

The Efficiency of Boosting on SVM

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Abstract

In this paper, we introduce SVM(support vector machine) developed to solve the problem of generalization of neural networks. We also introduce boosting algorithm which is a general method to improve accuracy of some given learning algorithm. We propose a new algorithm combining SVM and boosting to solve classification problem. Through the experiment with real and simulated data sets, we can obtain better performance of the proposed algorithm.

Key words : SVM, Boosting, Classification, Learning algorithm

1.

(parameter) (overfitting)
[5],
가 .
가 .
가 .
[5,6,10,11]. SVM VC
(penalty)
[1,5,9]. SVM (convex function)
가 .
가 .
Yoav Freun Robert Shapire[3]가 1995

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가

SVM

2

3

4

[4](Wisconsin breast cancer, German

credit data, Iris data)

2.

SVM

Vapnik

[1,5,6,9,10].

$\{(\mathbf{x}_i, y_i), i = 1, \dots, n\} \subset X \times \{-1, 1\}$ 가

가

X d 가

R^d 가 (Support Vector Classifier : SVC)

가

$$\min_{\mathbf{w}, b} \langle \mathbf{w} \cdot \mathbf{w} \rangle \tag{1}$$

$$s.t. y_i \{ \langle \mathbf{w} \cdot \mathbf{x}_i \rangle + b \} \geq 1$$

maximal margin classifier, $sign(f(\mathbf{x}))$, $f(\mathbf{x}) = \langle \mathbf{w} \cdot \mathbf{x} \rangle + b$

$$\langle \mathbf{x} \cdot \mathbf{y} \rangle \quad \mathbf{x} \quad \mathbf{y} \tag{1}$$

α_i

$$\max_{\alpha} L(\mathbf{w}, b, \alpha) = \frac{1}{2} \langle \mathbf{w} \cdot \mathbf{w} \rangle - \sum_{i=1}^n \alpha_i [y_i (\langle \mathbf{w}_i \cdot \mathbf{x}_i \rangle + b) - 1] \tag{2}$$

(2) \mathbf{w} b

$$\mathbf{w} = \sum_{i=1}^n y_i \alpha_i \mathbf{x}_i, \tag{3}$$

$$\sum_{i=1}^n y_i \alpha_i = 0$$

(3) (2)

$$\max_{\alpha} L(\mathbf{w}, b, \alpha) = \sum_{i=1}^n \alpha_i - \frac{1}{2} \sum_{i,j=1}^n y_i y_j \alpha_i \alpha_j \langle \mathbf{x}_i \cdot \mathbf{x}_j \rangle \tag{4}$$

$$s.t. \sum_{i=1}^n y_i \alpha_i = 0, \alpha_i > 0, i = 1, \dots, n.$$

(1) b α f \mathbf{w}

f α

$$f(\mathbf{x}) = \sum_{i=1}^n y_i \alpha_i \langle \mathbf{x}_i, \mathbf{x} \rangle + b \tag{5}$$

Karush-Kuhn-Tucker $\alpha^*, \mathbf{w}^*, b^*$ 가

$$\alpha_i^* [y_i (\langle \mathbf{w}_i^*, \mathbf{x}_i \rangle + b^*) - 1] = 0, \quad i = 1, \dots, n.$$

$$\begin{aligned} & \alpha_i^* = 0, \quad [y_i (\langle \mathbf{w}_i^*, \mathbf{x}_i \rangle + b^*) - 1] \neq 0 \\ & \alpha_i^* \neq 0, \quad [y_i (\langle \mathbf{w}_i^*, \mathbf{x}_i \rangle + b^*) - 1] = 0 \end{aligned}$$

(Support Vector, SV)

(5) f

$$f(\mathbf{x}) = \sum_{i \in SV} y_i \alpha_i \langle \mathbf{x}_i, \mathbf{x} \rangle + b$$

가 가 K C

$$f(\mathbf{x}) = \sum_{i \in SV} y_i \alpha_i K(\mathbf{x}_i, \mathbf{x}) + b, \quad 0 < \alpha_i < C, \quad i = 1, \dots, n.$$

[2,10]. RBF (Radial Basis Function)

$$K(\mathbf{x}, \mathbf{y}) = \exp \left\{ -\frac{\|\mathbf{x} - \mathbf{y}\|^2}{\sigma^2} \right\}$$

σ SVC

3. (Boosting)

1984 Valiant [8]

가 Freun Shapire가 1995 (AdaBoosting) [3]

Freun Shapire 1999 [4]

가 $(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)$,

가

가

[4].
 a_t 가 ε_t 가 1/2
 $a_t \geq 0$, ε_t a_t h_t (weak learner)
 T 가 T

```

1.  $(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_n, y_n)$   $(\mathbf{x}_i, y_i)$ 가
    $D_1(i) = \frac{1}{n}$ 
2. for  $t = 1, \dots, T$ 
    $D_t$  error 가
    $\varepsilon_t = \Pr_{i \sim D_t}[h_t(x_i) \neq y_i]$   $h_t$ 
3.  $a_t = \frac{1}{2} \ln \left( \frac{1 - \varepsilon_t}{\varepsilon_t} \right)$ 
4.  $D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-a_t} & \text{if } h_t(x_i) = y_i \\ e^{a_t} & \text{if } h_t(x_i) \neq y_i \end{cases}$ 
    $= \frac{D_t(i) \exp(-a_t y_i h_t(x_i))}{Z_t}$ ,
    $Z_t$ 
end
5.  $H(x) = \text{sign} \left( \sum_{t=1}^T a_t h_t(x) \right)$ 

```

가 a_t
 ([3],[4]).

4. (Simulation)

SAS Enterprise Miner 4.0
 (SVC on Boosting : SOB)
 MATLAB Release12 RBF
 Data, Iris Data) (Wisconsin Breast Cancer Data, German Credit
 (, , SVM, SOB)

(Training Error) 가 (Test Error) 가
 (Sampling) 50 가
 . SVC C
 C=10 SVC
 0.05, 0.2 1 가 . SOB σ 0.05,
 0.2, 1.2 . T 10 . 5,
 10 .

4.1 1

1 2 Wahba et al. [11],
 $\sin(4\pi x)/4 - y + 0.37 = 0$ $\sin(4\pi x)/4 - y + 0.63 = 0$
 . $\sin(4\pi x)/4 - y + 0.37 = 0$ 1 0.95 , -1
 0.05 . $\sin(4\pi x)/4 - y + 0.63 = 0$ -1
 0.95 , 1 0.05 . $\sin(4\pi x)/4 - y + 0.37 = 0$
 $\sin(4\pi x)/4 - y + 0.63 = 0$ 1 -1 0.5 .
 가 가 1000 .

Table 1. 1

		T	Training Error (/)	Test Error (/)
	-	-	0.2025 / 0.0075	0.1966 / 0.0214
	-	-	0.1425 / 0.0065	0.1886 / 0.0210
SVM	0.05	-	0.1365 / 0.0064	0.1928 / 0.0192
	0.2	-	0.2351 / 0.0073	0.2423 / 0.0227
	1	-	0.4330 / 0.0060	0.4376 / 0.0336
SOB	0.05	5	0.0375 / 0.0079	0.2213 / 0.0216
		10	0.0190 / 0.0052	0.2227 / 0.0215
	0.2	5	0.1510 / 0.0099	0.1829 / 0.0210
		10	0.1502 / 0.0109	0.1807 / 0.0223
	1	5	0.2007 / 0.0126	0.2173 / 0.0234
		10	0.1997 / 0.0126	0.2159 / 0.0290
	1.2	5	0.2030 / 0.0124	0.2189 / 0.0234
		10	0.2027 / 0.0121	0.2190 / 0.0235

1 SVC, , SOB 가
 . SVM

가 10 가 , SOB 4가 가 0.1807 가 0.2, T

4.3 2

2 $x^2 + y^2 = 0.85^2$ $x^2 + y^2 = 0.7^2$
 . $x^2 + y^2 = 0.85^2$ 1 0.95 , -1 0.05
 . $x^2 + y^2 = 0.7^2$ -1 0.95 , 1 0.05
 . $x^2 + y^2 = 0.85^2$ $x^2 + y^2 = 0.7^2$ 1 -1
 0.5 .

Table 2. 2

		T	Training Error (/)	Test Error (/)
	-	-	0.1433 / 0.0056	0.1566 / 0.0195
	-	-	0.1350 / 0.0025	0.1866 / 0.0200
SVC	0.05	-	0.0703 / 0.0063	0.2093 / 0.0214
	0.2	-	0.1955 / 0.0057	0.2113 / 0.0181
	1	-	0.3815 / 0.0070	0.3871 / 0.0302
SOB	0.05	5	0.0014 / 0.0011	0.2237 / 0.0214
		10	0.0001 / 0.0004	0.2278 / 0.0228
	0.2	5	0.1192 / 0.0107	0.1894 / 0.0214
		10	0.1059 / 0.0104	0.1893 / 0.0194
	1	5	0.1461 / 0.0074	0.1547 / 0.0139
		10	0.1462 / 0.0075	0.1548 / 0.0139
	1.2	5	0.1469 / 0.0084	0.1555 / 0.0149
		10	0.1468 / 0.0086	0.1555 / 0.0148

가 2 SVC , SOB
 가 1 가 SVC , SOB
 , SOB 4가 가
 T
 0.1547 가 1, T가 5 가 .

4.3 Wisconsin Breast Cancer Data

Wisconsin Breast Cancer Data 30 가
 . 30
 . 569 , 30 10가
 3가 . 10가
 , , -1.0), (), ()
 / -1.0), (), , ,
 30 가
 ,
 SAS E-miner Variable
 selection W_perimeter(-
), W_concave_point(-), M_concave_points(
), W_texture(-)4 가 ,
 W_perimeter, W_concave_point, M_concave_points, W_texture, W_area(
) 5 가
 5

Table3. Wisconsin Breast Cancer Data

Algorithm		T	Training Error (/)	Test Error (/)
	-	-	0.0263 / 0.0021	0.0352 / 0.025
	-	-	0.0131 / 0.0052	0.0529 / 0.024
SVC	0.05	-	0.0104 / 0.0030	0.0451 / 0.018
	0.2	-	0.1071 / 0.0057	0.1014 / 0.0224
	1	-	0.3702 / 0.0027	0.3489 / 0.0064
SOB	0.05	5	0.0000 / 0.0000	0.0406 / 0.0149
		10	0.0000 / 0.0000	0.0444 / 0.0160
	0.2	5	0.0001 / 0.0006	0.0471 / 0.0139
		10	0.0000 / 0.0000	0.0465 / 0.0139
	1	5	0.0296 / 0.0060	0.0332 / 0.0118
		10	0.0285 / 0.0056	0.0330 / 0.0122
	1.2	5	0.0313 / 0.0062	0.0343 / 0.0130
		10	0.0308 / 0.0071	0.0337 / 0.0127

Wisconsin Breast Cancer Data 가 SOB가 4가 가
 가 0.0332 가 1, T가 5 가

4.4 German Credit Data

German Credit Data 1000 , 20 . 20
 7 , 13 .
 (), (), (), (), (),
 (), (), (), , (),
 (), 가 (), (), () .
 700 , 300 .
 German Credit Data Wisconsin Breast Cancer Data 가
 11 .

Table 4. German Credit Data

		T	Training Error (/)	Test Error (/)
	-	-	0.2175 / 0.0012	0.2624 / 0.0392
	-	-	0.2330 / 0.0032	0.3024 / 0.0423
SVC	0.05	-	0.0000 / 0.0000	0.3105 / 0.0169
	0.2	-	0.0630 / 0.0052	0.2863 / 0.0118
	1	-	0.3000 / 0.0000	0.3000 / 0.0000
SOB	0.05	5	0.0001 / 0.0005	0.3281 / 0.0225
		10	0.0000 / 0.0000	0.3349 / 0.0217
	0.2	5	0.0000 / 0.0002	0.3193 / 0.0196
		10	0.0000 / 0.0000	0.3238 / 0.0186
	1	5	0.1541 / 0.0116	0.2972 / 0.0271
		10	0.1248 / 0.0122	0.2933 / 0.0215
	1.2	5	0.1832 / 0.0121	0.2888 / 0.0230
		10	0.1610 / 0.0116	0.2841 / 0.0223

4가 가 가 0.2624 가 .
 SOB가 가 0.2841 .

4.5 Iris Data

Iris Data 150 , 4 . 4
 3가 (Setosa, Versicolor, Viginica)
 (sepal length), (sepal width),
 (petal length) (petal width) . cm

Iris Data Setosa Versicolor 50 가
 SVC, SOB 가
 T
 가 1.2, T가 10
 0.0396

Table 5. Iris Data

		T	Training Error (/)	Test Error (/)
	-	-	0.0242 / 0.0130	0.0460 / 0.0211
	-	-	0.0256 / 0.0110	0.0600 / 0.0321
SVC	0.05	-	0.0000 / 0.0000	0.0431 / 0.0299
	0.2	-	0.1139 / 0.0119	0.1427 / 0.0531
	1	-	0.2973 / 0.0162	0.3147 / 0.0690
SOB	0.05	5	0.0000 / 0.0000	0.0476 / 0.0299
		10	0.0000 / 0.0000	0.0480 / 0.0247
	0.2	5	0.0000 / 0.0000	0.0578 / 0.0314
		10	0.0000 / 0.0000	0.0560 / 0.0267
	1	5	0.0173 / 0.0110	0.0524 / 0.0290
		10	0.0109 / 0.0105	0.0483 / 0.0290
	1.2	5	0.0225 / 0.0134	0.0472 / 0.0313
		10	0.0152 / 0.0122	0.0396 / 0.0295

SVC SOB가
 SOB 가 1 가

5.

SON (Wisconsin Breast Cancer Data, German Credit Data, Iris Data) . SOB

0 가 가
 SOB T () T
 가 SOB가
 CV(Cross Validation)

6.

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