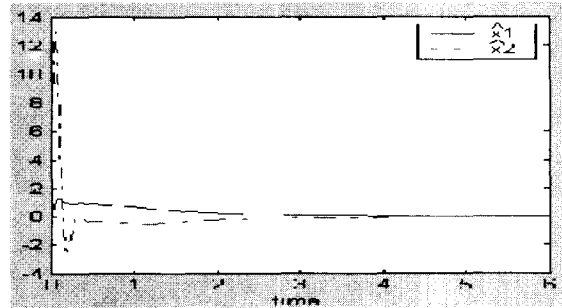
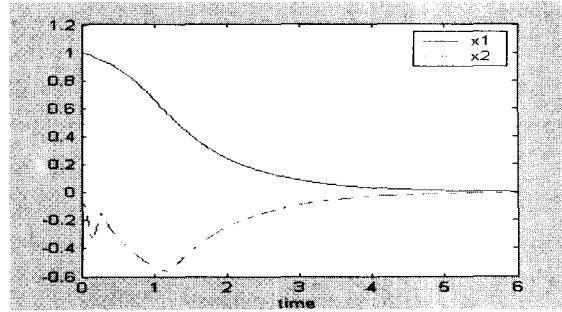


그림 2. 제안된 설계 방법에 대한 상태변수 와 상태변수 예측치

Fig. 2. The plot of states and estimates of states for the proposed design scheme.

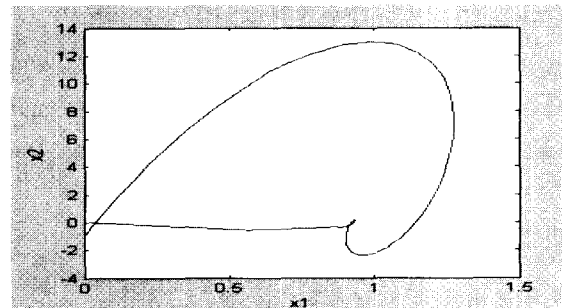
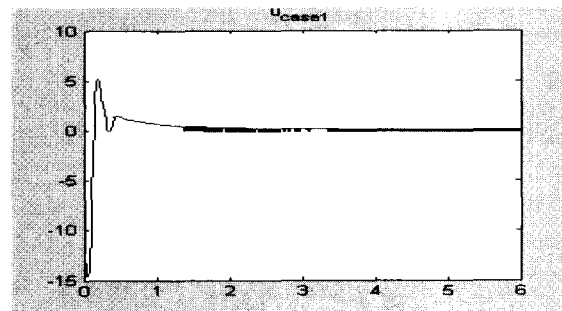
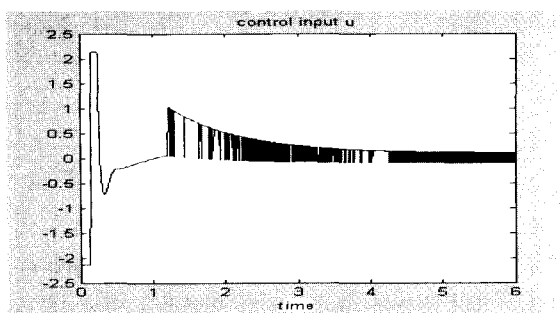


그림 3. 제안된 설계 방법에 대한 상 천이도

Fig. 3. The plot of phase portrait for the proposed design scheme.



(a)



(b)

그림 4. 경우 1 및 제안된 설계 방법에 의한 제어 입력 값((a) : 경우 1 (b) : 제안된 설계 방법)

Fig. 4. The plot of control input for case 1 and the proposed design scheme((a) : case 1 (b) : the proposed design scheme).

## V. Conclusion

We have designed a globally bounded control output feedback sliding mode control that stabilizes the a nonlinear system in the presence of modeling uncertainty. Even though we use a high-gain observer to estimate the states variables, the peaking does not exhibit in the states variables. We propose and justify a design method that give a more freedom in the globally bounded control design than that of the previous work. The magnitude of control input can be reduced from the proposed method due to the freedom of the design of a globally bounded control. We demonstrate the performance of the proposed design method via an example.

## References

[1] H.K. Khalil. *Nonlinear Systems, second edition.*

Prentice Hall, New Jersey, 1996.

- [2] S. Emelyanov, S. Korovin, A. Nersisyan, and Y. Nisenzov, "Output feedback stabilization of uncertain plants," *Int. J. of Contr.*, Vol. 55, No. 1 pp. 61~81, 1992.
- [3] S. Raghavan and J. Hedrick, "sliding compensator for a class of nonlinear systems," *Proc. American Contr. Conf.* pp. 1174~1179, 1990.
- [4] F. Esfandiari and K. H. Khalil. "Output feedback stabilization of fully linearizable systems," *Int. J. Contr.*, 56:1007-1037, 1992.
- [5] A. Atassi and H. Khalil. "A separation principle for the stabilization of a class of nonlinear systems," Vol. 44. No. 9, PP. 1672~1687, 1999.
- [6] A. Teel and L. Praly, "Global stabilizability and observability implies semi-global stability by output," *Syst. Contr. Lett.*, Vol. 22, pp. 313~325, 1994.
- [7] N. Mahmoud and H. Khalil, "Asymptotic regulation of minimum phase nonlinear systems using output feedback," *IEEE Trans. Automat. Contr.*, Vol. 41, pp. 1402~1412, 1996.
- [8] A. Isidori, "A remark on the problem of semiglobal nonlinear output regulation," *IEEE Trans. Automat. Contr.* Vol. 42, pp. 1734~1738, 1997.
- [9] S. Oh and H. Khalil. "Output feedback stabilization using variable structure control," *Int. J. Contr.*, 62:831~848, 1995.
- [10] A. Isidori, *Nonlinear control systems, 3rd ed.*, New York, Springer-Verlag, 1995.
- [11] I. Utkin, *Sliding modes in optimization and control*, Springer-Verlag, New York, 1992.

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