JOURNAL OF THE KOREAN ASTRONOMICAL SOCIETY
36: 67 ∼ 72, 2003

SOLAR SHORT-PERIOD OSCILLATIONS EXCITED BY A SMOOTH FORCE

HEON-YOUNG CHANG
Korea Institute for Advanced Study, Seoul, Korea 130-012
E-mail: hyc@ns.kias.re.kr
(Received May 9, 2003; Accepted June 6, 2003)

ABSTRACT

The basic objective of helioseismology is to determine the structure and the dynamics of the Sun by analysing the frequency spectrum of the solar oscillations. Accurate frequency measurements provide information that enables us to probe the solar interior structure and the dynamics. Therefore the frequency of the solar oscillation is the most fundamental and important information to be extracted from the solar oscillation observation. This is why many efforts have been put into the development of accurate data analysis techniques, as well as observational efforts. To test one's data analysis method, a realistic artificial data set is essential because the newly suggested method is calibrated with a set of artificial data with predetermined parameters. Therefore, unless test data sets reflect the real solar oscillation data correctly, such a calibration is likely incomplete and a unwanted systematic bias may result in. Unfortunately, however, commonly used artificial data generation algorithms insufficiently accommodate physical properties of the stochastic excitation mechanism. One of reason for this is that it is computaionally very expensive to solve the governing equation directly. In this paper we discuss the nature of solar oscillation excitation and suggest an efficient algorithm to generate the artificial solar oscillation data. We also briefly discuss how the results of this work can be applied in the future studies.

Key words : methods:data analysis – Sun:interior– Sun:oscillations

I. INTRODUCTION

The internal structure of the Sun has been of interest to put the theory of stellar structure and evolution to the test. Until relatively recent times, the criterion adopted to assess whether a solar model represents the real Sun well enough was simply whether the solar model could reproduce a few measured surface properties, which might not say anything about its interior properties directly. For example, the photon emitted from the surface was actually generated about 10^6 years ago. During the period the photon lost the information on the interior when it was generated. Even if it carries some of information, the story is about 10^6 years ago. More strong and more direct constraints of the internal properties of the Sun became available with the discovery of global solar oscillations and the observation of the neutrino flux from the core of the Sun (e.g., Gough & Toomre 1991; Bahcall, Gonzalez-Garcia, & Pena-Garay 2003). These two observations give rather direct and robust information about the interior of the Sun. The studies of solar neutrinos and of p-mode oscillations are largely complementary.

Helioseismology began with the discovery of oscillatory motions on the surface of the Sun by Leighton and his collaborators while they were studying the evolution of solar granules (Leighton, Noyes, & Simon 1962). Since then little progress was made in theoretical understanding the nature of the oscillations until it was suggested by Ulrich (1970), Leibacher & Stein (1971), and Ando & Osaki (1975, 1977) that the observed power of 5 minute oscillations should be confined to distinct ridges in a $k_h - \omega$ plane, the so called diagnostic diagram. Major differences of these oscillations from the classical stellar pulsations are that they are non-radial oscillations with small amplitudes and are multi-periodic with more than hundreds of thousands of modes. The amplitude of each oscillation is substantially less than 1 ms^{-1} for the Doppler velocity and less than about 10^{-6} in relative brightness fluctuation.

There have been many discussions of the excitation mechanism of the solar short-period oscillations (Ando & Osaki 1977; Goldreich & Keeley 1977; Goldreich & Kumar 1988, 1990; Kumar, Franklin, & Goldreich 1988; Goldreich, Murray, & Kumar 1994). According to accumulated observational evidence (e.g., Libbrecht, Popp, & Kaufman 1986; Libbrecht 1988a, 1988b) the oscillation modes are generally accepted to be linearly stable, and driven by the turbulent convection. The turbulent excitation model not only predicts the right order of magnitude for the p-mode energies, but it also naturally explains the observation that many modes are excited to nearly the same energy. A mode can be regarded as a damped simple harmonic oscillator.

The purpose in this paper is two fold. One is to suggest a more realistical model the solar p modes excited by the solar granulation, in the sense that common practice in generating artificial data is to simply add sinusoidal functions or at most their variations. The other is to provide an efficient way of generating the artificial signals. There are of order of $10^6 - 10^7$ granules at a time on the whole surface of the Sun. They last for from 10 minutes to half an hour. Therefore solar p modes whose mean period is about 5 minutes are experiencing about 10^6 kicks per period. And those stochastic hits result in a smooth function rather than a sparse impulse-like function. Unfortunately, simulations on helioseismic time-series often fail in satisfying both of these properties (cf. Kumar, Franklin, & Goldreich 1988; Schou & Brown 1994). This is either because it requires numerically quite expensive simulations, or because the high frequency of excitation has been overlooked. Even though previous models could reproduce a spiky power spectrum, as is seen in real observations, there is a danger that power at each frequency may not be independent when hits are sparse (cf. Baudin et al. 1996; Chaplin et al. 1997). Besides, simulations which do not contain proper excitation characters of the solar p modes may lead us to a wrong conclusion on the energy distribution of the modes or even very fundamental information such as the frequencies of the modes. Obviously a test of a new data-analysis technique with improper artificial data sets does not guarantee that the new data-analysis technique can extract any information from the real observation properly. Therefore, simulations for the stochastic excitation is of a great importance not only to understand the excitation mechanism but also to test any new data-analysis technique.

We shall formulate the problem in §2, and discuss the solution of the equation in §3. We then describe how one can efficiently generate artificial data which satisfy appropriate aspects of the solar p modes in §4. And finally we conclude by summarizing and making some comments in §5.

II. STOCHASTICALLY EXCITED SOLAR OSCILLATIONS

Lighthill (1952) shows that acoustic waves are due to acoustic quadrupole emissions and that only a small fraction of the turbulent energy can reach far outside from a source. Following the pioneering work on stratified atmospheres by Stein (1967), Goldreich & Kumar (1988, 1990) argued how the generation of acoustic waves by the turbulent convection is dominated by quadrupole emission because of destructive interference between monopole and dipole emission in the solar convection zone. For quadrupole emission at low Mach number the acoustic flux generation scales with the eighth power of the convective velocity. The turbulent eddy like the solar granule radiates as a quadrupole source at a rate

$$g \propto \rho u^3 l^2 M^{2k+1}, \tag{1}$$

with k=2, where ρ is the density, u and l are the characteristic velocity and length scale of the eddy respectively and M is the Mach number. Monopole radiation, $k=0$, occurs when there is mass source. Dipole radiation, $k=1$, is equivalent with a radiation due to external force. This strong velocity dependence implies that the source region of the acoustic flux is restricted to the narrow region where the convective velocity is close to its maximum. Thus, the acoustic flux is generated essentially in a very thin layer close to the solar surface, since the convective velocity increases towards the surface, as long as the solar luminosity is carried by the convection.

We model the stochastic excitation of the solar p modes by the turbulent convection as a damped harmonic oscillator excited by a random force. We set up the inhomogeneous wave equation that represents the p modes excited by the stochastic force. An incomplete understanding of the details of the turbulent convection may be a defect of this kind of model. However, as far as the purpose of this simulation is concerned, it should not be a serious matter. In other words, we have enough understanding, though not complete, about the turbulent convection for this purpose (Lee et al. 2001).

From the linearized perturbation equations governing oscillations of the Sun, assuming the Cowling approximation and ignoring the buoyancy frequency, N, one can begin with the forced wave equation modified by the acoustical cutoff frequency, ω_c, in order to derive the differential equation of a damped harmonic oscillator. The forced wave equation is given by

$$\left(\frac{\partial^2}{\partial t^2} + \omega_c^2\right)\Psi - c^2\nabla^2\Psi = \mathbf{F}(\mathbf{r}, t), \tag{2}$$

where $\mathbf{F}(\mathbf{r},t) = \Phi_k(t)\delta(r - R_\odot)\delta(\mu - \mu_k)\delta(\phi - \phi_k)$, δ represents a Dirac's δ-function, which can be identified as a forcing term due to a single granule k, c is the sound speed, $R_\odot$ is the position of granule in the radial direction, $\mu = \cos\theta$, θ being colatitude, and ϕ is an azimuthal angle. Since a typical size of a granule is much smaller than the characteristic wavelength of the p mode, the spatial variation of a granule can be safely ignored. The horizontal wave length of a mode is related to the degree l and the horizontal wave number k_h at radius r by

$$\lambda = \frac{2\pi}{k_h} = \frac{L}{r}, \tag{3}$$

where $L = \sqrt{l(l+1)}$. And we also assume the excitation occurs in a very thin layer near the surface as pointed out above. Therefore, the position of the granule k exciting a mode is located by $R_\odot, \mu_k$ and ϕ_k.

The growth and decay of a granule that causes the excitation is represented by a function, $\Phi_k(t)$, which is given by

$$\Phi_k(t) = \Lambda(t - t_k)^\alpha \exp(\frac{t - t_k}{\tau_k})^\beta, \tag{4}$$

where Λ is a constant, t_k is a time when an excitation begins, and α and β are constants which adjust a shape of the excitation function, $\Phi_k(t)$.

An arbitrary wave function $\Psi(\mathbf{r},t)$ can be expanded as

$$\Psi(\mathbf{r},t)=\sum_{nlm}A_{nlm}(t)\tilde{\Psi}_{nl}(r)Y_l^m(\theta,\phi), \quad (5)$$

where $\tilde{\Psi}_{nl}(r)$ is the radial part of eigenfunction, which is a solution of a homogeneous equation,

$$(\frac{\partial^2}{\partial t^2}+\omega_c^2)\Psi-c^2\nabla^2\Psi=0, \quad (6)$$

and $Y_l^m(\theta,\phi)=C_{lm}P_l^{|m|}(\mu)\cos m\phi$ or $C_{lm}P_l^{|m|}(\mu)\sin m\phi$, $P_l^{|m|}(\mu)$ being the Associated Legendre function, μ being $\cos\theta$, and C_{lm} being a normalization factor, n being the order of the mode, l being the degree, m being the azimuthal order of the mode. By substituting equation (5) into equation (2) and by inner product operation of spatial eigenfunctions with a different (n,l,m) mode one can derive

$$\ddot{A}_{nlm}+\omega_{nlm}^2A_{nlm}=\lambda'_{nlm}\Phi_k(t)\tilde{\Psi}_{nl}(R_\odot)P_l^{|m|}(\mu_k)\begin{pmatrix}\cos m\phi_k\\ \sin m\phi_k\end{pmatrix}, \quad (7)$$

where λ'_{nlm} is a constant due to the inner product operation, and ω_{nlm} is a frequency of the unforced free oscillator which is the one required from the inversion calculation. Here we assume that a damping term which would have been in equation (2) is a first order time derivative. In other words we assume that higher order derivatives are far less important than the first order term. Therefore we introduce a damping term in equation (7) such that equation (7) becomes

$$\ddot{A}_{nlm}+2\Gamma\dot{A}_{nlm}+\omega_{nlm}^2A_{nlm}=\lambda'_{nlm}\Phi_k(t)\tilde{\Psi}_{nl}(R_\odot)P_l^{|m|}(\mu_k)\begin{pmatrix}\cos m\phi_k\\ \sin m\phi_k\end{pmatrix}, \quad (8)$$

where Γ is a damping rate. Each $\Phi_k(t)$ has a finite width, and its amplitude is the fourth power of Gaussian variables such that its emission is proportional to the eighth power of the amplitude as the acoustic emission due to turbulence. Musielak et al. (1994) show that the dependence of the acoustic energy spectrum on the details of the turbulent energy spectrum is not sensitive. Therefore we take the Gaussian statistics because we do not have details of excitation and yet the Gaussian statistics is the most general. But raising such a high power is taken seriously since it may affect the final form of the forcing function and consequently the statistics of the amplitudes. In order to simulate stochastic excitation by many granules we add up in the force term many impulses which occur at different time t_k. The time interval $\Delta t_k\equiv t_{k+1}-t_k$ is given by independent Poisson variables. Therefore equation (8), which would represent the oscillations due to solar granules on the solar surface, finally reads

$$\ddot{A}_{nlm}+2\Gamma\dot{A}_{nlm}+\omega_{nlm}^2A_{nlm}=\lambda'_{nlm}\sum_k\Phi_k(t)\tilde{\Psi}_{nl}(R_\odot)P_l^{|m|}(\mu_k)\begin{pmatrix}\cos m\phi_k\\ \sin m\phi_k\end{pmatrix}. \quad (9)$$

Our artificial data consist of solutions of equation (9) with various frequencies at different life time and other parameters which we adopt from the observations.

III. DESCRIPTION OF THE SOLUTION

We solve equation (9) using the Green's function technique. A particular solution of equation (9) is therefore given by

$$A_{nlm}(t)=\tilde{\omega}^{-1}\exp(-\Gamma t)\int_{-\infty}^{t}\exp(-\Gamma t')\sin\tilde{\omega}(t-t')f(t')dt', \quad (10)$$

where $f(t)$ is the forcing term in the right-hand side in equation (9) and $\tilde{\omega}^2=\omega_{nlm}^2-\Gamma^2$. This technique has an advantage that the solution does not explicitly need an initial condition which we insufficiently know of. However, as the integral is to be performed from minus infinity to a particular time t, it is quite computationally expensive. In particular, since our simulation is aiming to add many source granules, a straight integral may cause serious computing time problem. We shall consider a method in the following section to represent the solution by an extremely cheaper way under a specific circumstance.

Now one may ask whether an expansion like equation (5) is adequate to represent the solution, or how many terms are required until the expansion is successfully converged to the solution. We have found that only a few hundred terms of n and l are enough to represent the solution. The dependence on n is much weaker than that on l. Therefore it is appropriate and effective to use the expansion to treat the problem. And it is also demonstrated that the function $A_{nlm}(t)$ can be given by

$$A_{nl}(t)=\begin{cases}0 & t<t_{\rm pro},\\ \exp(-\Gamma t)\cos(\tilde{\omega}_{nl}t+\delta) & t>t_{\rm pro},\end{cases} \quad (11)$$

where $t_{\rm pro}$ is the propagation time from the position of excitation to a position one observes the signal, and $\tilde{\omega}_{nl}$ and Γ are the frequency and the damping rate of the mode. We have tested how a signal propagates in an isothermal sphere. In an isothermal and homogeneous sphere the radial part of the eigenfunction is the spherical Bessel function. The result have been produced by a wave without damping. However, the main feature of the result does not change even when varying sound-speed and damping are taken into account. The signal is less sensitive to the number of terms in radial order n used in the expansion than it is to l. In other words, the expansion converges faster with respect to n than l. With increasing the number of angular degree, l, one can improve the resolution. It is not surprising because the angular degree is the one related to the surface structure. In practice, one truncates the expansion in another way, that is, in terms of eigen-frequency instead of the number of n and l. In geo-seismology the clear signal itself is required to simulate earthquakes. In a simulation in geo-seismology the expansion is truncated at fixed frequency, so that high frequency noise is filtered out naturally.

Another success of the expansion representation is that it is quite convenient to accommodate a spatial filter. For instance, we find that we need a small number of terms to generate signals in order to simulate whole-disk observation. To demonstrate that this is true, we have defined a spatial filter for a whole-disk observation. We fix the position of the source at the pole of the sphere in an arbitrary spherical coordinate system, which is regarded as fixed. Then we move the filter by the angles $\theta = \theta_0$, $\phi = \phi_0$, where θ and ϕ are measured in the fixed coordinate of the sphere, as the line-of-sight of an observer is inclined from the pole of the sphere by $\theta = \theta_0$. One may move the filter around and add signals observed through the filter. It is equivalent to the fact that one observes from a fixed position all the signals generated by various sources around the sphere. The filter from a direction of $\theta = \theta_0$, which is equivalent to an integration over the projected visible hemi-sphere from the direction of the filter, is given by

$$F_l = \frac{\int_0^\pi d\phi \int_0^{\frac{\pi}{2}-\theta_0} d\theta I_1 + \int_0^H d\phi \int_{\frac{\pi}{2}-\theta_0}^{\frac{\pi}{2}+\theta_0} d\theta I_1}{\int_0^\pi d\phi \int_0^{\frac{\pi}{2}-\theta_0} d\theta I_2 + \int_0^H d\phi \int_{\frac{\pi}{2}-\theta_0}^{\frac{\pi}{2}+\theta_0} d\theta I_2} \qquad 0 < \theta_0 \leq \pi/2 \qquad (12)$$

and

$$F_l = \frac{\int_0^H d\phi \int_{\theta_0-\pi/2}^{\frac{3\pi}{2}-\theta_0} d\theta I_1 + \int_0^\pi d\phi \int_{\frac{3\pi}{2}-\theta_0}^{\pi} d\theta I_1}{\int_0^H d\phi \int_{\theta_0-\pi/2}^{\frac{3\pi}{2}-\theta_0} d\theta I_2 + \int_0^\pi d\phi \int_{\frac{3\pi}{2}-\theta_0}^{\pi} d\theta I_2} \qquad \pi/2 < \theta_0 \leq \pi \qquad (13)$$

where

$$\begin{aligned} \cos H &= -\tan(\frac{\pi}{2} - \theta_0)/\tan\theta, \\ I_1 &= P_l(\cos\theta)\cos^2\gamma\sin\theta, \\ I_2 &= \cos\gamma\sin\theta, \\ \cos\gamma &= \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\phi - \phi_0) \quad (14) \end{aligned}$$

P_l being a Legendre polynomial. One can set $\phi_0 = 0$ without any loss of generality, since as far as ϕ_0 is concerned F_l is symmetric. We have tested that how fast $F_l S_l$ decays, where S_l is the signal. The decay is quite steep as expected, which implies that only a first few terms mainly contribute the spatially filtered signal for a whole-disk observation. In Fig. 1, we show the magnitude of a filtered mode $F_l S_l$ as a function of l in log-log scale.

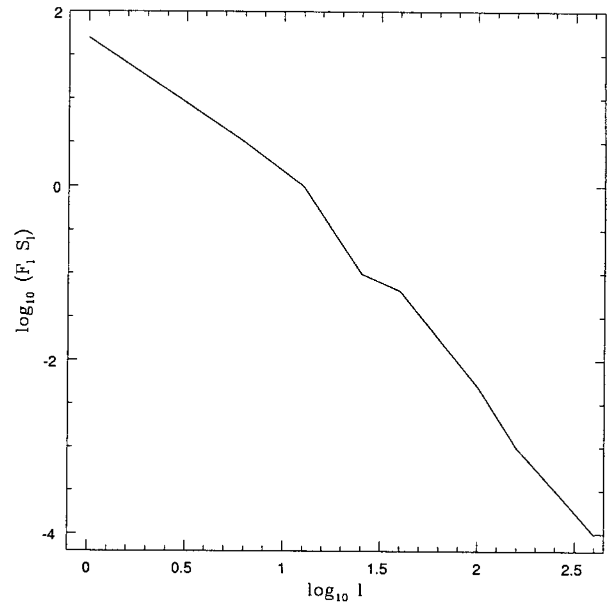

Fig. 1.— The magnitude of a filtered mode $F_l S_l$ as a function of l in log-log scale. The obtained slope from the least-square fit is -2.7.

IV. PARAMETERIZATION OF THE SUM OF SINUSOIDS

Formally a solution of the damped harmonic oscillator equation can be solved by the Green's function method which involves an integration from minus infinity to the present. Nonetheless, computationally it is quite expensive to generate time series data which represent a damped harmonic oscillator excited by a random force, when the Green's function technique is employed. A numerical integration itself is an expensive process. In order to avoid this expensive numerical integration one could divide the random force into a simpler form so that one can integrate it analytically. Note that the random force is itself a sum of analytical functions. Even if one could afford to have an analytical solution for a simple force term which comprises the random force, there are still many summing processes because the stochastic excitation is represented by the sum of all the effects of many single excitations.

Since the oscillator is being damped exponentially one could truncate a tail of an oscillator after some reasonably long time according to its required accuracy. However, there are still the enormous number of terms to add at a given time grid since the life-time of the mode in the solar case is very long, compared with the mean interval between the starts of excitations. The ratio of frequency to damping constant, ω_0/Γ, is a factor of thousands. Therefore it is impossible in practice to generate a long data set in a short time-scale to simulate the solar short-period oscillations by means of this direct summing process of analytical solutions.

One needs an efficient way to generate a data set, as long as at least the currently available data from real observations. Instead of adding transcendental functions directly, which is expensive, one can parameterize functions to be added and represent a new resulting function in terms of parameters of old functions. Let us consider the following equation:

$$\ddot{x} + 2\Gamma\dot{x} + \omega_0^2 x = f(t), \tag{15}$$

where Γ and ω_0 are damping rate and natural frequency of the mode respectively, $f(t) = \sum_j f_j(t-t_j)$, which is a stochastic function and each ingredient $f_j(t-t_j)$ has an analytical form which is sufficiently simple that one can solve the equation analytically. Since the equation is linear and each ingredient $f_j(t-t_j)$ is a simple analytical function, one can obtain $x_j(t-t_j)$ corresponding to $f_j(t-t_j)$ and subsequently calculate $x(t)$ by summing up $x_j(t-t_j)$. For the simulations, $f_j(t-t_j)$ is a product of exponential functions and polynomials as explained in the previous section.

Each solution $x_j(t-t_j)$ due to $f_j(t-t_j)$ can be written as $A(t)\cos(\omega t + \phi)$. We shall show how one can represent $A(t)$ and ϕ in terms of those in old functions to be added so that one can have a new resulting solution without an explicit adding operation. Let us consider two complex functions first,

$$\begin{aligned} \mathbf{x}_1 &= A_1 \exp[i(\omega t + \phi_1)], \\ \mathbf{x}_2 &= A_2 \exp[i(\omega t + \phi_2)], \end{aligned} \tag{16}$$

where A_1 and A_2 are slowly varying. Then the superposition of these two, $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2$, is expressed by

$$A\exp[i(\omega t + \phi)] = \Big(A_1\exp(i\phi_1) + A_2\exp(i\phi_2)\Big)\exp(i\omega t). \tag{17}$$

From this equation one can rewrite A and ϕ by

$$A = \Big[(A_1\cos\phi_1 + A_2\cos\phi_2)^2 + (A_1\sin\phi_1 + A_2\sin\phi_2)^2\Big]^{1/2} \tag{18}$$

$$\tan\phi = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}. \tag{19}$$

This can be simply shown in the phasor diagram.

The phasor of $\mathbf{x} = A\exp[i(\omega t + \phi)]$ is $A\exp(i\phi)$. Complex numbers can be represented by vectors in the phasor diagram, and they are added in the same way as the corresponding vectors. This fact can be used to show how a number of superposed oscillations which are not in phase with each other contribute to the sum. Since $\cos\phi$ is $\Re[\exp(i\phi)]$, one can use the phasor algebra to compute a new set of parameters. The resulting ϕ can be between 0 and 2π so that the same magnitude A can have two different phases, and so that the resulting A can be smaller than the greater of A_1 and A_2. Note that $|A_1 - A_2| < A < |A_1 + A_2|$.

V. SUMMARY AND DISCUSSION

The solar oscillation modes are generally accepted to be driven by the turbulent convection. A mode can be regarded as a damped simple harmonic oscillator. The strong velocity dependence of the acoustic emission implies that the source region of the acoustic flux is restricted to the narrow region in terms of the radius. Thus, the acoustic flux is generated essentially in a very thin layer close to the solar surface, and subsequently in the well-localised in the surface as well. On the other hand, there are of order of $10^6 - 10^7$ granules at a time on the whole surface of the Sun which last for a few 10 minutes (Sim et al. 2001). That is, solar p modes are experiencing about 10^6 kicks per period. Hence, regardless of the shape of the individual forcing function, those stochastic hits result in a smooth function rather than a sparse impulse-like function as commonly assumed. The equation of a damped simple harmonic oscillator which is excited by a random force of an arbitrary form can be solved by the formal Green's function technique. The practical problem of this approach is that it is computationally very much expensive though it may reflect all the physical properties. We have suggested one simple solution to this problem, that is, using the phasor diagram concept. One can parameterize functions to be added and represent a new resulting function in terms of parameters of old functions. In other words, the each solution due to a single granule can be represented in terms of those phasor parameters in old functions to be added so that one can have a new resulting solution without an explicit adding operation. With this approach of more realistic simulations the study concerning the amplitude, or even the correlation of the amplitudes, can be more reliable (cf. Roth 2001).

With the artificial data sets we generate by the method we suggest here, we have been investigating several aspects on the observational data, which will be discussed in elsewhere. Firstly, we are testing the claim that the life time of the mode can be determined by measuring the length of a bright spot obtained by the time-frequency analysis (Baudin et al. 1996). Different modes respond differently to the same excitation even when the force function is not particularly special. In other words, modes with different frequency react differently to a uniform force such that it looks that they are excited by independent forces. It becomes very clear when the contour is compared with the energy plot of the modes. Therefore it is of no use to measure the length of a bright spot to determine the life of the mode, since the bright spots are simply an interference pattern. It is not straightforward to estimate the life time by measuring the length of the bright spot, even in the case of $l = 0$ modes. Secondly, we are about to show that the excess in the distribution function of the energy is real and that our stochastic model could reproduce the excess. Then we ask what makes this excess in the stochastic process. One should bear in mind that our amplitude is raised by power of four, so that power is proportional to the eighth power of amplitude as a quadrupole emission. We believe the excess at higher energy tail is due to such a high power of the amplitude. Otherwise, one should expect a power-law behavior as a theory predicts, as long as the amplitude is the Gaussian. We expect that the excess in the distribution function of the energy is a normal behaviour of the stochastic process, and that it is caused by a high power force. This is a natural explanation of such an excess, instead of an exotic source for it, such as impacts of comets, or a sudden burst on the surface of the Sun (Chaplin et al. 1997; Foglizzo 1998). Lastly, we find that noisy patches are mainly due to the finite width of the forcing term. When we increased the characteristic width of our force, we had a much noisier power spectrum. This fact tells us that peaks in power spectrum between modes are not a pure noise. Those spiky peaks seem to contain much information about force terms. A wider forcing destroys the interference pattern of $l = 1$ modes quickly. Therefore, examining the pattern to determine the life of the modes is quite subject to the characteristic time of the force.

ACKNOWLEDGEMENTS

We would like to thank Douglas Gough, Takashi Sekii, Sasha Kosovichev, and Eva Novotny for useful discussions while the work is in progress, and the anonymous referee for critical comments which clarify points in the original manuscript.

REFERENCES

Ando, H., & Osaki, Y. 1975, Nonadiabatic nonradial oscillations - an application to the five-minute oscillation of the Sun, PASJ, 27, 581.

Ando, H., & Osaki, Y. 1977, The influence of the chromosphere and corona on the solar atmospheric oscillations, PASJ, 29, 221.

Bahcall, J. N., Gonzalez-Garcia, M. C., & Pena-Garay, C. 2003, Does the Sun shine by pp or CNO fusion reactions?, RhRvL, 90, 1301.

Baudin, F., Gabriel, A., Gibert, D., Palle, P. L., & Regulo, C. 1996, Temporal characteristics of solar p-modes, A&A, 311, 1024.

Chaplin, W. J., Elsworth, Y., Howe, R., Isaak, G. R., McLeod, C. P., Miller, B. A., & New, R. 1997, The observation and simulation of stochastically excited solar P modes, MNRAS, 287, 51.

Foglizzo, T. 1998, Are solar granules the only source of acoustic oscillations?, A&A, 339, 261.

Goldreich, P., & Keeley, D. 1977, Solar seismology. II - The stochastic excitation of the solar p-modes by turbulent convection, ApJ, 212, 243.

Goldreich, P., & Kumar, P. 1988, The interaction of acoustic radiation with turbulence, ApJ, 326, 462.

Goldreich, P., & Kumar, P. 1990, Wave generation by turbulent convection, ApJ, 363, 694.

Goldreich, P., Murray, N., & Kumar, P. 1994, Excitation of solar p-modes, ApJ, 424, 466.

Gough, D., & Toomre, J. 1991, Seismic observations of the solar interior, ARA&A, 29, 627.

Kumar, P., Franklin, J., & Goldreich, P. 1988, Distribution functions for the time-averaged energies of stochastically excited solar p-modes, ApJ, 328, 879.

Lee, H., Ryu, D., Kim, J., & Jones, T. W. 2001, Transonic magnetohydrodynamic turbulence, JKAS, 34, 321.

Leibacher, J., & Stein, R. 1971, A New description of the solar five-minute oscillation, ApL, 7, 191.

Leighton, R., Noyes, R., & Simon, G. 1962, Velocity fields in the solar atmosphere. I. preliminary report, ApJ, 135, 474.

Libbrecht, K. 1988a, The excitation and damping of solar oscillations, In *Seismology of the Sun and Sun-like Stars*, ed. E. J. Rolfe, ESA SP-286, p. 3.

Libbrecht, K. 1988b, Solar p-mode phenomenology, ApJ, 334, 510.

Libbrecht, K., Popp, B., & Kaufman, J. 1986, The excitation and damping of solar oscillations, Nature, 323, 235.

Lighthill, M. 1952, On sound generated aerodynamically. I. general theory, Proc. Roy. Soc. London , A211, 564.

Musielak, Z., Rosner, R., Stein, R., & Ulmschneider, P. 1994, On sound generation by turbulent convection: A new look at old results, ApJ, 423, 474.

Roth, M. 2001, Anticorrelation coupleing p-modes, ApJ, 559, 1165.

Schou, J. & Brown, T. 1994, Generation of artificial helioseismic time-series, Astron. Astrophys. Suppl., 107, 541.

Sim, K. J., Moon, Y.-J., Lee, C.-W., Chang, B. H., & Woo, H. S. 2001, The relative sunspot numbers in 2000, JKAS, 34, 119.

Stein, R. 1967, Generation of acoustic and gravity waves by turbulence in an isothermal stratified atmosphere, Solar Phys., 2 , 385.

Ulrich, R. 1970,The five-minute oscillations on the solar surface, ApJ, 162, 993.