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Warranty Models with Discrete Preventive Maintenance

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이산예방보수 정책을 고려한 보증모형분석

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Products which are sold with warranty, preventive maintenance actions by manufacturers and/or buyers have an impact on the total costs for both parties. In this paper, we develop the models to study the expected warranty cost for products with free repairable warranty with three types of discrete preventive maintenance. We deal with by utilizing the concept that preventive maintenance reduces the virtual age of the system. We assume that the maintenance planning horizon can be segmented into k discrete and equally sized periods. In such a scenario, numerical examples are presented.

Keywords: warranty, discrete preventive maintenance, virtual age, age reduction

1. Introduction

All products are unreliable in the sense that they eventually fail. An item failure can occur early in its life due to manufacturing defects or late in its life due to degradation of the item. The degradation is dependent on age and usage. Most products are sold with a warranty that offers protection to buyers against early failures over the warranty period. The warranty period offered has been progressively getting longer. For example, the warranty period for cars was three months in the early thirties and this changed to one year in the sixties and currently it varies from three to five years. With extended warranties, an item is covered for a significant part of its useful life. This implies that failures due to degradation can occur within the warranty period. The degradation of item can be controlled by preventive maintenance and this reduces the likelihood of failures. This implies that preventive maintenance becomes important when warranty periods are long.

Offering warranty implies additional costs to the manufacturer. This is the cost of repairing item failures (through corrective maintenance) over the warranty period. Preventive maintenance during the warranty period can reduce this cost. Since the buyer pays nothing for repairs during the warranty period, there is no incentive for him/her to invest any effort into preventive maintenance.

It is worthwhile for the manufacturer to carry out preventive maintenance only if the reduction in the warranty servicing cost is greater than the extra cost incurred with preventive maintenance. However, from the buyer's perspective, investment in preventive maintenance during the warranty period and after the warranty has expired can have a significant impact on the maintenance cost after the warranty

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has expired which is borne by the buyer. As a result, buyer's preventive maintenance actions (during the warranty period and afterwards) needs to be determined in the life cycle context.

In this paper we develop a framework to study preventive maintenance for items sold with warranty from both buyer and manufacturer perspective. Such a framework allows one to build alternate models to determine optimal preventive maintenance strategies. We carry out a literature review of models dealing with warranty and maintenance against this framework and then propose a new model formulation and carry out its analysis.

The outline of the paper is as follows. In Section 2, we give a brief overview of product warranty and maintenance so as to set the background for the main contribution of the paper. Following this, we develop a framework to study preventive maintenance for items sold with warranty in Section 3. In section 4, we carry out a review of the literature dealing with warranty and maintenance against this framework. Section 5 deals with a new model formulation and its analysis. Finally, we conclude with some discussions on topics for research in the future.

2. Warranties and Maintenance

2.1 Product Warranties

A warranty is a contract between buyer and manufacturer that becomes effective on the sale of an item. The purpose of a warranty is basically to establish liability in the event of a premature failure of an item, where "failure" is meant as the inability of the item to perform its intended function. The contract specifies the promised performance and if this is not met the means for the buyer to be compensated. The contract also specifies the buyer's responsibilities with regards to due care and operation of the purchased item.

There are many different types of warranties and they depend on the type of product. Products can be broadly divided into three categories—consumer durables, commercial and industrial products, and defense acquisition. For consumer durables, the most common warranties are the free replacement warranty, the pro-rata warranty and combinations of these two policies. In the free replacement warranty, failed items are either repaired or replaced by new ones at no cost to the buyer, and in the pro-rata warranty, it involves replacement at pro-rated cost. The above policies are also offered with commercial and industrial products sold individually. However, when they are sold in lots then cumulative and fleet warranties cover the lot as a whole as opposed to separate warranties for each item. The advantage of this is that it reduces the cost of administering the warranty. Warranties for items procured by the government include all of the above plus some special warranties, particularly in acquisition of defense products. The best known of these special warranties is the Reliability Improvement Warranty, which includes provisions for product development and improvement subsequent to the sale. For a taxonomy for warranty policies, see, Blischke and Murthy (1994).

There are many aspects to warranty. These have been studied extensively by researchers from different disciplines. Blischke and Murthy (1996) deal with these different aspects for consumer durables and industrial and commercial products. For more on RIW policies see Blischke and Murthy (1994). In this section we focus our attention on the cost analysis of warranties as they are relevant for later sections of the paper.

There are many issues involved in the cost analysis of a warranty. Two of these are—the perspective (buyer or manufacturer as the costs are different for each) and the basis on which the costs are to be assessed. There are a number of approaches to the costing of warranty. The following are some of the methods for calculating costs:

Cost to the manufacturer, per item sold. This per unit cost may be calculated as the total cost of warranty, as determined by general principles of accounting, divided by number of items sold.

Cost per item to the buyer, averaging over all items purchased plus those obtained free or at reduced price under warranty.

Life cycle cost of ownership of an item with or without warranty, including purchase price, operating and maintenance cost, etc., and finally including cost of disposal.

Life cycle cost of an item and its replacements, whether purchased at full price or replaced under warranty, over a fixed time horizon.

Cost per unit of time.

The selection of an appropriate cost basis depends on the product, the context and perspective. The type of customer, individual, corporation or government is important, as are many other factors. Cost models must be developed separately for manufacturer and buyer. Murthy and Blischke (1994) deal with the cost analysis for several warranty policies.

2.2 Maintenance

Maintenance can be defined as actions to (i) control the deterioration process leading to failure of a system and (ii) restore the system to its operational state through corrective actions after a failure. The former is called "preventive" maintenance and the latter "corrective" maintenance.

Corrective maintenance (CM) actions are unscheduled actions intended to restore a system from a failed state to a working state. This involves either repair or replacement of failed components. In contrast, preventive maintenance actions are scheduled actions carried out to either reduce the likelihood of a failure or prolong the life of the component. Preventive maintenance (PM) actions are divided into the following categories:

Clock-based maintenance: PM actions are carried out at set times. An example of this is the "Block replacement" policy.

Age-based maintenance: PM actions are based on the age of the component. An example of this is the "age replacement" policy.

Usage-based maintenance: PM actions are based on usage of the product. This is appropriate for items such as tires, components of an aircraft, and so forth.

Condition-based maintenance: PM actions are based on the condition of the component being maintained. This involves monitoring of one or more variables characterizing the wear process (eg, crack growth in a mechanical component). It is often difficult to measure the variable of interest directly and in this case, some other variable may be used to obtain estimates of the variable of interest. For example, the bearing wear in an engine can be measured by dismantling the bearing case of the engine. However, measuring the vibration, noise or temperature of the bearing case provides information about wear since there is a strong correlation between these variables and bearing wear.

Opportunity-based maintenance: This is applicable for multicomponent systems, where maintenance actions (PM or CM) for a component provides an opportunity for carrying out PM actions on one or more of the remaining components of the system.

Design-out maintenance: This involves carrying out modifications through re-designing the component. As a result, the new component has better reliability characteristics.

In general, preventive maintenance is carried out at discrete time instants. In cases where they are done fairly frequently (the mean time between maintenance actions is $<<$ the life of the item) then they can be approximated as occurring continuously over time. This results in the modeling and analysis becoming simpler.

Several review papers on maintenance have appeared over the last 30 years. These include McCall (1965), Pierskalla and Voelker (1976), Sherif and Smith (1976), Monahan (1982), Jardine and Buzzacot (1985), Thomas (1986), Gits (1986), Valdez-Flores and Feldman (1989), Pintelon and Gelders (1992), Dekker (1996) and Scarf (1997). Cho and Parlar (1991) and Dekker *et al.* (1997) deal with the maintenance of multi-component systems. These contain references to the large number of papers and books dealing with maintenance.

3. Framework for the Study of Warranty and Maintenance

As mentioned earlier, the cost analysis for the manufacturer is different from that for the buyer. For the manufacturer, it is the cost of servicing the warranty over the warranty period. This depends on the type of warranty offered. In the case of non-renewing warranty, the warranty period is fixed. In the case of renewing warranty, the warranty period is a random variable. The warranty servicing cost to the manufacturer is the cost of rectifying all failures over the warranty period. Blischke and Murthy (1994) deal with this topic in great detail.

From the buyer's perspective, the time interval of interest is from the instant an item is purchased to the instant when it is disposed or replaced. This includes the warranty period and the post-warranty period. The cost of rectification over the warranty period depends on the type of warranty. It can vary from no cost (in the case of free replacement warranty) to cost sharing (in the case of pro rata warranty). The cost of rectification during the post-warranty period is borne completely by the buyer. As such, the variable of interest to the buyer is the cost of maintaining an item over its useful life.

Preventive maintenance actions are carried out to either reduce the likelihood of a failure or to prolong the life of an item. Preventive maintenance can be perfect (restoring the item to "good-as-new") or imperfect (restoring the item to a condition that is between as "good-as new" and as "bad-as-old").

Corrective maintenance can be either minimal (repairing to back-as-old), imperfect or perfect as indicated earlier.

Preventive maintenance over the warranty period has an impact on the warranty servicing cost. It is worthwhile for the manufacturer to carry out this maintenance only if the reduction in the warranty cost exceeds the cost of preventive maintenance. From a buyer's perspective, a myopic buyer might decide not to invest in any preventive maintenance over the warranty period as failures over this period are rectified by the manufacturer at no cost to the buyer. Investing in maintenance is viewed as an additional unnecessary cost. However, from a life cycle perspective the total life cycle cost to the buyer is influenced by maintenance actions during the warranty period and the post warranty period. This implies that the buyer needs to evaluate the cost under different scenarios for preventive maintenance actions. This raises several interesting questions. These include the following:

1. Should preventive maintenance be used during the warranty period?
2. If so, what should be the optimal maintenance effort? Should the buyer or the manufacturer pay for this or should it be shared?
3. What level of maintenance should the buyer use during the post warranty period?

Preventive maintenance actions are normally scheduled and carried out at discrete time instant. When the preventive maintenance is carried out frequently and the time between the two successive maintenance actions is small, then one can treat the maintenance effort as being continuous over time. This leads to two different ways (discrete and continuous) of modeling maintenance effort.

Another complicating factor is the information aspect. This relates to various issues such as the state of item, type of distribution function appropriate for modeling failures, parameters of the distribution function etc. The two extreme situations are complete information and no information. Often, the information available to manufacturer and buyer is somewhere in between these two extremes and can differ. This raises several interesting issues such as the adverse selection and moral hazard problems. Quality variations (so that all items are not statistically similar) add yet another dimension to the complexity.

As such, effective study of preventive maintenance for products sold under warranty requires a framework that incorporates the factors discussed above. The number of factors considered and the nature of their characterization results in many different model formulations linking preventive maintenance and warranty. In the next section we review the models that have appeared in the literature.

4. Review of Literature Linking Warranty and Maintenance

In this section we carry out a chronological review of models dealing with warranty and preventive maintenance.

Ritchken and Fuh (1986) discuss an age replacement policy for a non-repairable item. The warranty offered is the pro-rata policy, hence any failure within the warranty period results in a replacement by a new one with the associated cost shared by the producer and the buyer. At the end of the warranty period, the item in use is preventively replaced after a period T (measured from the end of the warranty period) or on failure should it occur earlier. The optimal T^* is selected by minimizing the buyer's asymptotic expected cost per unit time using the renewal reward theorem.

Chun and Lee (1992) consider a model of a system with an increasing failure rate and subjected to periodic preventive maintenance actions during warranty period and after the warranty expired. They assume that the preventive maintenance is imperfect, that is the failure rate after maintenance is lower than that before maintenance but not as good-as-new. The hazard rate reduction is assumed to be equivalent to the reduction of the age of the system at preventive maintenance. The reduction is assumed to be the same for each maintenance action regardless of the age of the system and of a warranty period. The costs to the buyer consist of price of the system, cost during warranty period (portion of the preventive maintenance cost) and cost after the warranty has expired (all the maintenance costs). Any failures between preventive maintenance actions during the warranty period are repaired minimally by the manufacturer at no cost to the buyer. The optimal period between preventive maintenance actions is obtained by minimizing the buyer's asymptotic expected cost per unit time over an infinite period. An example is given for a system with Weibull failure distribution.

Chun (1992) deals with a model similar to the one in Chun and Lee (1992) but the focus is on the warranty cost to the manufacturer as opposed to the buyer. The optimal number of preventive maintenance actions, N^*, is obtained by minimizing the warranty cost over a finite horizon.

Jack and Dagpunar (1994) deal with the model studied by Chun (1992). They show that when the product has an increasing failure rate, a strict periodic policy for preventive maintenance action is not the optimal strategy. As a result, time intervals between successive preventive maintenance actions should not be identical. They derive the optimal preventive maintenance strategies to minimize the manufacturer's expected warranty cost over the warranty period. They show that for the policy to be strictly periodic, the preventive maintenance action must result in the product being restored to as good as new.

Dagpunar and Jack (1994) deal with a model similar to that in Jack and Dagpunar (1994). The cost of each preventive maintenance action is a function of the operating age and the effective age reduction resulting from the action. This cost is an increasing function of the age reduction. In this case, the optimal maintenance can result in the product not being restored to as good as new. The optimal number of preventive maintenance actions, N^*, operating age s^*, and age reduction x^* are obtained by minimizing the manufacturer's expected warranty cost.

Sahin and Polatoglu (1996) discuss a preventive replacement policy for repairable item following the expiration of warranty. Failures over the warranty period are minimally repaired at no cost to the buyer. The item is kept for a period T after the expiration of the warranty and replaced by a new item. Failures over this period are rectified minimally with the buyer paying the costs. They consider stationary and non-stationary strategies that minimize the long run average cost to the buyer.

Monga and Zuo (1998) deal with a model formulation where the components of a system are replaced under preventive maintenance action when their failure rate reaches some specified value. The model formulation includes warranty period and preventive maintenance action in addition to system design and burn-in period. The cost of rectifying failures under warranty is borne by the manufacturer and post warranty costs are borne by the buyer. The various decision variables (including preventive maintenance) are optimally selected by minimizing the system life cycle cost which is the sum of the manufacturing cost (including burn-in cost), installation and setup costs, warranty cost and post warranty cost.

Djamaludin *et al.*(2001) develop a framework to study preventive maintenance actions when items are sold under warranty and discuss a new model when the preventive maintenance is carried out continuously.

Finally, extended warranties can be viewed as maintenance service contracts. Padmanabhan (1995) and Murthy and Padmanabhan (1993) deal with extended warranties and Murthy and Ashgarizadeh (1998, 1999) deal with maintenance service contracts.

As can be seen from the models reviewed in this section, preventive maintenance is performed at discrete time instants. The cost of preventive maintenance actions during warranty period is borne by the manufacturer in order to reduce the warranty servicing cost.

5. Discrete Model Formulation

The following notations are used in this section.

L : life of product
W : warranty period
m : preventive maintenance level (decision variable) [$0 \leq m \leq M$] [$m=0$ implies no preventive maintenance]
M : Upper limit on maintenance level
$F_0(t)$: failure distribution function with no preventive maintenance
$f_0(t)$: failure density function with no preventive maintenance
$r_0(t)$: failure rate with no preventive maintenance
$r(t, m)$: failure rate function with preventive maintenance
$r_m(t)$: failure rate with preventive maintenance over $[0, L]$
C_R : cost of each repair
$C_m(t)$: maintenance cost for maintenance level m
β : shape parameter for Weibull distribution
θ_0 : scale parameter for Weibull distribution
$\delta(m)$: degree rejuvenation for maintenance level m
$\nu(t)$: the virtual age of the item at time t
C_{BX} : buyer's expected life cycle cost under Option X
C_{MX} : expected warranty cost per unit to manufacturer under Option X
X : Options: $=$A [no preventive maintenance], $=$ B [preventive maintenance over L], $=$ C [no preventive maintenance during the warranty period].

The model considers the following three options:

Option A : No preventive maintenance action over the life of the item

Option B : Discrete preventive maintenance over $[0,\ L)$

Option C : No preventive maintenance over the warranty period and discrete preventive maintenance over $[W,\ L)$

It examines the implications of this to the manufacturer's warranty servicing cost and the buyer's life cycle cost and then discusses the optimal formulation are as follows.

5.1 Product Warranty

The product is repairable and sold with a non-renewing free replacement warranty policy with a warranty period W. All failures in the warranty period $[0,\ W)$ are rectified by the manufacturer at no cost to the buyer. The product has a useful life L and the cost of rectifying failures in the interval $[W,\ L)$, subsequent to the expiry of the warranty is borne by the buyer.

5.2 Item Failures

Since the product is repairable, any failure can be rectified through repair. We confine our attention to rectification through minimal repair. Under minimal repair the failure rate after repair is nearly the same as that just before failure. We assume that the time to rectify a failure is small in relation to the mean time between failures and that it can be ignored.

We first consider the case with preventive maintenance [Option A]. Let $F_0(t)[f_0(t)]$ denote the product failure distribution [density] function. From Barlow and Hunter (1960) failures over the time are given by a non-homogeneous Poisson process with intensity function given by the failure rate function $r_0(t)$ given by

$$r_0(t) = \frac{f_0(t)}{1-F_0(t)}$$

5.3 Discrete Preventive Maintenance Actions

Preventive maintenance is carried out discretely over time, that is, preventive maintenance occurs at times $\tau_1, \tau_2, \cdots, \tau_k$. The maintenance is carried out periodically at Δ time between maintenance. The failure rate of the item is as before the maintenance. The maintenance level m is constrained by $0 \le m \le M$ where M denotes the upper limit. Larger value of m corresponds to greater maintenance effort (more frequent inspection and inspections of more components). Since each failure is minimally repaired, the item's virtual age following the jth preventive maintenance with maintenance effort level m is given by

$$\nu_j = \nu_{j-1} + \delta(m)(\tau_j - \tau_{j-1}) \tag{1}$$

where $\nu_0 = 0$, $\nu_0 = 0$ and $\delta(m) \in [0,1]$ represents the degree rejuvenation at a preventive maintenance with maintenance effort level m. Let $\delta(m) = (1+m)e^{-m}$, $m \ge 0$ and integer. Note that for $m = 0$, $\delta(0) = 1$, that is no preventive maintenance is carried out, hence no rejuvenation on the age of the item. But for $m = M$, $\delta(M) = 0$, gives that full preventive maintenance would restore the item to as new condition. Each preventive maintenance is assumed to reduce only the damage incurred since the previous preventive maintenance. Therefore, The item's virtual age at time t is given by

$$\nu(t) = \nu_{j-1} + t - \tau_{j-1},\quad \tau_{j-1} \le t \le \tau_j,\quad j = 1, 2, \cdots, k \tag{2}$$

where $\nu_{j-1} = \delta(m)\tau_{j-1}$, $\tau_j = j\Delta$

The complete intensity function for the point process of item failure is defined by Jack (1998). Therefore the chance of a failure at time t only depends on the item's virtual age, and is defined by

$$r[\nu(t)] = r(\nu_{j-1} + t - \tau_{j-1}),\quad \tau_{j-1} \le t \le \tau_j,\quad j = 1, 2, \cdots, k \tag{3}$$

We assume that n_1, n_2 and n_3 denote the number of PM actions during $[0,\ W]$, $[0,\ L]$ and $[W,\ L]$ respectively. n_1, n_2 and n_3 are the largest integer less than $[W/\Delta]$, $[L/\Delta]$ and $[L - W/\Delta]$, respectively. Let $r(t,\ m)$denote the failure rate for a given history of maintenance with level m. The characterization under different options is as follows:

<u>OPTION A</u>: The buyer carries out no PM over the life of the item. All failure is rectified through CM actions. In this case, $m = 0$ and as a result the failure rate is given by

$$r(t; 0) = r_0(t) \tag{4}$$

<u>OPTION B</u>: The buyer uses a maintenance level m throughout out the life of the item. Then the last PM occurs at time $\tau_{n_2} = \Delta n_2$. In this case, we have

$$r_m(t) = r_0(\nu_{j-1} + t - \tau_{j-1}),\ \tau_{j-1} \le t \le \tau_j,$$
$$j = 1, 2, \cdots, n_2$$
$$r_m(t) = r_0(\nu_{n_2} + t - \tau_{n_2}),\ \tau_{n_2} \le t \le L \qquad (5)$$

$[r_0(t) - r_m(t)]$ is an increasing function of m implying that the reduction in the age increases with the maintenance level. $r_m(t)$ is an increasing function of t implying that even with maximum maintenance effort, the failure rate is increasing with age.

OPTION C: The buyer carries out no PM over the warranty period and the maintenance effort subsequent to the expiry of the warranty is m. Then the first PM actions start at time $\tau_1 = W + \Delta$. Therefore the buyer carries out no PM until τ_1 after which PM starts. The failure rate is given by

$$r_m(t) = r_0(t),\ 0 \le t \le \tau_1$$
$$r_m(t) = [r_0(\tau_1) - r_m(\tau_1)] + r_0(\nu_j + t - \tau_j),$$
$$\tau_j \le t \le \tau_{j+1},\ j = 1, 2, \cdots, (n_3 - 1)$$
$$r_m(t) = [r_0(\tau_1) - r_m(\tau_1)] + r_0(\nu_{n_3} + t - \tau_{n_3}),$$
$$\tau_{n_3} \le t \le L \qquad (6)$$

<Fig 1> shows a plot of $r(t; m)$ for the three different options for a fixed m. For a given t, the failure rate under Option B is less than under Option C which in turn is less than under Option A.

6. Model Analysis

In this section we drive expressions for the expected life cycle cost to the buyer over the life of an item and the expected warranty serving cost to the manufacturer for the three options.

Since failures are rectified through minimal repair and the repair times are negligible, the expected number of failures over any interval is given by the integral of the failure rate function over the interval.

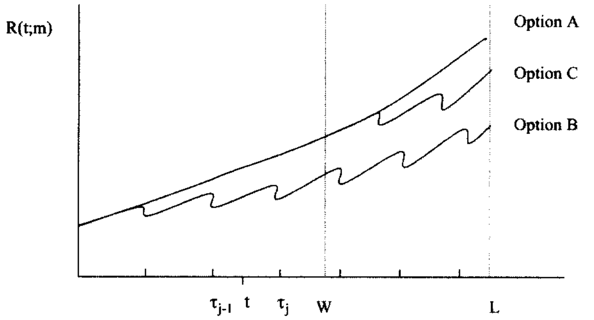

Figure 1. The failure rates for options A, B and C

As a result, the expected number of failures over warranty period is given by $\int_0^W r(t; m)dt$ and over the post warranty period by $\int_W^L r(t; m)dt$. $r(t; m)$ for Options A-C are given by (2), (3) and (6) respectively.

6.1 Buyer's Expected Life Cycle Cost

Let C_m denote the cost per unit time with maintenance level m and this increases with m. Therefore C_m could be modeled as a function of the maintenance effort level of m. The cost of corrective maintenance C_R depends on the number of failures over the interval $[W, L)$ and the cost of each repair.

OPTION A: Let $N_A(W, L)$ denote the expected number of failures during (W, L) for Option A, then

$$N_A(W, L) = \int_W^L r_0(t)dt = R_0(L) - R_0(W) \qquad (7)$$

Thus the buyer's expected life cycle cost under option A (C_{BA}) is given by

$$C_{BA} = C_R N_A(W, L) \qquad (8)$$

OPTION B: Let $N_B(W, L)$ denote the expected number of failures during (W, L) for Option B, then

$$N_B(W, L) = \int_W^{\tau_{n_1+1}} r_0(\nu_{n_1} + t - \tau_{n_1})$$
$$+ \sum_{j=n_1+1}^{n_2-1} \int_{\tau_j}^{\tau_{j+1}} r_0(\nu_j + t - \tau_j)\,dt$$
$$+ \int_{\tau_{n_2}}^{L} r_0(\nu_{n_2} + t - \tau_{n_2})\,dt \qquad (9)$$

The buyer's expected life cycle cost under Option B (C_{BB}) is given by

$$C_{BB} = C_m n_2 + C_R N_B(W, L) \qquad (10)$$

OPTION C: Let $N_C(W, L)$denote the expected number of failures during (W, L) for Option C. Then $N_C(W, L)$is given by

$$N_C(W, L) = \int_W^L r_m(t)dt = \int_W^{\tau_1} r_m(t)dt + \int_{\tau_1}^{\tau_{n_3}} r_m(t)dt$$
$$+ \int_{\tau_{n_3}}^{L} r_m(t)dt = \int_W^{\tau_1} r_0(t)dt$$
$$+ \sum_{j=2}^{n_3} \int_{\tau_{j-1}}^{\tau_j} [r_0(\tau_1) - r_m(\tau_1) + r_0(\nu_{j-1} + t - \tau_{j-1})]dt$$
$$+ \int_{\tau_{n_3}}^{L} [r_0(\tau_1) - r_m(\tau_1) + r_0(\nu_{n_3} + t - \tau_{n_3})]dt \qquad (11)$$

where $\tau_j = W + j\Delta,\ j = 1,\ 2,\ \cdots,\ n_3$

Then, the buyer's expected life cycle cost under Option C, C_{BC} is given by

$$C_{BC} = C_m n_3 + C_R N_C(W, L) \tag{12}$$

The optimal preventive level m for Options B and C can be obtained by minimizing the expected life cycle cost. As m is modeled as a discrete variable, then the optimal m is determined by evaluating the costs for the different values of m and then carrying out a relative comparison.

6.2 Manufacturer's Expected Warranty Serving Cost

The cost to the manufacturer is the warranty serving cost (the cost of rectifying failures under warranty). Under Options A and C, the expected warranty cost per unit is given by

$$C_{MA} = C_{MC} = C_R \int_0^W r_0(t)dt \tag{13}$$

Then under the Option B, the expected warranty serving cost per unit is given by

$$\begin{aligned} C_{MB} = {} & C_R \int_0^{\tau_1} r_0(t)dt \\ & + C_R \sum_{j=1}^{n_1-1} \int_{\tau_j}^{\tau_{j+1}} r_0(\nu_j + t - \tau_j)dt \\ & + C_R \int_{\tau_{n_1}}^{W} r_0(\nu_{n_1} + t - \tau_{n_1})dt \end{aligned} \tag{14}$$

Since $r_m(t) \leq r_0(t)$, the expected warranty serving cost to the manufacturer under Option B is smaller than that under Options A and C. Hence, the manufacturer would always prefer to have the buyer carry out preventive maintenance during the warranty period.

7. Numerical Example

Suppose that the failure distribution is given by the Weibull distribution with shape parameter β and scale parameter θ_0. Unlike in Kim *et al.* (2001), the scale parameter is not affected by the preventive maintenance. <Table 1> shows the relationship between maintenance level m and the degree rejuvenation $\delta(m)$ and the associated maintenance cost C_m.

7.1 Buyer's Expected Lifecycle Cost

Let $W = 2$(year), $L = 8$(year), $\beta = 2$ and $\theta_0 = 2$.

Table 1. Maintenance level m, $\delta(m)$ and C_m

Maintenance level m	$\delta(m)$	C_m
0	1.00	–
1	0.74	20
2	0.41	50
3	0.20	120
4	0.09	150
5	0.04	170

This implies that the mean time to first failure is 1.8 years. We assume that maintenance is carried out at discrete time τ_j, with $\Delta = 0.33$ year between maintenance. This implies that maintenance is carried out every 4 months.

<Table 2> shows the expected life cycle cost to the buyer under Options A, B and C with C_R varying from \$20 to \$500.

The result shows that for low corrective maintenance cost (< \$160), the optimal decision is Option A (no preventive maintenance during the life of the item). With the corrective maintenance cost increasing (but < \$300), the optimal decision is Option C, that is to carry out preventive maintenance after the warranty period has expired. The decision on the optimum preventive maintenance level depends on the ratio of the preventive maintenance and corrective maintenance costs.

For still higher repair costs (> \$320), the optimal decision is Option B (carry out preventive maintenance over the whole interval). The optimal preventive maintenance level depends on the ratio of the preventive maintenance and corrective maintenance costs. The corresponding results for the case when changes from 2 to 3 are shown in <Table 3>.

The results are similar. The expected costs are higher as to be expected. Note that the optimal strategy is Option A for $C_R \leq \$20$ and Option B for $20 \leq C_R \leq 500$.

7.2 Manufacturer's Expected Warranty Servicing Cost per Unit

For the same parameter values as in Section 6.1 and $\beta = 2$, the expected warranty cost per unit to the manufacturer under the three Options A (or C) and B are given in <Table 4> for C_R varying from \$20 to 500.

The results show that as the corrective maintenance cost increases, the expected warranty cost per unit to the manufacturer also increases. On the other hand, maintenance reduces the manufacturer expected

Table 2. The buyer expected life cycle costs for Options A, B and C, [$\beta=2$ and C_R varying]

	Option A	Option B			Option C		
C_R	$m=0$	$m=1$	$m=2$	$m=3$	$m=1$	$m=2$	$m=3$
20	300.0*	841.67	1369.84	2957.87	615.07	1099.01	2323.84
40	600.0*	1237.92	1550.22	3038.27	870.15	1298.02	2487.68
60	900.0*	1634.17	1730.59	3118.68	1125.22	1497.03	2651.52
100	1500.0*	2426.66	2091.34	3279.48	1635.37	1895.05	2979.20
140	2100.0*	3219.16	2452.09	3440.29	2145.52	2293.06	3306.88
160	2400.0*	3615.41	2632.46	3520.69	2400.59	2492.07	3470.71
180	2700.0	4011.65	2812.83	3601.09	2655.66*	2691.08	3634.55
200	3000.0	4407.90	2993.21	3681.50	2910.74	2890.09*	3798.39
240	3600.0	5200.40	3353.95	3842.30	3420.89	3288.11*	4126.07
260	3900.0	5596.64	3534.33	3922.71	3675.96	3487.12*	4289.91
280	4200.0	5992.89	3714.70	4003.11	3931.03	3686.13*	4453.75
300	4500.0	6389.14	3895.08	4083.51	4186.11	3885.14*	4617.59
320	4800.0	6785.39	4075.45*	4163.92	4441.18	4084.15	4781.43
340	5100.0	7181.64	4255.82	4244.32*	4696.25	4283.16	4945.27
360	5400.0	7577.88	4436.20	4324.72*	4951.33	4482.17	5109.11
440	6600.0	9162.87	5157.69	4646.33*	5971.62	5278.20	5764.47
500	7500.0	10351.62	5698.81	4887.54*	6736.84	5875.23	6255.98

Table 3. The buyer expected life cycle costs for Options A, B and C, [$\beta=3$ and C_R varying]

	Option A	Option B			Option C		
C_R	$m=0$	$m=1$	$m=2$	$m=3$	$m=1$	$m=2$	$m=3$
20	1260.0*	1481.88	1477.77	2949.14	1246.55	1417.65	2501.29
40	2520.0	2687.08	1789.69*	3022.32	2133.09	1935.30	2842.58
60	3780.0	3892.28	2101.62*	3095.50	3019.64	2452.95	3183.87
100	6300.0	6302.67	2725.48*	3241.85	4792.73	3488.24	3866.44
140	8820.0	8713.07	3349.34*	3388.20	6565.82	4523.54	4549.02
160	10080.0	9918.26	3661.27	3461.37*	7452.37	5041.19	4890.31
180	11340.0	11123.46	3973.20	3534.55*	8338.91	5558.84	5231.60
200	12600.0	12328.66	4285.13	3607.73*	9225.46	6076.49	5572.88
240	15120.0	14739.05	4908.98	3754.08*	10998.55	7111.78	6255.46
260	16380.0	15944.25	5220.91	3827.25*	11885.10	7629.43	6596.75
280	17640.0	17149.45	5532.84	3900.43*	12771.64	8147.08	6938.04
300	18900.0	18354.64	5844.77	3973.60*	13658.19	8664.73	7279.33
320	20160.0	19559.84	6156.70	4046.78*	14544.74	9182.38	7620.61
340	21420.0	20765.04	6468.63	4119.96*	15431,28	9700.03	7961.90
360	22680.0	21970.24	6780.56	4193.13*	16317.83	10217.68	8303.19
440	27720.0	26791.03	8028.27	4485.83*	19864.01	12288.27	9668.34
500	31500.0	30406.62	8964.06	4705.36*	22523.65	13841.22	10692.21

warranty cost, and greater preventive maintenance effort on the part of the buyer implies smaller warranty servicing cost.

7.3 Comment

The results of Sections 7.1 and 7.2 show the effect of preventive maintenance on the buyer's life cycle cost and manufacturer's warranty servicing cost. As the buyer's effort on preventive maintenance increases, the manufacturer warranty servicing cost decreases. For the case $C_R=\$500$ and $\beta=2$, from <Table 2> it is seen that optimal preventive maintenance results in lower costs to both the manufacturer and buyer. This is a win-win situation. If the buyer is myopic and does not invest in preventive maintenance

Table 4. The manufacturer's expected warranty serving cost per unit for Options A(or C) and B, [$\beta = 3$ and C_R varying]

	Option A(or C)	Option B				
C_R	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
20	20.0	14.49	8.99	5.54	3.74	2.89
40	40.0	28.97	17.97	11.08	7.49	5.78
60	60.0	43.46	26.96	16.61	11.23	8.68
100	100.0	72.43	44.94	27.69	18.72	14.46
140	140.0	101.40	62.91	38.77	26.21	20.24
160	160.0	115.88	71.90	44.31	29.96	23.13
180	180.0	130.37	80.89	49.84	33.70	26.03
200	200.0	144.85	89.87	55.38	37.45	28.92
240	240.0	173.82	107.85	66.46	44.94	34.70
260	260.0	188.31	116.83	72.00	48.68	37.59
280	280.0	202.79	125.82	77.54	52.43	40.49
300	300.0	217.28	134.81	83.07	56.17	43.38
320	320.0	231.77	143.80	88.61	59.92	46.27
340	340.0	246.25	152.78	94.15	63.66	49.16
360	360.0	260.74	161.77	99.69	67.40	52.05
440	440.0	318.68	197.72	121.84	82.38	63.62
500	500.0	362.13	224.68	138.46	93.62	72.30

during the warranty period then the manufacturer warranty servicing cost is higher. In this case, the manufacturer might get the buyer to invest in maintenance effort during the warranty period by offering monetary incentive as long as it is less than \$361.54 and the buyer chooses the level of maintenance $m=3$ as it gives the buyer the optimal saving.

8. Conclusion

In this paper we discussed preventive maintenance which is carried out by the buyer for item sold under warranty and formulated a simple model involving discrete preventive maintenance. We have also discussed for different maintenance level that is carried out during warranty period and after warranty expired as often the case in real life. The result shows that for large failure costs the optimal result is to carry out different maintenance levels during warranty and after warranty expired.

The model can be extended in several ways and we indicate a few.

1. The effect of maintenance often leads to the life of the item being extended. This implies that L increases with m. (ie, L increasing with m)
2. We have confined our analysis to the free replacement policy and failed item being always repaired minimally. The analysis of other types of warranty policies for example, pro-rata, combination is yet to be carried out.
3. We have not studied the different incentive schemes and the related moral hazard issues. This is a topic for considerable new research.
4. We have assumed that the maintenance level is constant. Often, this is not realistic and the maintenance effort changes with the age of the item -- less when it is new and more as it ages.
5. We have confined our attention to discrete preventive maintenance effort. Often, one employs both continuous and discrete (overhaul) preventive maintenance actions. This makes the problem more difficult and also interesting.

Some of these problems are currently under investigation by the author.

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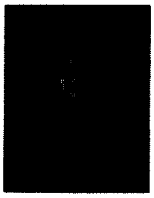

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