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## Performance Analysis of Strapdown Inertial Navigation System

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### Abstract

The purpose of this paper is to show and define the performance, the system mechanization and the algorithm of the Strapdown Inertial Navigation System(SDINS). First, navigation equations are derived in the Earth Fixed mechanization and this mechanization apply to the two kinds of inertial measurement units which consist of same fiber optic gyros and different accelerometers(SDINS-1 and SDINS-2). Those two accelerometers have the different bias. To evaluate its performance, two kinds of tests have been performed - static motionless test, and rectangle-route moving test. The results of the moving test are compared with the results of Differential GPS which has an accuracy with  $\pm 2.0\text{mm}$ . and are presented in this paper.

가 , 가 가  
 가 , FOG(Fiber Optic Gyro) (SDINS-1,  
 SDINS-2). 가 가 , 3  
 , 20 mm DGPS(Differential GPS)  
 : (inertial navigation system),  
 (strapdown INS), (differential GPS), (attitude  
 determination), (quaternion)

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### 1.

(Strapdown INS, SDINS)

1950

1966

Honeywell

PRIME  
1969 Apollo

가

가

가

(gimbal)

가

가

가

(Inertia Measurement Unit, IMU),  
(Electronic Unit, EU),  
(Navigation Computer Unit, NCU)

20mm DGPS (Differential GPS)

Laser Gyro

가

Fiber Optic Gyro (FOG)

## 2. Strapdown INS

1

(rotational motion)

(a<sub>b</sub>) 가 (gravity field) 가

(translation

g

motion)

가 (a<sub>b</sub>)  
가 (a<sub>n</sub>)  
(V) (R)

가

가

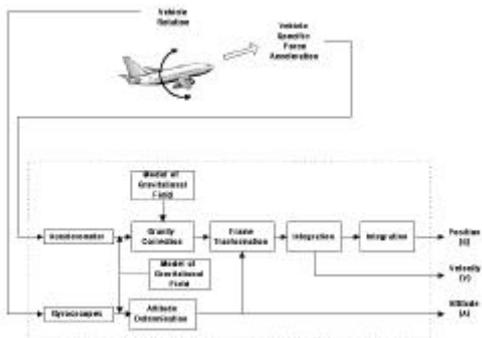
x, y, z

### 2.1

(quaternion, ) , 1

3

, singularity 가



1. SDINS

$$\lambda = \begin{bmatrix} \alpha \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} \cos \frac{\phi}{2} \\ \sin \frac{\phi}{2} \cdot \mathbf{n} \end{bmatrix} \quad (1)$$

$\lambda^T \cdot \lambda = 1$   
 , Rodrigez formula (2)  
 (Direction Cosine Matrix, A)

$$A(\lambda) = (\alpha^T - q^T \cdot q) \cdot I + 2[q \cdot q^T - \alpha \cdot q \times] \quad (2)$$

(t) , (3)

$$\frac{d}{dt} A = -\omega \times A, \quad \frac{d}{dt} \lambda = \frac{1}{2} \Phi(\omega) \cdot \lambda \quad (3)$$

$$\Phi(\omega) = \begin{bmatrix} 0 & -\omega^T \\ \omega & -\omega \times \end{bmatrix}, \quad \omega \times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad (4)$$

(4) Peano series

(5)

$$\Phi(\delta \lambda) = I + \frac{1}{2} \int_0^1 \Phi(\omega(\tau_1)) d\tau_1 + \frac{1}{4} \int_0^1 \Phi(\omega(\tau_1)) \int_0^1 \Phi(\omega(\tau_2)) d\tau_2 d\tau_1 + \dots$$

$$\delta \lambda = \left[ \frac{1}{2} \int_0^1 \omega(\tau_1) d\tau_1 \right] + \frac{1}{4} \int_0^1 \Phi(\omega(\tau_1)) \int_0^1 \omega(\tau_2) d\tau_2 d\tau_1 + \dots \quad (5)$$

2.2

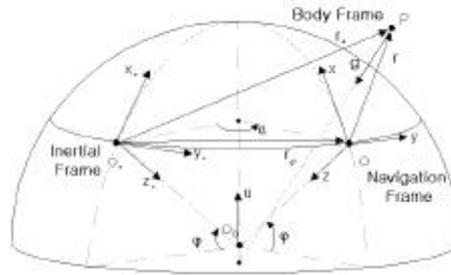
$$\dots \quad (10)$$

가

(inertial frame,  $O_{*x_*y_*}$ )

(navigation frame,  $Oxyz$ )

(body frame, P) 2



2.

( $u_e$ ) ( $v_e$ ), 가 ( $w_e$ )

$$u_e = \Omega [\cos \varphi \quad 0 \quad -\sin \varphi]^T \quad (6)$$

$$v_e = R \cdot \Omega \cdot \cos \varphi [0 \quad 1 \quad 0]^T \quad (7)$$

$$w_e = u_e \times v_e = R \cdot \Omega^2 \cdot \cos \varphi [\sin \varphi \quad 0 \quad \cos \varphi]^T \quad (8)$$

(latitude),

, R

(A)

$$\lambda^{\ddot{}}(t) = \frac{1}{2} \Phi(\omega) \cdot \lambda(t), \quad A^{\ddot{}}(t) = -\omega \times A(t) \quad (9)$$

$$v_e^{\ddot{}}(t) = A^T(t) a(t) + g(r_e), \quad r_e^{\ddot{}}(t) = v_e(t) \quad (10)$$

$r^*$ ,  $v^*$

(Law of Coriolis)

$$v_e(t_0) = v(t_0) + u_e \times r(t_0) + v_e, \quad r_e(t_0) = r(t_0) \quad (11)$$

$$\mathbf{r}_i(t) = \mathbf{r}_i(t_0) + \Delta t \cdot \mathbf{v}_i(t_0) + \mathbf{r}_i(t) \quad (11)$$

$$\mathbf{v}_i(t) = \mathbf{v}_i(t_0) + \mathbf{v}_i(t) \quad (12)$$

$$\mathbf{v}_i(t) = \int_{t_0}^t [\mathbf{A}^T(\tau)\mathbf{a}(\tau) + \mathbf{g}(\mathbf{r}_i, \tau)] d\tau \quad (13)$$

### 3.

(SDINS)

(H/ W)

( 가 )

(Inertial Measurement System,

IMU)

가

( )

#### 3.1

x, y, z

3 1

FOG 3 1 가 (A-12) 3 3 가  
(CXL02LF3) mounting

KVH 1 FOG (model no:RA2100) 3

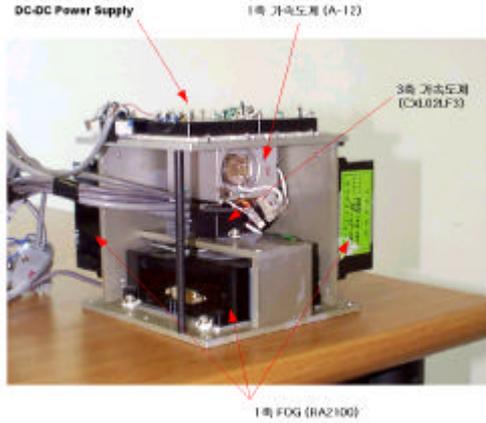
FOG 1 interferometric

FOG

+18V), 170mA DC +12V(+9 ±100. / s

0.34 kg ,

-40 +70



3. (IMU)

#### 1. FOG

Performance		RA2100	Unit
Performance	Range	± 100	° / sec
	Resolution Rate	0.014	° / sec
	Scale Factor	20	mV/ ° / sec
	Nonlinearity	0.5	%, rms
	Full TsmP	2	%, p-p
	Bias Stability	0.002	° / sec, 1
	Angle Random Walk(noise)	5	° / hr/ rt-Hz
	Bandwidth	100	Hz
	Turn on Time	1	sec
Electrical	Input Voltage	+9 +18	Vdc
	Sensor Output	20	mV/ deg/ s
Physical	Weight	0.34	Kg
	Operating Temperature	-40 +75	
	Size	112 × 108 × 41	mm

2. 3 가 (CXL02LF3)

Performance		CXL02LF3	Unit
Perforamnce	Range	± 2	g
	Sensitivity	1	V/ g
	Bandwidth	125	Hz
	Noise	1.5	mg, rms
	Noise density	130	μg/ √Hz
	Nonlinearity	± 1	% FS
	Zero g-output	2.5 ± 0.15	mV
Environmental	Temperature Range	-40 +85	
	Temperature Sensitivity	± 3.5	% FS
	Shock	2000	g
	Input Voltage	5	Vdc

Environmental	Zero Signal Drift (during one hour)	< 25 μg
	Torquer Resistance	150
	Operational Temperature	75 ± 2
	Temperature Range	-60 +80
	Shock	7g, 16 20 ms
	Weight	38 gram

가 가  
 , 가  
 Corssbow CXL02LF3 ,  
 가 Ramenskoye  
 Design Bureau A-12 .  
 CXL02LF3 silicon micro-machined  
 ±2g 가 , ±30mg  
 . DC 5V, 12mA ,  
 2.5V ±2V .  
 2 CXL02LF3 3 가

가 (A-12) 1  
 torque- feedback 가  
 . 가  
 ±15V DC, 50 mA ,  
 ±20g 0.5mg .  
 3.6 가  
 가 . 3 1 가  
 . 가

3. 1 가 (A-12)

TSF	Nominal Torquer Scale Factor (TSF)	1.7 ± 0.5 mA/ g
	TSF Repeatability (during one year)	< 200 ppm
	TSF-non linearity	< 30 μg/ g <sup>2</sup>
	TSF-temperature sensitivity	< 20 ppm/
IAE	Input Axis Error (IAE) (direct component)	< 10 mR
	IAE Repeatability (during one year)	< 100 μR
	IAE temperature sensitivity	< 25 μR/
Bias	Bias (direct component)	< 17 mg
	Bias Repeatability (during one year)	< 0.3 g
	Bias temperature sensitivity	< 30 μg/

32 AD 가 16-bit (resolution)

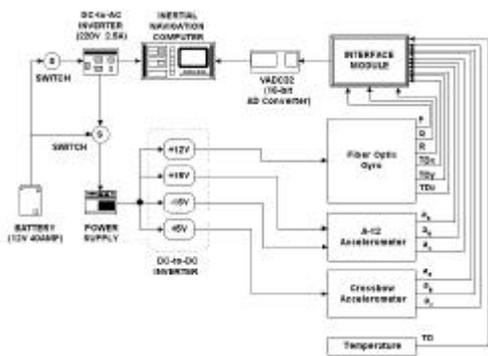


4.

AD Industrial/ Rugged 가  
 PC Or Industrial Computer 0  
 . ( 4) 1 20mm Differential  
 PC Internal Clock GPS  
 (sampling frequency)  
 20, 25, 40, 50, 67, 100Hz

4.1

5 3 3 , SDINS-1 SDINS-2 3 6  
 A-12 가 CXL02LF3 가 3  
 가 6 1 = = = 0° (north)  
 16 3



5.

4. 가

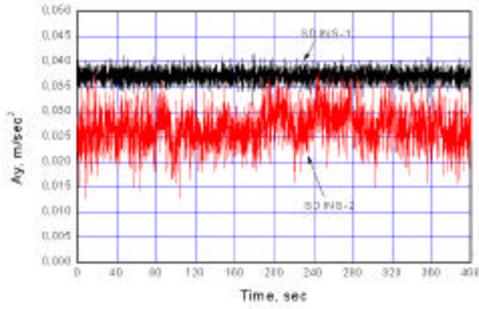
(SDINS) 가 3 (3-axis motion table) (static test) (trajectory moving test) 가  
 가 (A-12) FOG 가  
 SDINS-1 (CXL02LF3) FOG 가



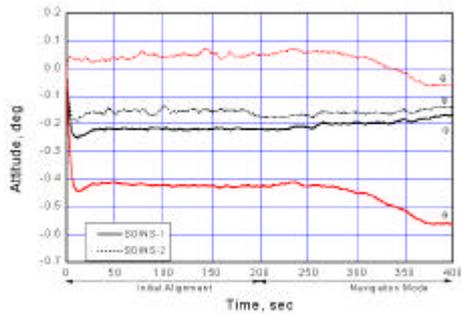
6. 3

( )

1 1



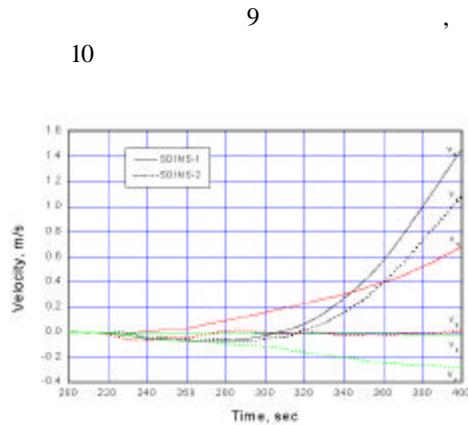
7. 가 ( )



8. ( )

4.

Attitude	SDINS-1	
	Mean(deg)	Std(deg)
Roll( )	-0.2171	0.00137
Pitch( )	-0.4255	0.00125
Attitude	SDINS-2	
	Mean(deg)	Std(deg)
Roll( )	-0.1633	0.0142
Pitch( )	0.0342	0.0113



9. ( )

8

200

SDINS-1 -0.21

7°, 0.00137°, SDINS-2

-0.163°, 0.0142°

SDINS-1 -0.426°, 0.0012

5°, SDINS-2 0.032°, 0.0113°

SDINS-1 가 가 10

5.

Velocity	SDINS-1	
	Mean(m/s)	Std(m/s)
$v_x$	-0.451	2.392
$v_y$	0.213	1.188
$v_z$	0.005	0.021
$\sqrt{v_x^2 + v_y^2}$	2.363	1.059
Velocity	SDINS-2	
	Mean(m/s)	Std(m/s)
$v_x$	-0.554	2.456
$v_y$	0.070	1.392
$v_z$	0.011	0.262
$\sqrt{v_x^2 + v_y^2}$	2.489	1.160

3

SDINS-1 0.02

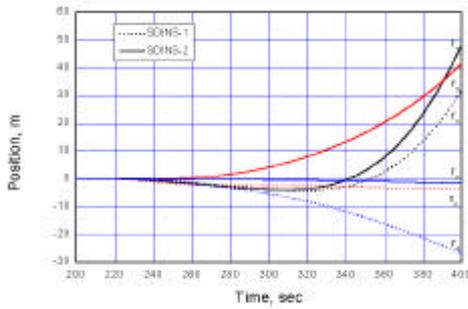
1°, SDINS-2 0.014°, 0.029°, 0.035°

가 10

4

3 SDINS-1  
 $v_x=0.451\text{m/s}$ ,  $v_y=0.2131\text{m/s}$ ,  $v_z=0.005\text{m/s}$   
 , SDINS-2  $v_x=-0.554\text{m/s}$ ,  $v_y=$   
 $0.070\text{m/s}$ ,  $v_z=0.011\text{m/s}$   
 $(\sqrt{v_x^2 + v_y^2})$  2.363 m/s,  
 2.489m/s 가 ,

x y 가 가  
 가 가 가  
 , FOG  
 ,  
 가 가  
 0 가 가  
 가 가  
 10 6 10



10. ( )

가  
 FOG  $(\sqrt{r_x^2 + r_y^2})$  2.0cm  
 SDINS-1 127.977m SDINS-2  
 136.60m  
 , SDINS-1  
 가  
 z  
 1.508m, 23.669m 가 12  
 z 가  
 SDINS-1  
 가 가 ,  
 z 가

6.

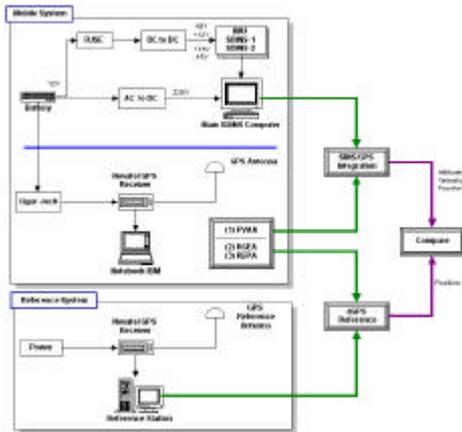
Position	SDINS-1	
	Mean(m)	Std(m)
$r_x$	-27.111	131.187
$r_y$	11.977	66.664
$r_z$	0.430	1.508
$\sqrt{r_x^2 + r_y^2}$	127.977	66.298
Position	SDINS-2	
	Mean(m)	Std(m)
$r_x$	-31.111	137.323
$r_y$	-1.444	83.480
$r_z$	0.3444	23.669
$\sqrt{r_x^2 + r_y^2}$	136.60	76.694

4.2

1.5km , 7 가  
 , 200 ,

Differential GPS  
 ,  
 Differential GPS  
 ,  
 GPS

11 ,  
 12  
 ,  
 GPS 13, 14 .



11.



14. GPS

4.3

(starting point, A)

( =90° )

(F)



12.

15

SDINS-2 D , SDINS-1

DGPS

SDINS-1

16

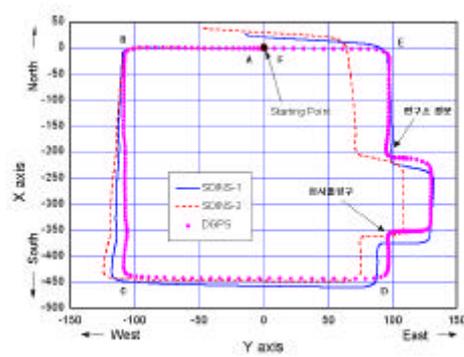
SDINS-1

SDINS-2

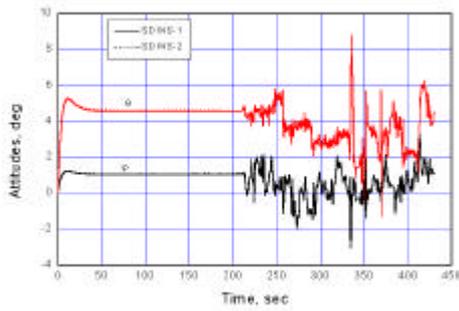
- SDINS-1 : =1.053°, =-4.577°, =-90.011°
- SDINS-2 : =1.063°, =-4.576°, =-90.004°



13.



15.



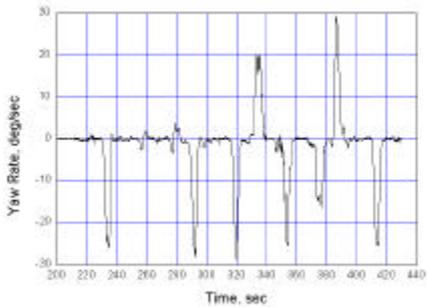
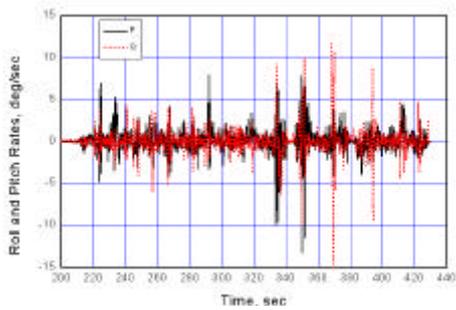
16.

16 330 , 350 , 370

17

18

가



$\pm x$  17.

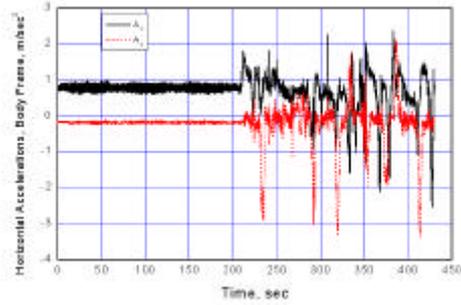
( )

19

B, C, D, E

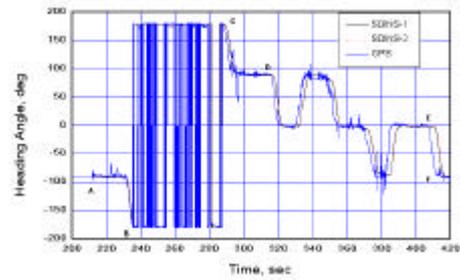
D-E 4

$=0^\circ$ ,  $=180^\circ$ ,  $=9^\circ$   
 $0^\circ$ ,  $=-90^\circ$



18.

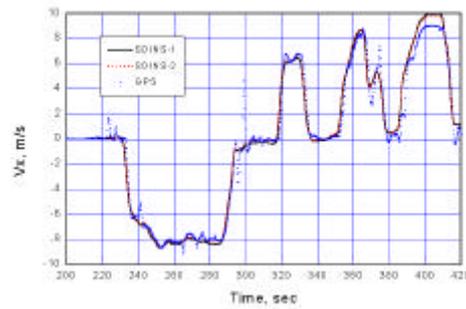
가



19.

15 x, y, z

, 20  
 , 21





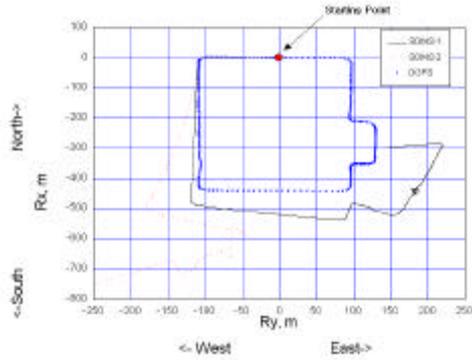
30.363m, SDINS-2                      64.379m  
 SDINS-1                      가  
 7                      DGPS                      2

(starting point)

, SDINS-

7.

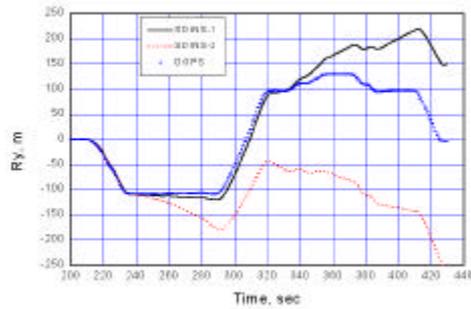
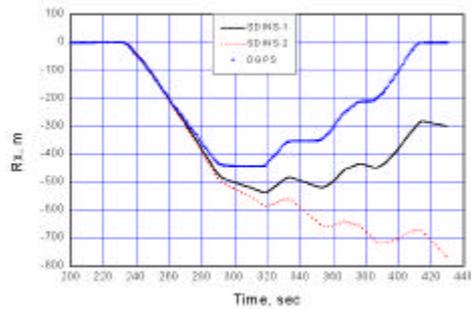
		SDINS-1		
		$r_x$ (m)	$r_y$ (m)	$r_z$ (m)
A-B		0.0488	1.5756	-0.0770
		0.4577	2.0482	0.1077
B-C		1.8607	-2.4181	-0.4308
		3.1230	2.9502	0.5231
C-D		-6.7609	-7.8743	-0.7574
		7.1503	8.1004	0.7659
D-E		-11.6316	-0.3055	1.4679
		12.2182	3.7464	2.5151
E-F		8.0527	1.8773	5.5804
		9.9199	5.6787	5.6103
		SDINS-2		
		$r_x$ (m)	$r_y$ (m)	$r_z$ (m)
A-B		0.0720	1.8190	0.4411
		0.4549	2.2733	0.5955
B-C		2.6238	-3.7402	4.8898
		3.2018	4.9936	5.6045
C-D		-24.4912	-12.2858	7.8086
		2.7262	12.5147	7.8195
D-E		-3.8055	-11.5774	5.4438
		5.3535	11.5774	6.8384
E-F		15.2026	-15.0228	-1.7298
		15.8335	16.1613	1.8716



22.

23                      x, y, z

24



23.

(                      )

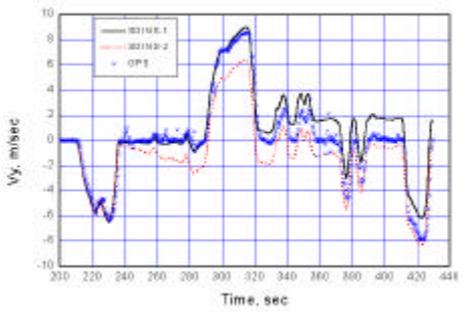
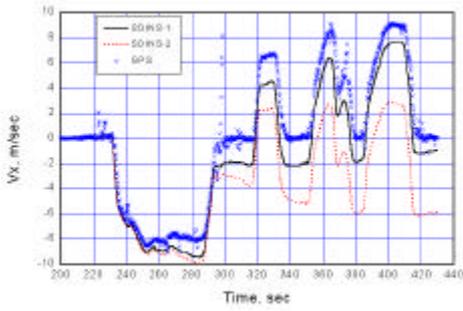
4.4

가

가

22

SDINS-2



±x 24. ( )

5.

KARI-TN-FCGD-1999-001

(Development

Algorithm for Strapdown Inertial Navigation System) (1999 9 )

(Differential GPS) 가

가 가 (A-12)

FOG (SDINS-1) 가  
(CXL02LF3) FOG (SDINS-2)

SDINS 0.1 0.2 °, 0.1 0.4 °  
, 3 SDINS-1  
v<sub>x</sub>=-0.451m/s, v<sub>y</sub>=0.2131m/s, v<sub>z</sub>=0.005m/s  
, SDINS-2 v<sub>x</sub>=-0.554m/s,  
v<sub>y</sub>=-0.070m/s, v<sub>z</sub>=0.011m/s  
( (√(r<sub>x</sub><sup>2</sup>+r<sub>y</sub><sup>2</sup>)) ) SDINS-1  
127.977m SDINS-2 136.60m

SDINS-1 가

0.042 ° (SDINS-1), 0.049 ° (SDINS-2)  
, 0.09 °  
(SDINS-1), 0.102 ° (SDINS-2)

SDINS-1 v<sub>x</sub>, v<sub>y</sub>, v<sub>z</sub>  
1.414, -0.670, 0.241m/s , SDINS-2 1.433,  
-0.860, -0.284m/s  
SDINS-1 r<sub>x</sub>, r<sub>y</sub>, r<sub>z</sub> 27.078, -13.737, 12.668m  
, SDINS-2 41.471, -49.243, -5.455m

1. , , , , , KARI-FCGD-TN-1999001-v.1-rev.1, 1999
2. , , , , , KARI-AD-TN-2000001-v.1- rev.1, 1999
3. 8 , (GNSS)  
2 , & , 1999.12

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