실제 교량의 시스템 신뢰성해석에 기초한 수명예측

Lifetime Prediction of Existing Highway Bridges Using System Reliability Approach

양 승 이[1)]

Yang, Seung Ie

요　　약 : 이 논문에서는 교량의 수명을 예측하기 위한 시스템 신뢰성 이론이 설명되고, 생애 분포 함수를 이용하여 현존하는 교량의 잔존 수명을 예측하는 방법이 설명된다. 시스템 이론과 생애함수 (survivor functions) 를 이용하여 LIFETIME 이라는 프로그램을 개발하였다. Survivor functions은 주어진 시간 t에 대해 신뢰성을 산출한다. 이 프로그램을 이용하여 콜로라도주에 있는 교량의 수명을 예측하였다. 이 교량은 직렬과 병렬로 구성된 시스템으로 컴퓨터 모델링 되었으며 이 모델을 이용하여 시스템 파괴 확률을 시간에 대해 계산하였다.

ABSTRACT : In this paper, the system reliability concept was presented to predict the lifespan of bridges. Lifetime distribution functions (survivor functions) were used to model real bridges to predict their remaining life. Using the system reliability concept and lifetime distribution functions (survivor functions), a program called LIFETIME was developed. The survivor functions give the reliability of component at time t. The program was applied to an existing Colorado state highway bridge to predict the failure probability of the time-dependent system. The bridge was modeled as a system, with failure probability computed using time-dependent deteriorating models.

핵 심 용 어 : 교량, 시스템신뢰성, 잔존수명, 생애함수

KEYWORDS : bridges; system reliability; remaining life; lifetime function

1. Introduction

The civil infrastructures are designed to serve the public. And no matter how well these are designed, they are deteriorating with time. Especially, the bridges are one of the important civil infrastructures. With the maintenance of almost 600,000 U.S. highway bridges funded by the federal government alone [O'Connor and Hyman 1989], the annual cost of inspection and repair of bridges is significant. With ever increasing budgetary constraints and the continuing decay of the nation's infrastructure, it is more important to use this fund effectively. In order to avoid the high cost of rehabilitation, the rating (evaluation) of these bridges must correctly report the actual load-carrying capacity. The specifications [Manual 1983, Condition 1994] are used for bridge rating. These specifications use level zero (allowable stress design) and level one (load factor design) safety checking requirements. Level zero safety checking requirement is a deterministic method. Level one method uses characteristic values of basic random variable. In these safety checking methods (level zero and level one methods), the single component is used to check safety and to compute rating value. However, the bridges are a system of all components.

For over three decades, researchers have been investigating application of system reliability concepts and techniques to structural design and evaluation [Cornell 1967, Moses 1982, Ang and Tang 1984, Thoft-Christensen and Murotsu 1986, Ditlevsen and Bjerager 1986, Karamchandani 1990].

1) 미국 콜로라도대학교 토목환경공학과 박사과정

본 논문에 대한 토의를 2002년 10월 31일까지 학회로 보내주시면 토의 회답을 게재하겠습니다.

Resistances and loads are not constant with time. This is because ductility and strength of materials deteriorate with time and are affected by previous loading history, and loads on structures vary with time. Time invariant reliability analysis of structural systems may provide unconservative reliability estimation because it considers only the initial variability of random variables, which may increase with time [Iizuka and Frangopol 1991].

As a new approach, a computer program is developed by using system reliability concept and lifetime function. One of the lifetime functions, survivor function, is used to predict time variant system failure probability. The program is applied to an existing Colorado state highway bridge (E-17-AH).

2. System Reliability Analysis

Structure function [Leemis 1995] is a useful tool to describe the state of system with n components. Structure function defines the system state as a function of the component state. A system is assumed to be a collection of n components [Ghosn and Frangopol 1999]. In addition, it is assumed that both components and the system can either be functioning or failed. The state of component i, x_i, is assumed as

$$x_i = \begin{cases} 0 & \text{if component } i \text{ has failed} \\ 1 & \text{if component } i \text{ is functioning} \end{cases} \quad (1)$$

for i = 1, 2, ..., n

The n component system can be expressed as a system state vector as following.

$$\mathbf{x} = \{x_1, x_2, \ldots, x_n\} \quad (2)$$

Structure function, $\phi(\mathbf{x})$, expresses the system state vector $\mathbf{x}$ to zero or one. The structure function $\phi(\mathbf{x})$ for a given system state vector is

$$\phi(\mathbf{x}) = \begin{cases} 0 & \text{if the system has failed} \\ 1 & \text{if the system is functioning} \end{cases} \quad (3)$$

The most common system is the series and parallel system. For series system, since the any one component failure in the system causes the system failure, the series system is expressed as

$$\begin{aligned} \phi(\mathbf{x}) &= \begin{cases} 0 & \text{if there exists an } i \text{ such that } x_i = 0 \\ 1 & \text{if } x_i = 1 \text{ for } all \ i = 1, 2, \ldots, k, \ldots, n \end{cases} \\ &= \min\{x_1, x_2, \ldots, x_k, \ldots, x_n\} \\ &= \prod_{i=1}^{n} x_i \end{aligned} \quad (4)$$

For parallel system, all component failures in a system cause the system failure, the parallel system is expressed as

$$\begin{aligned} \phi(\mathbf{x}) &= \begin{cases} 0 & \text{if } x_i = 0 \text{ for } all \ i = 1, 2, \ldots, k, \ldots, n \\ 1 & \text{if there exists an } i \text{ such that } x_i = 1 \end{cases} \\ &= \max\{x_1, x_2, \ldots, x_k, \ldots, x_n\} \\ &= 1 - \prod_{i=1}^{n} (1 - x_i) \end{aligned} \quad (5)$$

As an example, the structure function is obtained for a 5-component system shown in Fig. 1. Also, Fig. 1 shows the reduction steps. These reduction steps are also expressed as functions through Eq. (6) to Eq. (9).

The first reduction step is a parallel system between components 2 and 3. By first reduction, the subsystem 1 is obtained and expressed as following.

$$\phi_{s2}(\mathbf{x}) = 1 - (1 - x_2)(1 - x_3) \quad (6)$$

Where

x_i = State of component i

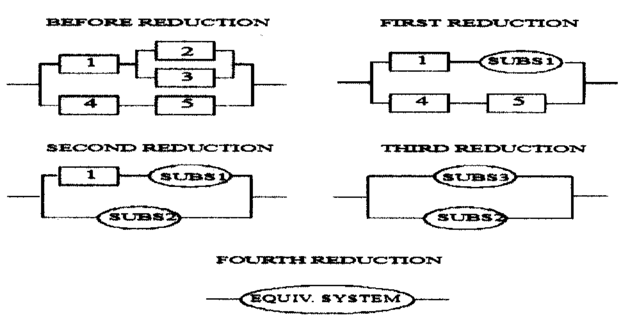


Fig. 1 Sequential Reduction Procedure

The second reduction is a series system between components 4 and 5. This is expressed as following.

$$\phi_{S2}(\mathbf{x}) = x_4 x_5 \qquad (7)$$

The third reduction is also series system between subsystem 1 and component 1.

$$\phi_{S3}(\mathbf{x}) = x_1 \phi_{S1} \qquad (8)$$

By fourth reduction, the structure function for this 5-component system is obtained.

$$\phi(\mathbf{x}) = 1 - \{1 - x_1[1-(1-x_2)(1-x_3)]\}(1 - x_4 x_5) \qquad (9)$$

The structure function is deterministic. This assumption may be unrealistic for certain types of components or system. So, reliability functions [Leemis 1995] are necessary to model the structures which are in use. x_i was defined to be the deterministic state of component i. Now, x_i is a random variable. The probability that component i is functioning is given by

$$p_i = P[x_i = 1] \qquad (10)$$

Where

p_i = Probability that component i is functioning

In order to obtain the reliability function for a 5-component system shown in Fig. 1, the same procedure is necessary. But the component reliability function, p, is used in each step instead of component state x.

3. Lifetime Distribution

Reliability function gives the reliability of components or system at specific time t. In this section, the probability of failure is generalized to be a function of time with lifetime distribution. There are several lifetime functions to describe the evolution of the probability of failure. In this paper, one lifetime function is introduced called "Survivor function". The lifetime function applies to both discrete and continuous lifetime and is used to describe the distribution of system lifetime, as well as of its components.

The survivor function is the generalization of reliability because the survivor function gives the reliability that a component or system is functioning at one particular time. The survivor function is expressed

$$S(t) = P[T \geq t] \qquad t \geq 0 \qquad (11)$$

It is assumed that when $t \leq 0, S(t)$ is one. The survivor function has to satisfy three conditions. These are

1) $S(0) = 1$
2) $\lim_{t \to \infty} S(t) = 0$
3) $S(t)$ is non-increasing without any maintenance

Several distributions are used as survivor functions. The exponential distribution, Weibull distribution, Log-Logistic distribution, and Exponential Power distribution are used in this paper. These survivor functions are shown in table 1.

Table 1 Survivor Function

Distribution	Survivor function
Exponential	$\exp(-\lambda t)$
Weibull	$\exp(-(\lambda_s t)^k)$
Log-logistic	$\frac{1}{1+(\lambda_s t)^k}$
Exponential- power	$\exp(1-\exp(\lambda_s t)^k)$

Where

λ = Failure rate
λ_s = Scale factor
κ = Shape factor
t = Time, t≥0

Exponential survivor function is only one parameter distribution and has constant failure rate. Others have two parameters (failure rate and shape factor). Depending on shape factor κ, survivor functions have an increasing failure rate or constant failure rate or decreasing failure rate.

In order to find out the lifetime function for a system, the concept explained in section 2 and section 3 are used. To make lifetime function for a system, the component survivor functions are used as arguments. As an example, if there is a three-component series system with independent relation for each component, the system reliability function is

$$S(t) = S_1(t)S_2(t)S_3(t) \quad (12)$$

Where

$S_i(t)$ = Survivor function of component i

For three-component parallel system, the system lifetime function is

$$S(t) = 1-(1-S_1(t))(1-S_2(t))(1-S_3(t)) \quad (13)$$

4. Application to Existing Colorado Bridge

4.1 Program Flowchart

The program LIFETIME is developed using the system reliability concept and lifetime distributions explained in the previous sections. For random parameters of lifetime distributions, Monte Carlo simulation is used. The flowchart is shown in Fig. 2. Because of space limitation, it is impossible to show in detail.

In the figure, ICDF menas "Inverse Cumulative Distribution Function". Depending on the simulation numbers, computing time is decided. Failure rate and scale factor can have six random distributions (Uniform, Triangular, Log-normal, Gamma, Exponential, and Beta distributions). Because the generated random numbers should be positive, these distribution types shown in Fig. 2 are selected. Although Uniform survivor distribution

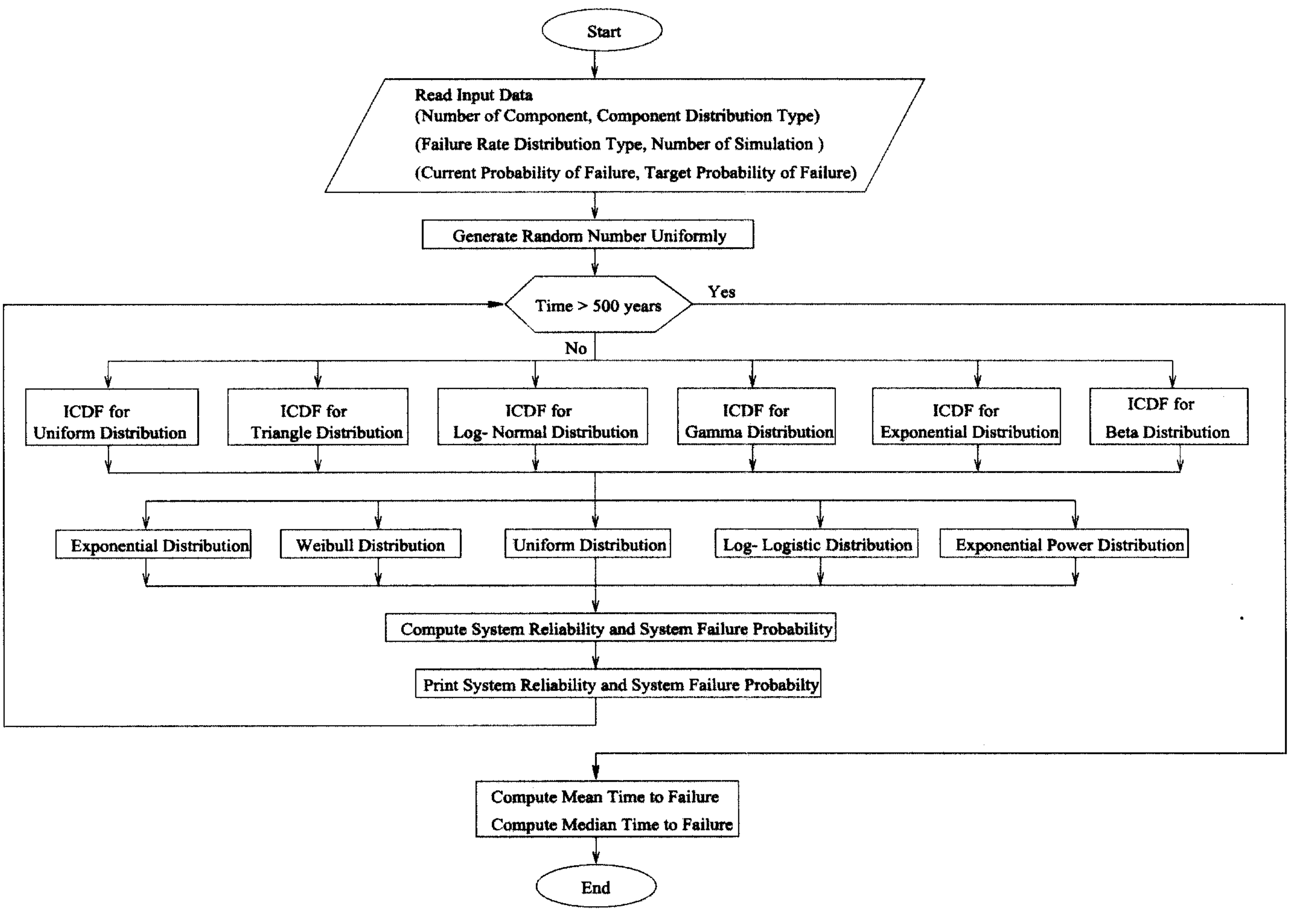


Fig. 2 Flowchart of the Program

is programmed, it is not explained in the previous section because it is not used very frequently.

4.2 Application of Program LIFETIME

In this section, the applications of LIFETIME are shown.

The Fig. 3 shows the time dependent system failure probability of series systems. It is assumed that each component is independent and has the same deterministic failure rate. And the survivor function of each component is exponential. It can be seen that for a series system, increasing components makes the system dangerous. The parallel systems are shown in Fig. 4. The failure rate of each component is the same and has random distribution (uniform: a = 0.00413/year and b = 0.00586/year). It is assumed that all components are independent. The Fig. 5 and Fig. 6 show the time dependent system failure probability of arbitrary 3-component and 4-component systems. All components of each system are independent. The 3-component system has 0.005/year failure rate of components 2 and 3. Fig. 5 shows the effect of changing failure rate of component 1. In the case of 4-component system, the failure rate of each component has random variable whose distribution type is uniform (a = 0.00413/year and b = 0.00586/year).

4.3 Colorado State Bridge E-17-AH

In this section, the Colorado State Bridge E-17-AH is explained. Bridge E-17-AH is located on 40th Avenue (State Highway 33) between Madison and Gardfield Streets in Denver. The bridge has three simple spans

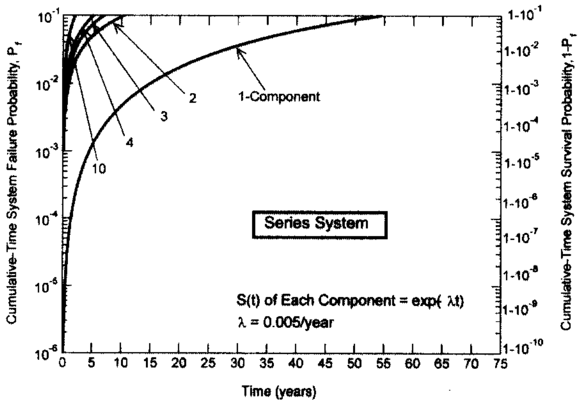


Fig. 3 Cumulative-Time System Failure Probability of Series System

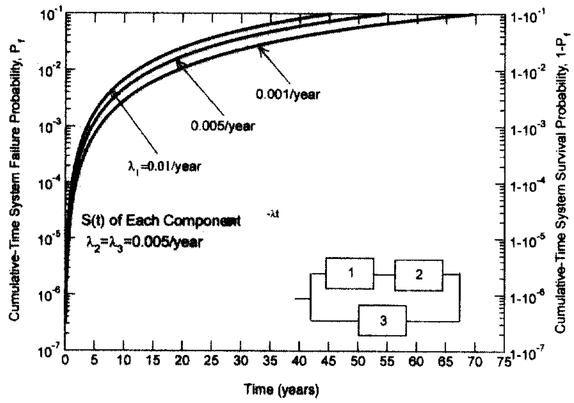


Fig. 5 Cumulative-Time Failure Probability of 3-Component System

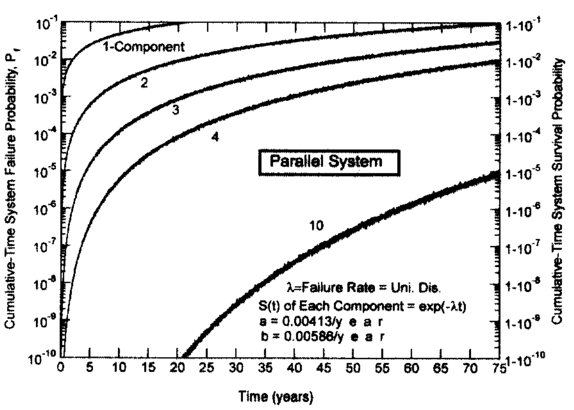


Fig. 4 Cumulative-Time System Failure Probability of Parallel System

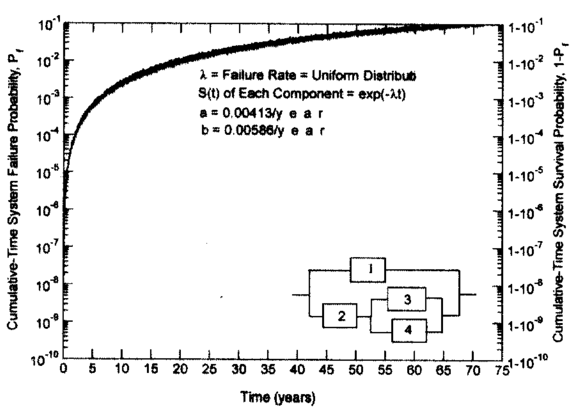


Fig. 6 Cumulative-Time Failure Probability of 4-Component System

of equal length (43.83 ft) and a total length of 137.3 ft. The deck consists of 9.0 in of reinforced concrete and a 3.0 in surface layer of asphalt. The east-west bridge has two lanes of traffic in each direction with an average daily traffic 8,500 vehicles. The roadway width is 40 ft with 5 ft pedestrian sidewalks and handrailing on each side. The bridge offers 22.17 of clearance for the railroad spur that runs underneath. There is no skew or curvature. The slab is supported by nine standard-rolled, compact, and non-composite steel girders. The girders are stiffed by end diaphragms and intermediate diaphragms at the third points. Each girder is supported at one end by a fixed bearing and an expansion bearing at the other end. The bridge is shown in Fig. 7 and 8.

4.4 Data Collection

The lifetime distribution functions were explained in section 3. Each lifetime distribution has each parameter (failure rate, scale factor and shape factor) and this should be obtained from data analysis to predict the probability of failure of real bridges. In this paper, the data from Maunsell Ltd [Maunsell 1999] for Highway Agency are used for bridge components. In this report, the serviceable life is defined to be the time taken for a significant defect requiring attention to be recorded at an inspection. Detailed information of the condition of each component at the time of inspection is recorded and this data are transferred to the National Structures Database. According to defect severity, four levels were classified.

- Severity1 : no significant defects
- Severity2 : minor defects of a non urgent nature
- Severity3 : defects which shall be included for attention within the next annual maintenance program
- Severity4 : the defect is severe and urgent action is needed

Therefore, the serviceable life from this report is defined to be the time taken for a structural defect to be recorded for attention in next annual year or the time taken for a structural defect to be needed for urgent action.

In the state-of-the-art Rilem 14 on the Durability of Concrete Structures [Rilem 1996] the RILEM Technical Committee states that the probability density of service life generally pecks rapidly before decreasing gently towards zero when approaching an infinitely long service life. This type of distribution must be selected.

Because the curve fitting process in data analysis was generally confined to initial part of the continuous distribution, it was impossible to fit exact shape of the distribution.

Weibull distribution was selected as best fit of the data and summarized in the report [Maunsell 1999]. Table 2 and 3 contain the parameters of Weibull distribution for Severity 3 and 4.

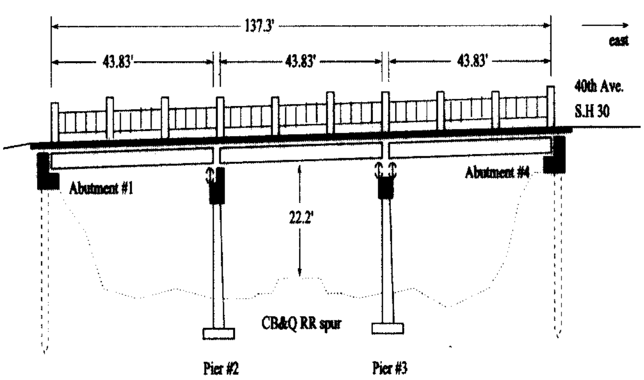


Fig. 7 Colorado State Bridge E-17-AH :Profile (Estes 1997)

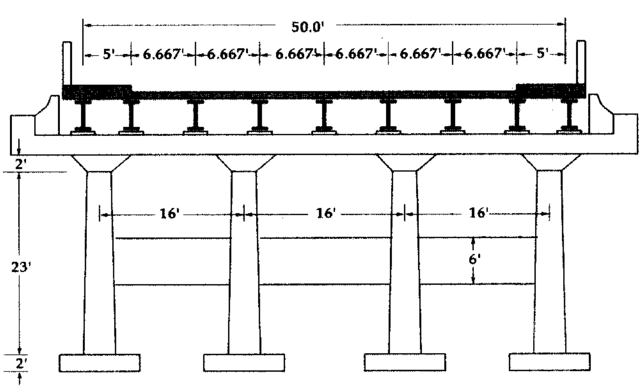


Fig. 8 Colorado State Bridge E-17-AH :Cross Section (Estes 1997)

4.5 Modeling of Colorado State Bridge E-17-AH

Due to nonlinearity in multi-girder bridge types, single girder failure doesn't cause the bridge failure.

Table 2 Parameters of Weibull Distributions of Serviceable Life for Severity 3 Defects(Adapted from Maunsell 1999)

Category **Structure forms**	**Description**	x **Years**	$1/\lambda_s$	**MODE**
A1	Arches, concrete	2.44	41.28	33
A2	Slab decks	1.40	56.12	22
A3A	RC beam and Slab, slabs	2.98	27.73	24
A3B	RC beams and Slab, Beams	2.88	30.26	26
A4A	Composite, slabs	2.84	37.72	32
A4B	Composite, beams	1.47	26.66	12
A5A	Pretensioned slabs	1.70	68.74	40
A5B	Pretensioned beams	1.41	69.45	28
A6A	Post tensioned, slabs	2.62	51.55	42
A6B	Post tensioned, beams	3.29	23.97	21

Table 3 Parameters of Weibull Distributions of Serviceable Life for Severity 4 Defects(Adapted from Maunsell 1999)

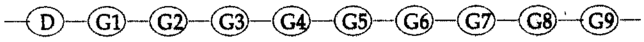
Category **Structure forms**	**Description**	x **Years**	$1/\lambda_s$	**MODE**
A1	Arches, concrete	Insufficient Data for Analysis		
A2	Slab decks	2.37	130.50	103
A3A	RC beam and Slab, slabs	3.76	83.36	76
A3B	RC beams and Slab, Beams	1.66	228.50	119
A4A	Composite, slabs	2.91	98.98	85
A4B	Composite, beams	2.86	94.70	81
A5A	Pretensioned slabs	1.90	223.39	119
A5B	Pretensioned beams	3.19	80.23	71
A6A	Post tensioned, slabs	3.03	104.20	91
A6B	Post tensioned, beams	2.60	100.83	83

When one girder fails on bridge, the load redistribution takes place and the bridge is capable to carry additional loads. The multi-girder bridges are modeled as combination of series and parallel systems in reliability analysis. The following failure modes are considered.

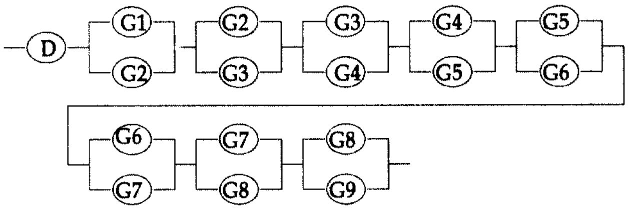
- Any one girder failure or deck failure causes the bridge failure.
- Any two adjacent girder failures or deck failure cause the bridge failure.
- Any three adjacent girder failures or deck failure cause the bridge failure.
- Failure of any external girder or any two adjacent internal girders or deck failure cause the bridge failure.

These failure models are shown in Fig. 9 for Bridge E-17-AH. With these failure modes, the reliability analysis will be performed.

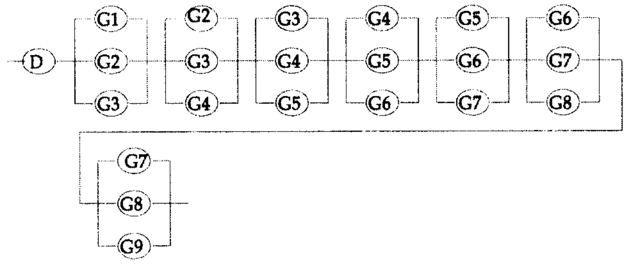
Because the data of severity 3 are too conservative, the results are unrealistic. So, in this paper, the time dependent system failure probability is shown for each failure mode by using severity 4. The results are shown as following.



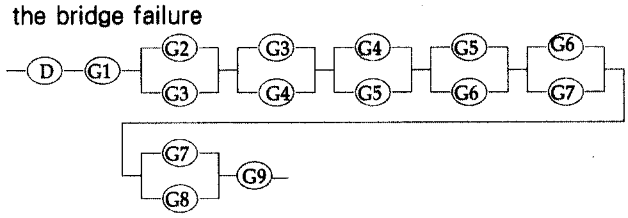
Any one girder failure or deck failure causes the bridge failure



Any two adjacent girder failures or deck failure cause the bridge failure



Any three adjacent girder failures or deck failure cause the bridge failure



Failure of any external girder or any two adjacent internal girders or deck failure cause the bridge failure

Where
D = Deck failure
G1 and G10 = Exterior girder failure
G2, G3, G4, G5, G6, G7, G8, and G9
= Interior girder failure

Fig. 9 Failure Modes

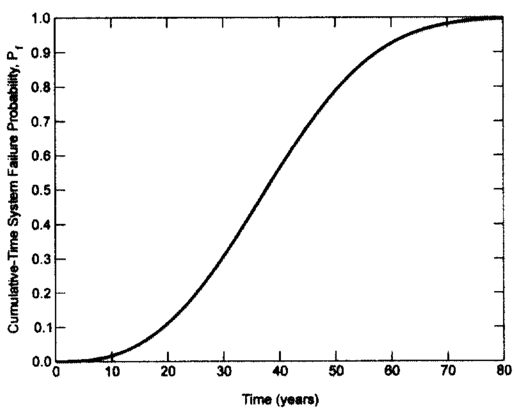


Fig. 10 Cumulative-Time System Failure Probability for the First Failure Mode

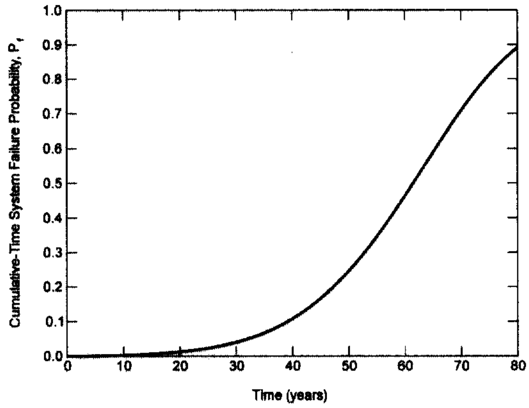


Fig. 11 Cumulative-Time System Failure Probability for the Second Failure Mode

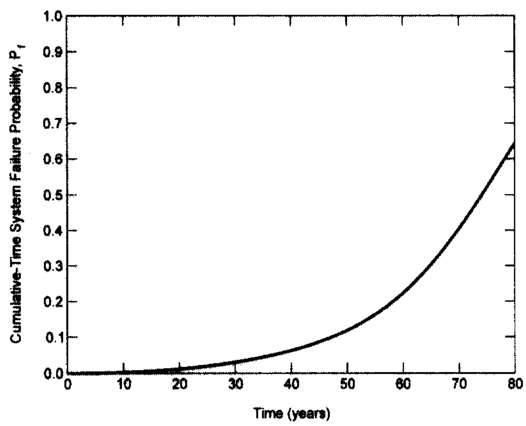


Fig. 12 Cumulative-Time System Failure Probability for the Third Failure Mode

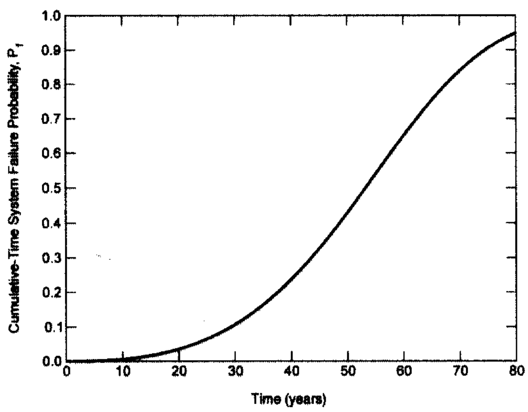


Fig. 13 Cumulative-Time System Failure Probability for the Fourth Failure Mode

Fig. 10 to Fig. 13 show time-dependent system failure probability for each failure mode. It is assumed that each component for each failure mode is independent. For the deck, the scale factor and shape factor are

0.0077/year and 2.37, respectively. And, for the interior and exterior girders, the scale factor and shape factor are 0.0106/year and 2.86.

5. Conclusion

In this paper, the system reliability concept and lifetime distribution functions were used to develop the program "LIFETIME". The main purpose of program "LIFETIME" is to predict the time dependent system failure probability. In order to apply the program "LIFETIME" to an existing bridge, very important data were obtained in this paper. The program "LIFETIME" may be applied to other structures which can be modeled as a combination of series and parallel systems. Because the program "LIFETIME" gives the time dependent system failure probability, the result can be used for making the plan of the repair or maintenance with a target system failure probability.

References

1. American Association of State Highway Transportation Officials, (1983). Manual for Maintenance Inspection of Bridges, Washington, D.C., p 53.
2. American Association of State Highway Transportation Officials, (1994). Manual for Condition Evaluation of Bridges, Washington, D.C., p 153
3. Estes, A. C., (1997). A system reliability approach to the lifetime optimization of inspection and repair of highway bridges, Ph.D. thesis, University of Colorado, p 741.
4. Ghosen, M and Frangopol, D.M., (1999). Bridge Reliability: Components and Systems, Chapter 4 in Bridge Safety and Reliability, D.M. Frangopol, ed., ASCE, Reston, Virginia, pp 83-112.
5. Iizuka, M., and Frangopol, D.M. (1991). Time Invariant and Time Variant Reliability Analysis and Optimization of Structural System , Technical Report No. 91-02, University of Colorado at Boulder, p 369.
6. Leemis, L. M., (1995). Reliability, probabilistic models and statistical methods, Prentice-Hall, New Jersey, p 319.
7. RILEM Report 14, (1996). Durability Design of Concrete Structures, Report of RILEM Technical Committee 130-CSL, Edited by Sarja, A and Vesikari, E., London.
8. O'Connor, D.S., and Hyman, W.A., (1989). Bridge Management System, Technical Report FHWA-DP-71-01R, Federal Highway Administration, Washington D.C.
9. Cornell, C.A., (1967). Bounds on the Reliability of Structural System, Journal of Structural Division, Proceedings of ASCE, 93:171-200.
10. Moses, F., (1982). Reliability Development in Structural Engineering, Structural Safety, 1:3-13.
11. Ang, A.H. and Tang, W.H., (1984). Probability Concepts in Engineering Planning and Design Vol I and II, John Wiley and Son, New York.
12. Thoft-Christensen, P. and Murotsu, Y., (1986). Application of Structural System Reliability Theory, Springer-Verlag, Berlin.
13. Ditlevsen, O. and Bjerager, P., (1986). Methods for Structural System Reliability, Structural Safety, 3:195-229.
14. Karamchandani, A., (1990). New Methods in System Reliability, Ph.D. thesis, Stanford University.
15. Highway Agency, (1999). Serviceable Life of Highway Structures and Their Components, Maunsell Ltd, Birmingham, P 32.

(접수일자 : 2001년 10월 22일)