가상 확장된 배열 안테나를 이용한 근접 입사신호의 분해능 향상 기법

Enhanced Resolution of Spatially Close Incoherent Sources using Virtually Expanded Arrays

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요 약

본 논문에서는 임의 배열 안테나에 입사하는 협대역 인코히런트 신호의 도래각(DOA)을 추정하기 위한 분해능 향상 방안을 제시한다. 도래각 추정 알고리즘의 분해능은 배열 안테나의 크기에 의존하므로 배열 안테나의 크기를 증가시키면 분해능을 향상시킬 수 있지만, 실환경 하에서 배열 안테나의 물리적인 크기를 증가시키는 것은 실용적이지 못하다. 따라서 원 배열 안테나의 소자 간격을 가상적으로 확장시켜 크기가 서로 다른 가상 확장 배열 안테나들을 구성하고 이들의 공간 스펙트럼을 평균함으로서 분해능을 향상시키는 방안을 제안한다. 등간격 원형 배열 안테나에 입사된 인코히런트 신호에 표준 MUSIC 알고리즘과 제안 방식을 적용하여 제안 방식이 우수한 분해능 성능을 나타냄을 컴퓨터 시뮬레이션을 통하여 보였다.

ABSTRACT

In this paper, we propose a resolution enhancement method for estimating direction-of-arrival (DOA) of narrowband incoherent signals incident on a general array. The resolution of DOA algorithm is dependent on the aperture size of antenna array. But it is very impractical to increase the physical size of antenna array in real environment. We propose the method that improves resolution performance by virtually expanding the sensor spacing of original antenna array and then averaging the spatial spectrum of each virtual array which has a different aperture size. Superior resolution capabilities achieved with this method are shown by simulation results in comparison with the standard MUSIC for incoherent signals incident on a uniform circular array.

Key words : direction finding, virtual array, resolution.

I. Introduction

The resolution of DOA algorithm depends on the aperture size of antenna array, the number of sensors, the number of snapshots, and SNR [1]. The aperture size of antenna array has a major effect on the resolution capability among these parameters. The effective aperture size is determined by the carrier frequencies of incident signals. In prac-

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tical situation, the effective aperture size is reduced when the carrier frequencies of the incident signals are small because the physical aperture size is fixed. It is therefore necessary to increase the aperture size so as to improve the resolution capability. However, it is impractical to increase the array's physical size in real environment because of the mechanical structure of antenna array and strong wind. Especially in case of mobile DOA system, large aperture size can not be allowed because it has very limited installation space for antenna array.

We can increase the aperture size not physically but virtually in order to solve these problems. The virtual expansion means that the aperture size of original antenna array is increased mathematically. We can make numerous virtual arrays from original antenna array and these virtual arrays have different aperture size from each other. The virtual arrays can show better resolution performance than original antenna array and the resolution capability is proportional to the expansion size. But there is a limitation to the virtual expansion size because spurious peaks appear in the spatial spectrum of DOA algorithm if we increase the aperture size without increasing the number of sensors. It is not economical way to increase the number of sensors because each sensor needs receiver in order to receive signals. If we apply the virtual expansion method to the original array, then we can obtain the output signals of virtually expanded array by transforming the output signals of original array into the output signals of virtually expanded array. These transformed signals can be applied to existing DOA algorithms such as MUSIC [2].

We propose a new approach which improves the resolution capability. The proposed method expands the original antenna array not physically but virtually and averages many spatial spectrums of virtual arrays whose aperture sizes are different from each other to reduce spurious peaks. The main feature of our proposed method is that it is applicable to any array geometry in conjunction with the existing DOA algorithms in order to improve the resolution capability.

Ⅱ. Mathematical Model

Let us consider a general array composed of M identical sensors located at the points $\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_M$ in real three dimensional space and receiving $N(N<M)$ narrowband signals coming from directions $\{\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_N\}$. The vector $\boldsymbol{k}_j$ is a unit direction vector. Under this conditions, the output from mth sensor is found to be

$$x_m(t) = \sum_{n=1}^{N} s_n(t)\, e^{j(\omega_0 \kappa_n^T z_m / c + \Phi_n)} + \eta_m(t) \tag{1}$$

where c is the propagation velocity, ω_0 is the center radian frequency, Φ_n is a random initial phase of the nth signal, $\eta_m(t)$ is the additive noise at the mth sensor and T denotes the transpose operator. The outputs of m sensors can be represented in the vector format [2]

$$\boldsymbol{x}(t) = \boldsymbol{A}(\boldsymbol{\kappa})\, \boldsymbol{s}(t) + \boldsymbol{\eta}(t) \tag{2}$$

where $s(t)$ is a N-dimensional source vector, $\eta(t)$ is the M-dimensional additive sensor noise vector and $\boldsymbol{A}(\kappa)$ is the M×N steering matrix whose columns are steering vectors and written as

$$\boldsymbol{A}(\boldsymbol{\kappa}) = [\boldsymbol{a}(\boldsymbol{\kappa}_1), \boldsymbol{a}(\boldsymbol{\kappa}_2), \ldots, \boldsymbol{a}(\boldsymbol{\kappa}_N)] \tag{3}$$

$$\boldsymbol{a}(\boldsymbol{\kappa}_n) = [e^{j\omega_0 \kappa_n^T z_1 / c}, e^{j\omega_0 \kappa_n^T z_2 / c}, \ldots, e^{j\omega_0 \kappa_n^T z_M / c}]^T \tag{4}$$

In the analysis to follow, it is assumed that signals and additive noises are zero mean wide-sense stationary and ergodic complex-valued Gaussian random processes which are pairwisely uncorrelated. From our assumption, it follows that the covariance matrix of original array $\boldsymbol{R}_x$ is given by

$$\boldsymbol{R}_x = \boldsymbol{A}(\boldsymbol{\kappa})\boldsymbol{R}_s\boldsymbol{A}(\boldsymbol{\kappa})^H + \sigma^2\boldsymbol{R}_\eta \tag{5}$$

in which $R_s = E\{s(t)s(t)^H\}$, $\sigma^2\boldsymbol{R}_\eta = E\{\eta(t)\eta(t)^H\}$. In this expression, the E and H denote the expectation and Hermitian transpose operator, respectively.

Ⅲ. Averaged Spatial Spectrum of the Virtually Expanded Arrays

3-1 Virtual Expansion of Antenna Arrays

The resolution capability of DOA algorithm mainly depends on the aperture size of antenna array. But it is very impractical to increase the aperture size because the mechanical strength of antenna array structure is limited and mobile DOA system has a very limited installation space for antenna array. We can increase the aperture size not physically but virtually. The virtual expansion is the procedures that increase the aperture size of original antenna array mathematically. If we apply the virtual expansion method to the original array, then we can obtain the output signals of virtually expanded array from that of the original array. If we apply these output signals of virtually expanded array to the DOA algorithms, then enhanced resolution performance can be obtained.

In the virtual expansion procedures, the geometrical shape of original array must be maintained. That is to say, the direction of sensor location vector $\boldsymbol{z}_k$ must be maintained. Only the length of the sensor location vector is virtually increased and the virtual expansion ratio of all sensor location vectors must be same. The array whose radius is virtually enlarged h times as large as that of the original array is shown in Fig. 1. The steering matrix of the virtually expanded array which is a function of κ and h is expressed as

$$\boldsymbol{B}(\boldsymbol{\kappa},h) = [\boldsymbol{b}(\boldsymbol{\kappa}_1,h), \boldsymbol{b}(\boldsymbol{\kappa}_2,h), \ldots, \boldsymbol{b}(\boldsymbol{\kappa}_N,h)] \in C^{M\times N} \tag{6}$$

where $b(\kappa_n, h)$ is the steering vector of virtually expanded array and is given by

$$\boldsymbol{b}(\boldsymbol{\kappa}_n,h) = [e^{j\omega_0 h\kappa_n^T z_1/c}, e^{j\omega_0 h\kappa_n^T z_2/c}, \ldots, e^{j\omega_0 h\kappa_n^T z_M/c}]^T \tag{7}$$

where h denotes the array expanding factor. If $h=1$, then $\boldsymbol{B}(\kappa, h)$ corresponds to the original steering matrix $\boldsymbol{A}(\kappa)$. The output vector of the virtually expanded array is found to be

$$\boldsymbol{y}(t) = \boldsymbol{B}(\boldsymbol{\kappa},h)\, s(t) + \boldsymbol{\eta}_B(t) \tag{8}$$

where $\eta_B(t)$ is the M-dimensional noise vector of the expanded array.

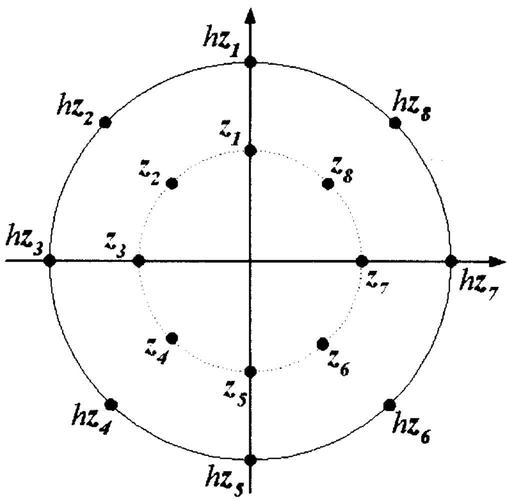


Fig. 1. Virtual expansion of antenna array.

3-2 Transformation Matrix

If matrix $\boldsymbol{A}(\kappa)$ and $\boldsymbol{B}(\kappa, h)$ have full rank of N, then there exists the M×M transformation matrix $\boldsymbol{T}(\kappa, h)$ such that

$$\boldsymbol{B}(\boldsymbol{\kappa},h) = \boldsymbol{T}(\boldsymbol{\kappa},h)\,\boldsymbol{A}(\boldsymbol{\kappa}) \tag{9}$$

The proof of (9) is given in [3]. From equation (9), the covariance matrix $\boldsymbol{R}_y$ of $\boldsymbol{y}(t)$ is given by

$$\boldsymbol{R}_y = \boldsymbol{T}(\kappa,h)\boldsymbol{R}_x\boldsymbol{T}(\kappa,h)^H \quad (10)$$

Up to this point, we have assumed that κ is the same as true angle. Since κ is unknown, however, it must be estimated in practical situation. Wang and Kaveh introduced the preliminary processing to obtain good estimates for κ[3]. We also use the same scheme of this preliminary processing method under the assumption that the preliminary estimates $\{\hat{\kappa}_1, \hat{\kappa}_2, \dots, \hat{\kappa}_M\}$ are in the neighborhood of the true angles.

To estimate the transformation matrix $\boldsymbol{T}(\kappa, h)$, the M×M modified steering matrices $\widetilde{\boldsymbol{B}}(\kappa, h)$ and $\widetilde{\boldsymbol{A}}(\kappa, h)$ are specified by

$$\widetilde{\boldsymbol{B}}(\kappa,h) = [\boldsymbol{b}(\hat{\kappa}_1,h), \boldsymbol{b}(\hat{\kappa}_2,h), \dots, \boldsymbol{b}(\hat{\kappa}_M,h)] \quad (11)$$

$$\widetilde{\boldsymbol{A}}(\kappa) = [\boldsymbol{a}(\hat{\kappa}_1), \boldsymbol{a}(\hat{\kappa}_2), \dots, \boldsymbol{a}(\hat{\kappa}_M)] \quad (12)$$

The choice of the transformation matrix plays an important role in establishing the proposed algorithm effectively. We can optimally select the transformation matrix in the least squares sense. We now consider the least squares problem of determining the transformation matrix $\boldsymbol{T}(\kappa, h)$ so that the Frobenius norm of the residual matrix $\boldsymbol{E} = \widetilde{\boldsymbol{B}}(\kappa, h) - \boldsymbol{T}(\kappa, h)\widetilde{\boldsymbol{A}}(\kappa)$ is minimum. Mathematically, this problem can be expressed as

$$\min_{\boldsymbol{T}(\kappa,h)} \left\| \widetilde{\boldsymbol{B}}(\kappa,h) - \boldsymbol{T}(\kappa,h)\widetilde{\boldsymbol{A}}(\kappa) \right\|_F \quad (13)$$

where $\|.\|_F$ denotes the Frobenius norm.
Two kinds of transformation matrix can be considered. One is a nonunitary matrix, the other is an unitary matrix. We shall herein consider the problem as that of selecting an unitary transformation matrix $T(\kappa, h)$ which minimizes $\|E\|_F$ with $T(\kappa, h)$ preserving the Frobenius norm, that is, $\left\|\boldsymbol{T}(\kappa,h)\widetilde{\boldsymbol{A}}(\kappa)\right\|_F = \left\|\widetilde{\boldsymbol{B}}(\kappa,h)\right\|_F$. The solution of this constraint minimization problem is given by [4]

$$\boldsymbol{T}(\kappa,h) = \boldsymbol{V}\boldsymbol{U}^H \quad (14)$$

where the unitary matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ correspond to the singular value decomposition of $\widetilde{\boldsymbol{A}}(\kappa)\widetilde{\boldsymbol{B}}(\kappa,h)^H$, that is, $\widetilde{\boldsymbol{A}}(\kappa)\widetilde{\boldsymbol{B}}(\kappa,h)^H = \boldsymbol{U}\Sigma\boldsymbol{V}^H$. We can estimate the virtual arrays covariance matrix $\boldsymbol{R}_y$ from equation (10) using the transformation matrix of equation (14) and covariance matrix of original array $\boldsymbol{R}_x$. This covariance matrix $\boldsymbol{R}_y$ can be applied to the existing DOA algorithms such as MUSIC in order to improve the resolution capability.

3-3 Averaged Spatial Spectrum of Virtual Arrays

The spatial spectrums of virtually expanded arrays show spurious peaks because the sensor spacing is larger than the original array without increasing the number of sensors. It is very difficult to discern the real peaks from these spurious peaks when the magnitude of spurious peaks are high. The magnitudes and occurring frequencies of spurious peaks are proportional to the aperture size. Therefore we can increase the aperture size to a certain extent when we use only one virtual array. But if we use several virtual arrays with different expanding factor h, then we can solve this problem.

The positions and magnitudes of spurious peaks of virtual array depend on the expanding factor h because the locations of sensors are changed by the expanding factor. If we average the spatial spectrums of virtual arrays which have different expanding factor from each other, then the averaged spatial spectrum shows reduced spurious peaks. Therefore we can discern real peaks from spurious peaks. The averaged spatial spectrum can be represented as follows

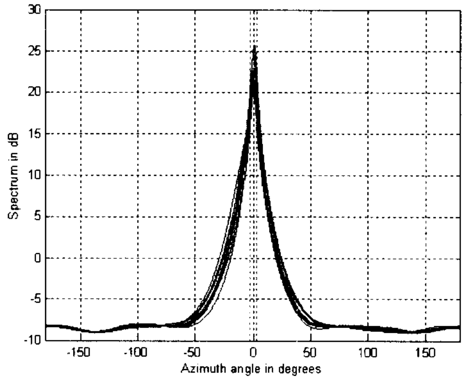
$$P_{av}(\hat{\mathbf{e}}) = \frac{1}{Q}\sum_{i=1}^{Q} P_i(\hat{\mathbf{e}}) \tag{15}$$

where Q is the average number, $P_i(\kappa)$ is the spatial spectrum of ith virtual array which has the expanding factor h_i.

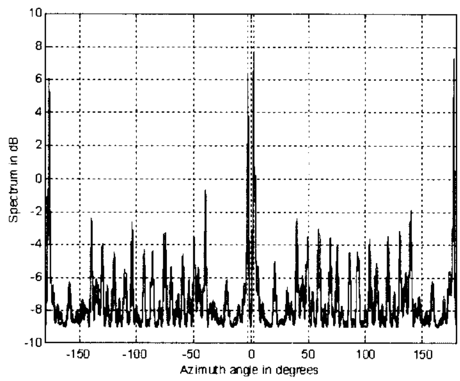
Ⅳ. Simulations

To test the effectiveness of the proposed methods, in resolving closely spaced plane waves, the case of two narrowband incoherent sources incident on a uniform circular array with M=8 sensors is considered. The radius of the original array is 0.5λ. Nine virtual arrays are used to enhance the resolution capability and the array expanding factors of virtual arrays are $\boldsymbol{h}$= [9, 10, 11, 12, 13, 14, 15, 16, 17]. We used MUSIC as a spectrum estimation algorithm. The sensor noise vector $\eta(t)$ is taken as a complex valued additive white Gaussian process whose components have identical variances and are statistically independent of the source signals.

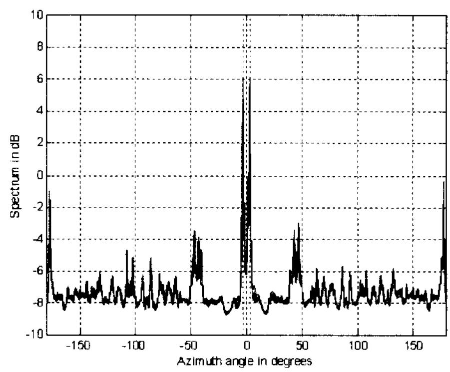
Two incident plane waves are taken to have bearing angles $\theta_1 = -3.0°$ and $\theta_2 = 3.0°$. The sensor noise variance is selected so that the SNR of 5 dB is obtained. In employing the proposed methods, a set of initial angles is chosen to be {−3.5°, −3.0°, −2.5°, −2.0°, 2.0°, 2.5°, 3.0°, 3.5°}. This experiment was repeated ten times and the resulted ten spectral estimates are shown in superimposed fashion in Figs. 2(a), (b) and (c). From these estimates, it is evident that the MUSIC spectrum of original array fails to resolve the two signals in any of ten trial runs and the MUSIC spectrum of single virtual array with $\boldsymbol{h}$= 17 partially resolves two signals. The averaged MUSIC spectrum of nine virtual arrays, however; achieved a consistent resolution and has small



(a) Spatial spectrum of original array (h=1, SNR=5 dB)



(b) Spatial spectrum of virtually expanded array (h=17, SNR=5 dB)



(c) Averaged spatial spectrum of virtually expanded arrays(h=[9, 10, 11, 12, 13, 14, 15, 16, 17], SNR =5 dB)

Fig. 2. Ten statistically independent superimposed bearing estimates for two incoherent plane waves at azimuth angles of −3.0° and 3.0°.

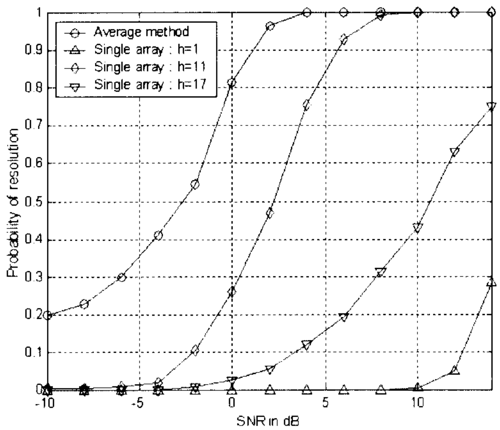


Fig. 3. Probability of resolution versus SNR(azimuth=[-3.0°, 3.0°]).

spurious peaks compared with the single virtual array case.

To investigate the comparative resolution capability of the proposed method, we carry out computer simulation with the signal to noise ratio (*SNR*) changed. Two hundred indepentent trials are made. In this case, azimuth angles of sources are {-3.0°, 3.0°} and the preliminary estimates of azimuth angles are taken to be {-3.5°, -3.0°, -2.5°, -2.0°, 2.0°, 2.5°, 3.0°, 3.5°}.

The probability of resolution is herein defined as the probability that one source is estimated in the interval [-3.0° -2.0°, 3.0° +2.0°] and simultaneously the second source in the interval [3.0° -2.0°, 3.0° +2.0°]. Fig. 3 shows the probability of resolution when *SNR* is changed. From the results of Fig. 3, we know that the averaged spectrum method with nine virtual arrays has a better resolution performance than the virtual expansion method with single array.

V. Conclusions

A new algorithm has been presented for improving the resolution capability of sensor array on which multiple narrowband incoherent sources are impinging. The fundamental concept is based on the virtual expansion of the original array and average process of spatial spectrums of virtual arrays which have different aperture size. From the simulation results, it has been found that the proposed algorithms provide superior resolution performance relative to that of the standard MUSIC.

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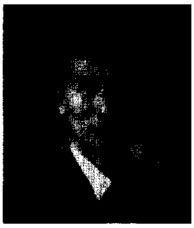

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