한국도시방재학회논문집
제2권 4호 2002년 12월
pp. 97 ~ 116

Formulation of Dynamic Vehicle-Bridge Interaction Problems

Gyu Sei Yi*

Abstract

In this papers, a finite element formulation is proposed for dynamic analysis of vehicle-bridge interaction problems under realistic loading conditions. Although the formulation presented in this paper is based on the consideration of only a single traversing vehicle, it can be extended to include several different bridge configurations. The traversing vehicle and the vibrating bridge superstructure are considered as an integrated system. Hence, although material and geometric nonlinearities are excluded, this introduces nonlinearity into the problem. Various vehicle models, including those with suspension systems, are considered. Traveling speed of the vehicle can be varied. The finite element discretization of the bridge structure permits the inclusion of arbitrary geometrical configurations, and surface and boundary conditions. To obtain accurate solutions, time integration of the equation of vehicle-bridge motion is carried out by using the Newmark method in connection with a predictor-corrector algorithm.

Key words :

Introduction

In this paper, a finite element based procedure is formulated for the analysis of vehicle-bridge interaction problems under general traffic conditions. An accurate dynamic response of bridge structures is obtained by modeling both the vehicle and the bridge structure in a relatively realistic manner. The accuracy in solution is obtained by taking into account (a) the interaction between vehicles and bridge and that among vehicles, (b) bridge type and characteristics, (c) vehicle characteristics and operating conditions, and (d) roadway surface roughness. Such an analysis procedure may be considered unnecessarily complex to implement into a conventional bridge design process.

Conventional bridges have been successfully designed on the basis of existing code procedures. Traditionally, a so-called impact factor is used to take into account the dynamic effects induced by traversing vehicles. This is done by predicting the static response of the bridge superstructure on the basis of equivalent live loads representing several different truck loading conditions. Then, static stress resultants are amplified by using an impact factor. These amplified values constitute the basis for design. The impact factor is established to represent the ratio of the difference between the maximum live load dynamic and static responses to maximum live load static response. However, there is no clear definition of impact factor, and its interpretation widely varies in the literature. For this reason, this paper uses the term dynamic amplification factor (DAF).

* Member, Assistant Professor, Dept. of Civil Engineering, SunMoon University

It is noted that, although an amplified static analysis process may provide adequate and safe designs, it cannot reflect the true characteristics of bridge dynamics and performance. On the other hand, a realistic analysis based design method, in connection with an optimization process, can provide sufficient information about the true behavior of bridge structures. Clearly, this provides true safety and economy in design.

Although a realistic analysis procedure is more extensive and effort consuming, it permits the consideration of general traffic conditions, rather than a limited number of equivalent vehicle loadings. Such an analysis may supplement experiments since it is usually not practical to operate several experimental trucks simultaneously. This type of extensive analysis may be an excellent tool in evaluating existing bridges for additional as well as new loading conditions. Furthermore, it may help provide appropriate speed and truck load limits, and establish guidelines on evaluating truck suspension systems. Hence, efforts in developing an analysis method such as the one described in this paper, are essential.

A pioneering work involving an analytical approach for determining the dynamic response of highway bridges is presented by Fleming and Romualdi (1961). Relatively updated reviews of the literature concerning vehicle-guideway interaction problems are provided by Ting and Yener (1983), and Yener and Chompooming (1991a). Traditionally, in formulating bridge behavior, the interaction between the vehicle and the bridge is neglected. This approach, commonly referred to as the moving force approximation, physically corresponds to neglecting the dynamic effects of the traversing vehicle on the vibrating bridge. On the other hand, the procedure which includes the dynamic effects of the vehicle is generally referred to as the moving mass approximation (Yener and Chompooming 1991a). The solution algorithm proposed in this paper takes into account both of these approximations by considering a number of different vehicle models in the problem formulation. The vehicle speed can be varied arbitrarily. It is noted that although, for brevity and clarity, the formulation presented in this paper is restricted to the consideration of one vehicle only, multiple vehicles and arbitrary traversing directions can easily be accommodated into the problem formulation. It is assumed that, while traversing the bridge, the vehicle remains in contact with the bridge surface. In this paper, the roadway surface is assumed to be smooth.

Although the material and geometric linearities are preserved in the formulation, consideration of the vehicle and bridge superstructure as an integrated system introduces nonlinearity into the problem. Roadway surface irregularities, as well as multiplicity of vehicles, traffic lanes, and traversing directions have already been considered (Yener, Chompooming and Yi 1993). Multiple spans, and material and geometric nonlinearities will be the topics of future publications. The implementation of the proposed solution methodology is described in (Yener, Chompooming and Yi 1993a).

BRIEF DESCRIPTION OF SOLUTION PROCEDURE

As mentioned earlier, the consideration of interaction between the vehicle and bridge results in a nonlinear structural response. This is due to the fact that, in each time step, interaction force vector Fin, hence structure force vector F, varies in accordance with the vehicle-bridge coupling motion. Therefore, the solution procedure adopted is incremental in time.

At the beginning of each discrete time step, the structural degrees of freedom (DOF's) are predicted on the basis of previously determined quantities in accordance with the Newmark method (Newmark 1959). Then, the vehicle

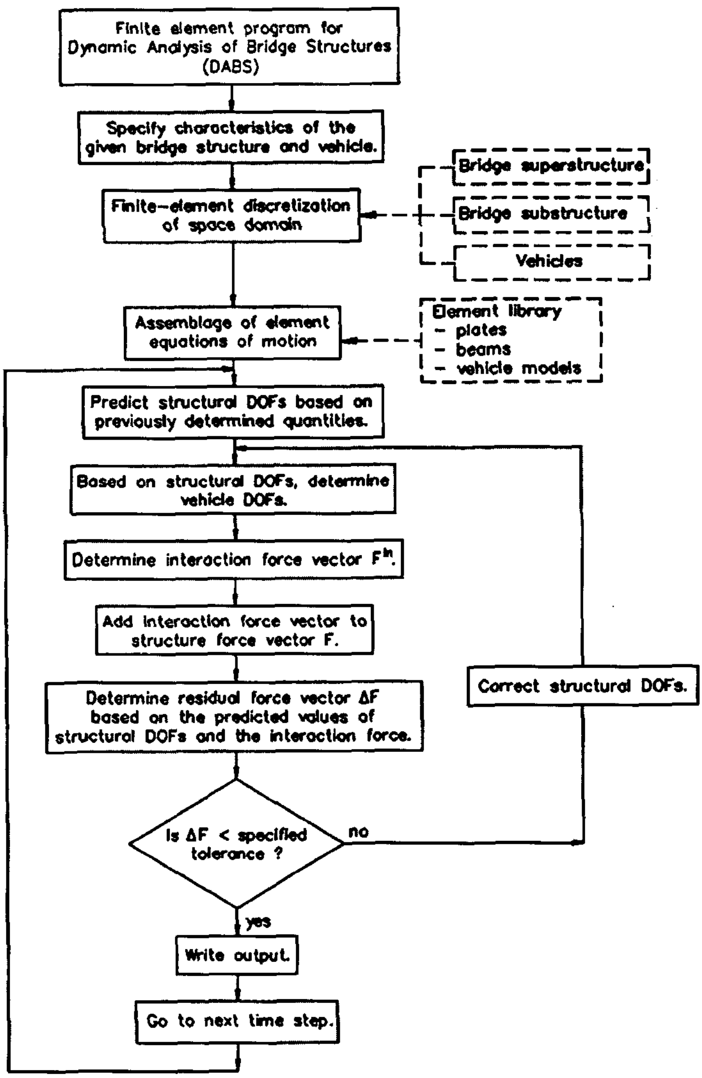

Fig 1. Major Tasks in the Structural Analysis Module

DOF's are determined based on the predicted structural DOF's. The interaction force vector, due to the vehicle-bridge system, is evaluated and added to the structure force vector. Based on the predicted values of structural DOF's and the interaction force, the governing matrix equation of vehicle-bridge motion is evaluated. If the governing equation is satisfied within a specified tolerance, the iterative process is completed for the current time step. Otherwise, the structural DOF's are corrected and the iterative process within the current time step is continued. If desired, once the nodal displacements, velocities, and accelerations are determined, stresses in the elements can be computed.

A depiction of the solution algorithm is given in Fig. 1. The input data primarily consist of bridge geometry, bridge boundary conditions, vehicle characteristics, finite element discretization data, and material properties. The finite element discretization of space domain is carried out on the basis of the discretization

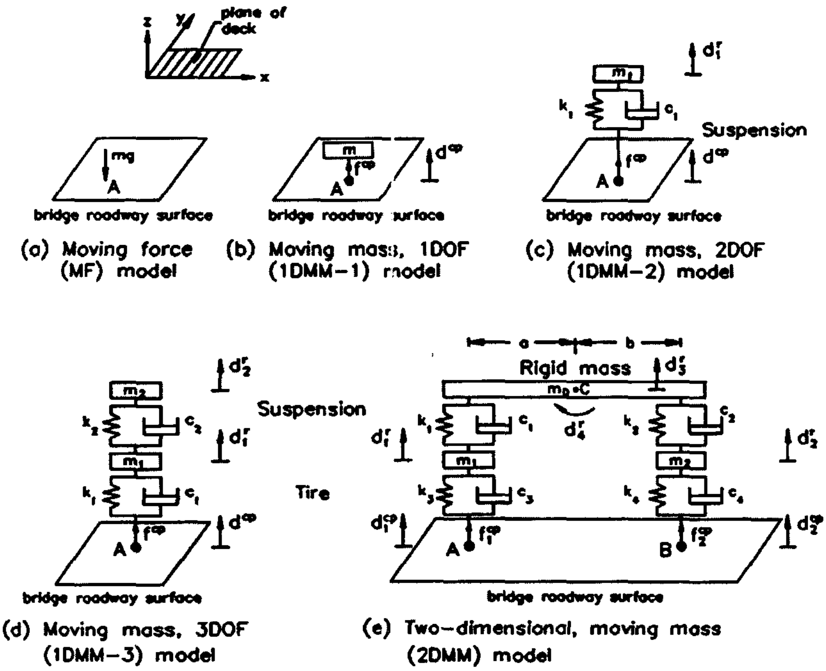

Fig 2. Various Vehicle Models

input data. The matrix equations of motion are set up for the elements and vehicle. These equations are assembled to obtain the structure matrix equation of motion on the basis of element connectivity. Clearly. consideration of such a procedure requires a discussion, as well as the formulation, of vehicle, structure, and interaction models.

VEHICLE MODELING

Most heavy vehicles consist of several major components, such as tractors, trailers, and suspension systems. These characteristics of vehicles are generally modeled by a set of springs and dampers. Fig. 2 illustrates the vehicle models used in this investigation, which range from a simple moving force model to a relatively elaborate two-dimensional model. Suspension systems are idealized by a finite number of elements, consisting of rigid mass, linear spring, and viscous damper elements. As indicated in Figs. 2d and 2e, tires are similarly modeled by an assemblage of spring and damper elements. The spring and damper coefficients are generally determined from experimental data.

The moving force model (MF) shown in Fig. 2a is included for comparison with moving mass models of varying complexity. The simple moving mass model (1DMM-1) in Fig. 2b has one DOF which represents the vertical displacement of the tire contact point A directly under the vehicle axle. The moving mass-damped spring models (1DMM-2 and 1DMM-3) in Figs. 2c and 2d have two and three DOF's, respectively. In the two-dimensional six DOF's model (2DMM, Fig. 2e), the interaction between tractor and trailer is considered directly through the motion of rigid mass m0. Furthermore, appropriate combinations of models 1DMM-3 and 2DMM can be used to represent a variety of multi-axle heavy vehicles. It should be noted that three-dimensional vehicle models with additional DOF's can also be considered in the

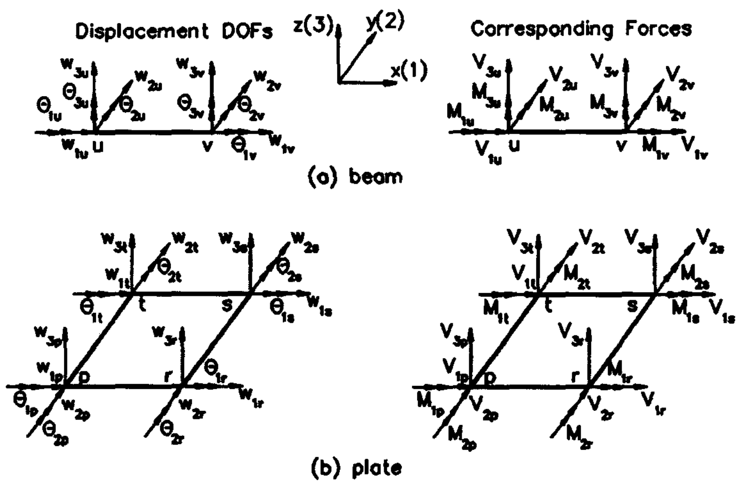

Fig 3. Beam and Plate Elements for Bridge Analysis

problem formulation. However, in general, the effects of roll motion of the vehicle on bridge dynamics are of relatively little importance as compared to those of pitch and vertical motion (Huang 1976).

MODELING OF BRIDGE STRUCTURE

Bridge Superstructure

For illustrative purposes, highway bridge superstructures consisting primarily of a concrete deck slab and steel girders are considered in this study. Bridge girders and, if included, transverse diaphragms are considered as an assemblage of beam elements. On the other hand, the deck slab is modeled as an assemblage of plate elements. The flexural, torsional, membrane, and shear modes of vibration are considered in the formulation of both beam and plate elements, Fig. 3 shows the DOF's, as well as the corresponding nodal forces, for beam and plate elements. The modeling scheme permits the consideration of any number of lanes on the deck.

Bridge Substructure

The bridge substructure can be modeled as either a rigid mass or an assemblage of plate and/or beam elements resting on a stable foundation. When the substructure is considered as a rigid mass, it is implied that the explicit consideration of substructure is not included in the analysis. When the bridge substructure is represented as an assemblage of plate and/or beam elements, the interaction between the superstructure and substructure is implicitly taken into account. Due to the nature of the finite element algorithm used in this study, bridge bearings are assumed not to permit longitudinal movement of the superstructure with respect to the substructure.

VEHICLE-BRIDGE INTERACTION KINEMATIC COUPLING

A difficulty in analyzing vehicle-bridge interaction problems involves finding an appropriate procedure for treating the kinematic coupling of the vehicle-superstructure system. To illustrate the kinematic coupling, we consider the following generic form of the equation of motion of the simplified vehicle-bridge system shown in Fig. 4 at vehicle contact point A.

$$M \ddot{w} + C \dot{w} + K w = f^{cp} \qquad (1)$$

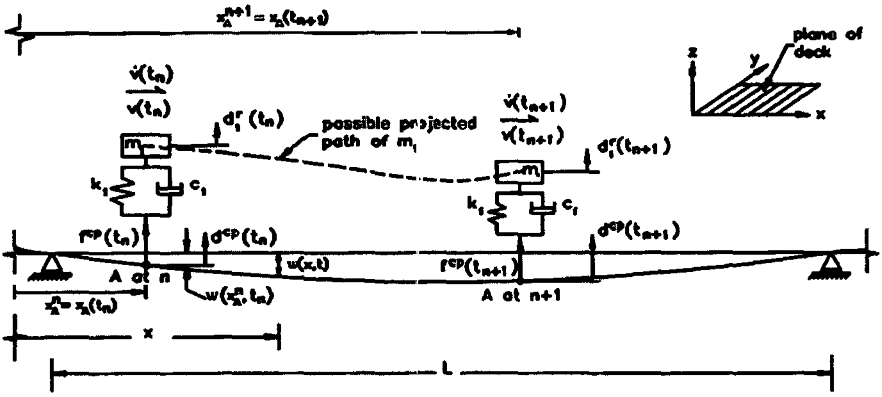

Fig 4. Vehicle Traversing on a Simple Bridge Model

in which the superstructure properties are represented by M(mass), C(damping), K(stiffness), and w(displacement). The displacement function w represents the vertical displacement of the superstructure at the contact point at time t, i.e., at $x = x_A(t)$. The notations $\dot{w}$ and $\ddot{w}$ represent the corresponding velocity and acceleration, respectively, in which superposed dots denote time derivatives. Since the position of vehicle contact point A varies with time t, it is implied that any quantity, which is a function of the position of the vehicle contact point, is also an implicit function of time. It is noted that the bridge is modeled as a one-dimensional simple structure in Fig. 4 for the sole purpose of clarity in presenting the following argument.

The term f^{cp} in (1) represents the interaction force between the vehicle and the superstructure, which can be viewed as the force exerted by the moving vehicle on the bridge surface. Hence, our objective appears to be conventional in that we seek, for an externally applied time dependent load f^{cp}, the solution of the linear, second order ordinary differential equation given in (1) for w. However, for a moving mass problem such as that illustrated in Fig. 4, interaction force f^{cp} is a function of the vehicle motion, which in turn depends on the superstructure motion at the vehicle contact point. This interdependency of motions is generally referred to as the kinematic coupling of the vehicle-bridge system. Consequently, it can be concluded that f^{cp} is a function of among others, w and its derivatives. Hence, at each time step, f^{cp} in (1) is not known a priori, and rather is a function of unknowns w and its derivatives, as well as the vehicle characteristics and DOF's. Such consideration of the kinematic coupling in the determination of interaction force f^{cp} renders (1) nonlinear, and introduces response nonlinearity. Clearly, when the kinematic coupling is neglected, f^{cp} simply is an externally applied force being a function only of time. The resulting problem is of the moving force type.

Prior to discuss the details of the mathematical formulation, it would be beneficial to briefly mention the characteristics of the generalized governing differential equation of motion of a structural system.

SEMIDISCRETIZED MATRIX DIFFERENTIAL EQUATION OF MOTION

The general form of the differential equation of motion of a structural system can be semi-discretized with respect to space coordinates, and written in matrix form as (Hughes 1987)

$$M \ddot{D} + C \dot{D} + K D = F \qquad (2)$$

where M, C, and K are the mass, damping, and stiffness matrices for the system, respectively. $D = D(t)$ is the nodal displacement vector of semi-discretized system, and $\dot{D}$ and $\ddot{D}$ are the corresponding nodal velocity and acceleration vectors, respectively.

F in (1) is the structure force vector, which is a function of time t and consists of external force components. The interaction force produced by the traversing vehicle, and gravity loads of the vehicle and bridge structure, are considered as external loads. Furthermore, an equivalent uniformly distributed wind load can be imposed on the superstructure if required, as well as the equivalent static force when bridge structure is located in the region where earthquakes may be anticipated. Wind loads and other types of loads on both super and substructures, and on traversing vehicles, may also be considered.

EQUATIONS OF MOTION OF VEHICLE MODELS

A general form of the vehicle equation of motion can be written as

$$m \ddot{d} + c \dot{d} + k d = f \qquad (3)$$

where m, c, and d are the mass, damping, and stiffness matrices of the lumped vehicle model, respectively. d is the vector of DOF's representing the motion of the vehicle model. $\dot{d}$ and $\ddot{d}$ are the corresponding velocity and acceleration vectors, respectively, and f is the corresponding vector of external forces. The vehicle DOF's can be partitioned as

$$d = \left(\frac{d^{r}}{d^{cp}} \right) \text{ and } f = \left(\frac{f^{r}}{f^{cp}} \right) \qquad (4)$$

where the superscript cp refers to the DOF's at the contact point and r represents the remaining DOF's.

One-Dimensional Heavy Vehicle Models

One-dimensional models discussed in this section refer to those shown in Figs. 2a through 2d. In the equations of vehicle motion, m denotes the lumped mass of the vehicles, g the acceleration of gravity, k the spring constant, c the damping constant, and f^{cp} the force exerted on the superstructure at the contact point by a moving vehicle.

MF Model - In Fig. 2a, the MF model is represented by a single force. This force, produced by the static weight of the vehicle, can be calculated from

$$f^{cp} = m g \qquad (5)$$

1DMM-1 Model - Since neither damping nor spring is used for 1DMM-1 model in Fig. 2b, (3) becomes

$$m \ddot{d}^{cp} = f^{cp} - m g \qquad (6)$$

1DMM-2 Model -For 1DMM-2 model shown in Fig. 2c, (3) can be written as (Yener and Chompooming 1991b)

$$\begin{bmatrix} m_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{d}_1^{r} \\ \ddot{d}^{cp} \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix} \begin{Bmatrix} \dot{d}_1^{r} \\ \dot{d}^{cp} \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} d_1^{r} \\ d^{cp} \end{Bmatrix} = \begin{Bmatrix} -m_1 g \\ f^{cp} \end{Bmatrix} \qquad (7)$$

1DMM-3 Model - For 1DMM-3 model shown in Fig. 2d, (3) takes the form (Yener and

Chompooming 1991b)

$$\begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{d}_2^r \\ \ddot{d}_1^r \\ \ddot{d}^{cp} \end{Bmatrix} + \begin{bmatrix} c_2 & -c_2 & 0 \\ -c_2 & c_2+c_1 & -c_1 \\ 0 & -c_1 & c_1 \end{bmatrix} \begin{Bmatrix} \dot{d}_2^r \\ \dot{d}_1^r \\ \dot{d}^{cp} \end{Bmatrix} + \begin{bmatrix} k_2 & -k_2 & 0 \\ -k_2 & k_2+k_1 & -k_1 \\ 0 & -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} d_2^r \\ d_1^r \\ d^{cp} \end{Bmatrix} = \begin{Bmatrix} -m_2 g \\ -m_1 g \\ f^{cp} \end{Bmatrix} \quad (8)$$

It should be noted that, in the absence of m_2, k_2, and c_2, (8) can be reduced to (7).

EQUATION OF MOTION OF BRIDGE MODEL

The equation of motion for plate elements is formulated based on the Reissner-Mindlin plate theory, in which the kinematic relationships contain shear deformations (Yener and Chompooming 1991b). Membrane contributions are considered in the determination of plate stiffness. The equation of motion for beam elements is formulated based on the Timoshenko beam theory, which takes into account the shear deformations. Axial and torsional contributions are considered in the determination of beam stiffness.

The semi-discretized equation of motion for plate and beam elements can be written as

$$M^e \ddot{D}^e + C^e \dot{D}^e + K^e D^e = F^e \quad (9)$$

where M^e and K^e are the element mass and stiffness matrices, respectively. C^e is the viscous damping matrix, and F^e the element nodal force vector.

Plate Elements

Nodal Displacement Vector - We define the nodal displacement vector D^e for plate element e as

$$D^e = (w_{11}^e, w_{21}^e, w_{31}^e, \theta_{11}^e, \theta_{21}^e, \ldots, w_{1a}^e, w_{2a}^e, w_{3a}^e, \theta_{1a}^e, \theta_{2a}^e, \ldots, w_{1n_{en}}^e, w_{2n_{en}}^e, w_{3n_{en}}^e, \theta_{1n_{en}}^e, \theta_{2n_{en}}^e)^T \quad (10)$$

where a=1,...,n_{en}, in which nen is the number of element nodes. Element nodes are numbered in the counterclockwise direction. As implied in (10), there are 5 DOF's at each node of element e. The total number of DOF's for element e is then equal to $5n_{en}$. For example, for the 4-node plate element (a=1,...,4), the 20×1 element nodal displacement vector can be written as

$$D^e = (w_{11}^e, w_{21}^e, w_{31}^e, \theta_{11}^e, \theta_{21}^e, w_{12}^e, w_{22}^e, w_{32}^e, \theta_{12}^e, \theta_{22}^e, w_{13}^e, w_{23}^e, w_{33}^e, \theta_{13}^e, \theta_{23}^e, w_{14}^e, w_{24}^e, w_{34}^e, \theta_{14}^e, \theta_{24}^e)^T$$

Fig. 5 shows each component of D^e for a 4-node, bilinear plate element in the local coordinate system.

Stiffness Matrix - The element stiffness matrix K^e can be written as a combination of bending stiffness K_b^e, shear stiffness K_s^e, and membrane stiffness K_m^e (Yener and Chompooming 1991b).

$$K^e = K_b^e + K_s^e + K_m^e \quad (11)$$

where

$$K_b^e = \int_{A^e} B^{b^T} D^b B^b dA, \quad K_s^e = \int_{A^e} B^{s^T} D^s B^s dA, \quad K_m^e = \int_{A^e} B^{m^T} D^m B^m dA \quad (12)$$

In these expressions, A^e is the area of the plate element. The constitutive matrices D^b, D^s and D^m, and strain-displacement matrices B^e, B^s,

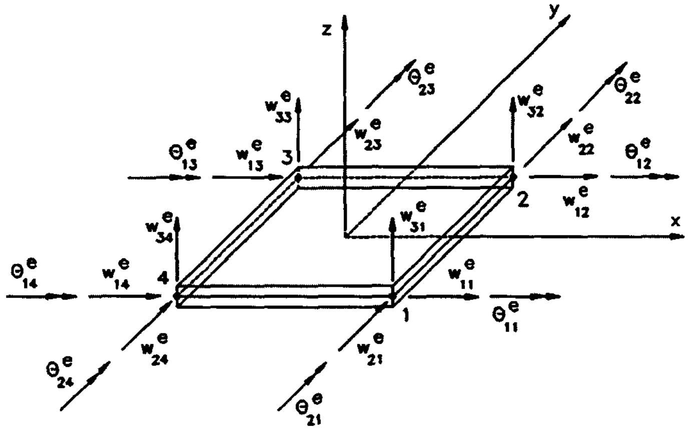

Fig. 5 Nodal Displacement Components for the Plate element

and B^m.

Mass Matrix - The element mass matrix M^e can be written as (Yener and Chompooming 1991b)

$$M^e = [M^e_{pq}] \tag{13}$$

where

$$M^e_{pq} = \delta_{ij} \int_{A^e} N_a m_p N_b dA \tag{14}$$

and N_a is the element displacement shape function associated with note a, and δ_{ij} is the Kronecker delta. Furthermore, p=5(a-1)+i, and q=5(b-1)+j. Indices a and b range from 1 to n_{en}, and I and j range from 1 to 5. The notation m_p is equal to ρt_p when i=j=1,2,3, corresponding to transverse and in-plane inertia forces, and $m_p = \rho(t_p)^3/12$ when i=j-4,5, corresponding to rotary inertia. ρ is the element mass density, and t_p the plate element thickness.

Force Vector - The element nodal force vector F^e can be written as (Yener and Chompooming 1991b)

$$F^e = \{F^e_j\} \tag{15a}$$

where

$$F^e_j = \int_{A^e} N_a F_i dA \text{ corresponding to DOF } j = 5a - 5 + i$$

$$F^e_j = \int_{A^e} N_a C_i dA \text{ corresponding to DOF } j = 5a - 2 + k \tag{15b}$$

in which F_i, for i=1,2,3, are the total applied force components per unit area in the x, y, and z directions, and C_k for k=1,2, are the total applied coupling force components per unit area about the x and y axes.

Beam Elements

Nodal Displacement Vector - We define the nodal displacement vector D^e for beam element e as

$$D^e = (w^e_{11}, w^e_{21}, w^e_{31}, \theta^e_{11}, \theta^e_{21}, \theta^e_{31}, \ldots, w^e_{1a}, w^e_{2a}, w^e_{3a}, \theta^e_{1a}, \theta^e_{2a}, \theta^e_{3a}, \ldots, w^e_{1n_{en}}, w^e_{2n_{en}}, w^e_{3n_{en}}, \theta^e_{1n_{en}}, \theta^e_{2n_{en}}, \theta^e_{3n_{en}})^T \tag{16}$$

There are 6 DOF's at each node of element e.

The total number of DOF's is equal to $6n_{en}$. Fig. 6 shows the components of D^e for a 2-node, linear beam element in the local coordinate system.

Stiffness Matrix - The element stiffness matrix K^e can be written as a combination of bending stiffness K_b^e, shear stiffness K_s^e, torsional stiffness K_t^e, and axial stiffness K_a^e matrices (Yener and Chompooming 1991b).

$$K^e = K_b^e + K_s^e + K_t^e + K_a^e \tag{17}$$

where

$$K_b^e = \int_0^{\ell^e} B^{b^T} D^b B^b dx, \quad K_s^e = \int_0^{\ell^e} B^{s^T} D^s B^s dx \tag{18a}$$

$$K_t^e = \int_0^{\ell^e} B^{t^T} (GJ^e) B^t dx, \; K_a^e = \int_0^{\ell^e} B^{a^T} (EA^e) B^a dx \tag{18b}$$

where E and G are the material Young's modulus and shear modulus, respectively. The constitutive matrices are D^b and D^s, and the strain-displacement matrices B^b and B^s. The notation ℓ^e is the length of the beam element, A^e the cross-sectional area, and $J^e = I_2^e + I_3^e$, where I_2^e and I_3^e are the moments of inertia about y and z axes, respectively.

Mass Matrix - The element mass matrix M^e can be written as (Yener and Chompooming 1991b)

$$M^e = [M_{pq}^e] \tag{19}$$

where

$$M_{pq}^e = \delta_{ij} \int_0^{\delta^e} N_a m_b N_b dx \tag{20a}$$

in which p = 6(a-1)+i, and q = 6(b-1)+j, with indices a and b ranging from 1 to n_{en}, and I and j from 1 to 6. The values of m_b are given as

$$m_b = \rho A^e \text{ when } 1 \le i = j \le 3 \tag{20b}$$

$$m_b = \begin{cases} \rho J^e & when \quad i=j=4 \\ \rho I_2^e & when \quad i=j=5 \\ \rho I_3^e & when \quad i=j=6 \end{cases} \tag{20c}$$

Force Vector - The element nodal force vector F^e is (Yener and Chompooming 1991b)

$$F^e = \{F_j^e\} \tag{21a}$$

where

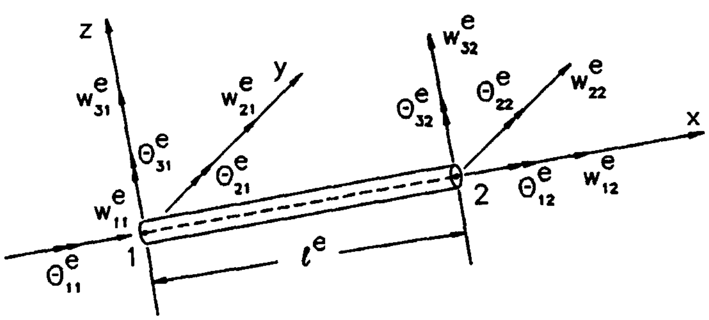

Fig 7. Nodal Displacement Components for the Beam Element

$$F_j^e = \int_0^{\ell^e} N_a F_i dx \text{ corresponding to DOF}$$
$$j = 6a - 6 + i$$
$$F_j^e = \int_0^{\ell^e} N_a C_i dx \text{ corresponding to DOF}$$
$$j = 6a - 3 + k \quad (21b)$$

In (21b), a=1,..., n_{en}, and F_i, for i=1,2,3 are the applied force components per unit length in the x, y, and z directions. C_1 is the applied twisting moment per unit length, and C_2 and C_3 are the applied bending moments per unit length about the y and z axes, respectively.

Viscous Damping in Bridge Structures

In this paper, the viscous damping, for which the induced damping force is assumed to be proportional to the relative velocities, is used to take into account the energy dissipation phenomenon that occurs during vibration. The viscous damping matrix, for both the plate and beam elements, is assumed to be proportional to the element mass and stiffness matrices (Hughes 1987).

$$C^e = C_M M^e + C_K K^e \quad (22)$$

where C_M and C_K are the parameters to be selected to produce desired damping characteristics.

SOLUTION ALGORITHM FOR THE VEHICLE-BRIDGE SYSTEM

Definition of the Problem

The finite element matrices of beam and plate elements, and equivalent nodal forces due to interaction and other external loads, can be assembled to obtain the semi-discretized matrix equation of motion for the discretized structural system in the form of (2). It should be noted that, at this stage, the nodal boundary conditions including prescribed nodal displacements, forces, and moments are already imposed into (2).

The current initial-value problem consists of finding a displacement vector D=D(t) which satisfies (2) and the given initial conditions

$$D(0) = \boldsymbol{u}_0, \quad \dot{D}(0) = \dot{\boldsymbol{u}}_0 \quad (23)$$

where $\boldsymbol{u}_0$ and $\dot{\boldsymbol{u}}_0$ are the vectors of specified initial values of nodal displacements and velocities, at time t=0, respectively.

To obtain an approximate solution, the semi-discretized matrix equation of motion is further discretized with respect to time on the basis of the Newmark method in connection with predictor-corrector scheme (Fox and Mayers 1987). A detailed description of this solution procedure is given in the following section.

Time Discretization of the System Equation

For brevity, the following discussion is based on vehicle model 1DMM-2 in Fig. 2c, which has only one contact point where the vehicle mass is not in direct contact with the roadway. We note that the time discretized form of the governing matrix differential equation (2) is algebraic. In order to reflect this characteristic of the governing equation, in the following expressions, $D(t_n)$, $\dot{D}(t_n)$, and $\ddot{D}(t_n)$ are replaced by $\boldsymbol{D}_n$, $\boldsymbol{V}_n$, and $\boldsymbol{A}_n$, respectively. t_n is the time at step n, which indicates the discrete time index. Furthermore, notations $\boldsymbol{d}_n$, $\boldsymbol{v}_n$, and $\boldsymbol{a}_n$ denote time discretized values of vehicle DOFs at t_n, i.e., $\boldsymbol{d}_n = \boldsymbol{d}(t_n)$, $\boldsymbol{v}_n = \dot{\boldsymbol{d}}(t_n)$, and $\boldsymbol{a}_n = \ddot{\boldsymbol{d}}(t_n)$.

As the solution is advanced to the next time step, the time discretized vehicle-bridge equation of motion can be written as

$$\boldsymbol{M}\boldsymbol{A}_{n+1} + \boldsymbol{C}\boldsymbol{V}_{n+1} + \boldsymbol{K}\boldsymbol{D}_{n+1} = \boldsymbol{F}_{n+1} \quad (24)$$

In this equation, A_{n+1}, V_{n+1}, and D_{n+1} are the nodal DOF's of the bridge structure at time t_{n+1}, which we refer to as the current time step. The nodal force vector F_{n+1} is constructed by adding the interaction force vector, $F^{\in}$, induced by the vehicle-bridge coupling motion, to the specified external load vector, F^{sp}. F^{sp} may be composed of dead weight of the bridge structure, as well as equivalent wind and earthquake loadings. $F^{\in}$ is the equivalent nodal force vector due to the interaction force developed at contact points along the vehicle traversing path.

For 1DMM-2 vehicle model, the interaction force at time t_{n+1}, f^{cp}_{n+1}, can be determined from (7) and written as

$$f^{cp}_{n+1} = k_1(d^{cp} - d^{r}_{1})_{n+1} + c_1(v^{cp} - v^{r}_{1})_{n+1} \tag{25}$$

In (25), d^{cp} and v^{cp} are the vehicle DOF's at the contact point, and d^{r}_{1} and v^{r}_{1} the DOF's of vehicle mass m_1, indicating the remaining vehicle DOF's. f^{cp} is the interaction force on superstructure at the contact point.

Since DOF's at the vehicle contact point d^{cp}, v^{cp}, and a^{cp} are interrelated to the superstructure DOFs, the structure force vector can be written symbolically as

$$F_{n+1} = oldF^{sp}_{n+1} + F^{\in}_{n+1}(D^{cp}_{n+1}, V^{cp}_{n+1}, A^{cp}_{n+1}, d_{n+1}, a_{n+1}) \tag{26}$$

It is emphasized that $F^{\in}_{n+1}$ is a function of vehicle DOF's d, v, and a, as well as the corresponding structural DOF's of the plate element containing the contact point D^{cp}, V^{cp}, and A^{cp}.

The finite-difference representation of displacement and velocity vectors at time t_{n+1}, based on the Newmark method, can be written as

$$D_{n+1} = D_n + \Delta t\, V_n + \frac{1}{2}\Delta t^2[(1-2\beta)A_n + 2\beta A_{n+1}] \tag{27a}$$

$$V_{n+1} = V_n + \Delta t[(1-\gamma)A_n + \gamma A_{n+1}] \tag{27b}$$

where Δt is the time increment from time t_n to t_{n+1}, and β and γ are the parameters governing the stability and accuracy of the algorithm. Substituting (27) into (24) permits us to write (24), in a symbolic form, in terms of the primary unknown A_{n+1}, as

$$MA_{n+1} + CV_{n+1}(A_{n+1}) + KD_{n+1}(A_{n+1}) = F_{n+1}(A^{cp}_{n+1}, d_{n+1}, V_{n+1}, a_{n+1}) \tag{28}$$

The terms containing subscript n represent values determined at the previous time step, and therefore are not explicitly included in (28). (28) is written in symbolic form to emphasize that the interaction problem is nonlinear in nature, and may best be solved using an iterative process. In this paper, a multi predictor-corrector procedure, in connection with the Newmark method, is employed in the solution of (28). In this process, the currently corrected values are treated as predictors in the proceeding step, and hence predictor-corrector procedure is repeated until the desired accuracy is obtained.

Multi Predictor-Corrector Solution Scheme

To initiate the multi predictor-corrector process within the current time step, a value for A_{n+1} in (28) needs to be assumed. This value may be designated as A^{0}_{n+1} and can be taken as zero, where superscript 0 is the initial value of predictor-corrector iteration index i. On this basis, from (27), V^{0}_{n+1} and D^{0}_{n+1} can be computed as

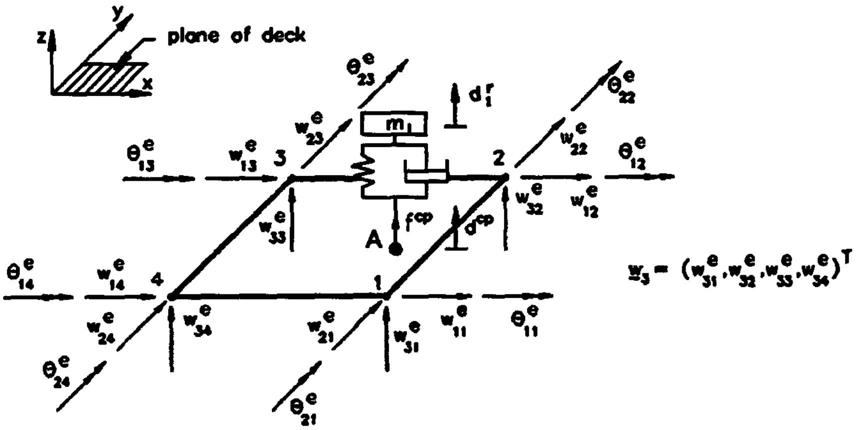

Fig 8. Vehicle Model 1DMM-2 on a Plate Element

$$D^0_{n+1} = D_n + \Delta t\, V_n + \frac{1}{2}(1-2\beta)\Delta t^2 A_n \quad (29a)$$

$$V^0_{n+1} = V_n + (1-\gamma)\Delta t\, A_n \quad (29b)$$

It should be emphasized that $A^0_{n+1} = 0$, V^0_{n+1} and D^0_{n+1} in (29) are described in terms only of the previously determined quantities at t_n. With values of A^0_{n+1}, V^0_{n+1} and D^0_{n+1}, it is now necessary to determine the corresponding fore vector in (28). At this point, from an implementation perspective, it becomes convenient to update the iteration index i, and start the predictor-corrector loop. Furthermore, to emphasize that the initial predicted values of the vehicle-bridge system consist not only of A^0_{n+1}, V^0_{n+1}, D^0_{n+1}, but also of F^0_{n+1}, the force vector is designated as F^{i-1}_{n+1} in terms of A^{i-1}_{n+1}, V^{i-1}_{n+1} and D^{i-1}_{n+1}. As indicated in (26), F^{i-1}_{n+1} is a function of interaction force basis of the predicted values of the structural $(F^{\epsilon}_{n+1})^{i-1}$, the determination of which, on the DOF's, is discussed below.

Based on D^{i-1}_{n+1} and V^{i-1}_{n+1}, the vehicle DOF's at the contact point, d^{cp}_{n+1} and V^{cp}_{n+1}, can be approximated through the use of element displacement shape functions of the plate element containing the contact point (Fig. 7). It is noted in passing that for vehicle model 1DMM-1, DOF's at the contact point also include a^{cp}. Knowing the approximated values of d^{cp}_{n+1} and v^{cp}_{n+1}, the remaining vehicle DOF's, d^r_{n+1}, v^r_{n+1}, and a^r_{n+1}, can be determined from the equation of vehicle motion, which can be written in the following form.

$$\begin{bmatrix} m^r & m^{rcp} \\ m^{rcp^T} & m^{cp} \end{bmatrix} \begin{Bmatrix} a^r_{n+1} \\ a^{cp}_{n+1} \end{Bmatrix}^{i-1} + \begin{bmatrix} c^r & c^{rcp} \\ c^{rcp^T} & c^{cp} \end{bmatrix} \begin{Bmatrix} v^r_{n+1} \\ v^{cp}_{n+1} \end{Bmatrix}^{i-1} + \begin{bmatrix} k^r & k^{rcp} \\ k^{rcp^T} & k^{cp} \end{bmatrix} \begin{Bmatrix} d^r_{n+1} \\ d^{cp}_{n+1} \end{Bmatrix}^{i-1} = f = \begin{Bmatrix} f^r_{n+1} \\ f^{cp}_{n+1} \end{Bmatrix}^{i-1} \quad (30)$$

where superscripts r, cp, and rcp denote matrix partitions of matrices m, c, and k, and vector f in the equations of vehicle motion given in (7) through (8). It needs to be mentioned that for all moving mass vehicle models in Fig. 2 except for 1DMM-1 the vehicle mass corresponding to a^{cp} is zero. We note that in order not to interrupt the flow of discussion, the procedure for determining d^{cp}, v^{cp}, d^r, v^r, and a^r is detailed in the following section.

The interaction force at the contact point $(f^{cp}_{n+1})^{i-1}$, which is currently unknown, can be determined from the lower partitioned equations of vehicle motion in (30). Once the interaction force is evaluated, it can be added to the interaction force vector $(\boldsymbol{F}_{n+1}^{\in})^{i-1}$ in the form of equivalent nodal forces. This completes the formation of the force vector $\boldsymbol{F}_{n+1}^{i-1}$ in (26) and the governing equation of the system given in (28).

After this initial predictor step is completed, the process is continued values of $\boldsymbol{A}_{n+1}^{i-1}$, $\boldsymbol{V}_{n+1}^{i-1}$, $\boldsymbol{D}_{n+1}^{i-1}$, and $\boldsymbol{F}_{n+1}^{i-1}$ do not identically satisfy the governing equation (28). Based on the values of $\boldsymbol{D}^{i-1}$, $\boldsymbol{V}^{i-1}$, and $\boldsymbol{A}^{i-1}$, and the interaction fore vector $(\boldsymbol{F}^{\in})^{i-1}$ at the current time step, the structure governing equation is evaluated to determine whether there is any residual force in the system. From (28), the residual force vector can be determined as

$$\Delta \boldsymbol{F} = \boldsymbol{F}_{n+1}^{i-1} - (\boldsymbol{MA}_{n+1}^{i-1} + \boldsymbol{CV}_{n+1}^{i-1} + \boldsymbol{KD}_{n+1}^{i-1}) \quad (31)$$

If $\Delta \boldsymbol{F}$ is within a specified tolerance, the process is stopped. Otherwise, the correction procedure is continued by determining the additional acceleration vector to balance the excessive force. This is done by rewriting the governing equation in an incremental form as

$$\boldsymbol{M}\Delta \boldsymbol{A} + \boldsymbol{C}\Delta \boldsymbol{V} + \boldsymbol{K}\Delta \boldsymbol{D} = \Delta \boldsymbol{F} \quad (32)$$

where, with (27),

$$\Delta \boldsymbol{A} = \boldsymbol{A}_{n+1}^{i} - \boldsymbol{A}_{n+1}^{i-1} \quad (33a)$$

$$\Delta \boldsymbol{V} = \boldsymbol{V}_{n+1}^{i} - \boldsymbol{V}_{n+1}^{i-1} = \Delta t \gamma \Delta \boldsymbol{A} \quad (33b)$$

$$\Delta \boldsymbol{D} = \boldsymbol{D}_{n+1}^{i} - \boldsymbol{D}_{n+1}^{i-1} = \Delta t^2 \beta \Delta \boldsymbol{A} \quad (33c)$$

and $\Delta \boldsymbol{F}$ is given by (31).

(32) can be written in terms of only $\Delta \boldsymbol{A}$, by substituting (33b) and (33c), as

$$\Delta \boldsymbol{A} = [\boldsymbol{M}^*]^{-1} \Delta \boldsymbol{F} \quad (34)$$

where $\boldsymbol{M}^*$ is referred to as the effective structure mass matrix and is given as

$$\boldsymbol{M}^* = \boldsymbol{M} + \Delta t \gamma \boldsymbol{C} + \Delta t^2 \beta \boldsymbol{K} \quad (35)$$

Once $\Delta \boldsymbol{A}$ is determined, the extreme right hand sides of (33b) and (33c) can be used to determine the corresponding incremental values $\Delta \boldsymbol{V}$ and $\Delta \boldsymbol{D}$, respectively. The predicted values of $\boldsymbol{A}_{n+1}^{i-1}$, $\boldsymbol{V}_{n+1}^{i-1}$, and $\boldsymbol{D}_{n+1}^{i-1}$ can then be corrected in accordance with (33).

At this point, index i is advanced and $\boldsymbol{F}_{n+1}^{i}$ corresponding to the current predicted values evaluated, and the process continued. Once the desired accuracy is obtained, we then proceed to the next time step.

SOLUTION ALGORITHM FOR THE VEHICLE MOTION

Kinematic Conditions at Vehicle Contact Point

The vertical DOF d^{cp} of the contact point A in Fig. 7 can be written as

$$d^{cp} = d^{cp}(\boldsymbol{x}, t) = w_3(\boldsymbol{x}, t) \quad (36)$$

As indicated in Fig. 7, $w_3(\boldsymbol{x}, t)$ is the DOF corresponding to the global vertical translation of the bridge deck at point $\boldsymbol{x}$ and time t.

From (36), the velocity of the vehicle corresponding to the vertical DOF of the contact point A can be written as

$$\dot{d}^{cp}(x, t) = \frac{dd^{cp}}{dt} = \frac{\partial w_3}{\partial x}\frac{dx}{dt} + \frac{\partial w_3}{\partial Y}\frac{dY}{dt} + \frac{\partial w_3}{\partial t} \quad (37a)$$

Because $\partial w_3/\partial Y$ and dy/dt are small, the effect of the product of $\partial w_3/\partial Y$ and dy/dt is negligible. Hence, (37a) can be reduced to

$$\dot{d}^{cp}(x,t) = \frac{\partial w_3}{\partial x} v(t) + \frac{\partial w_3}{\partial t} \quad (37b)$$

where v(t) denotes the vehicle velocity at time t along the global x direction. Fig. 4 identifies the global axes system with respect to the longitudinal direction. The vehicle position vector $\boldsymbol{x}_A$ is allowed to have components in the y as well as the x directions. The direction of the traveling vehicle is determined by the sign of vehicle velocity. The x component of the vehicle path is taken along the positive x direction when v(t) is positive, and along the negative x direction when v(t) is negative. The same is valid also for the y component of the vehicle position.

Based on (37b), the vehicle acceleration corresponding to the vertical DOF of the contact point A can be written as

$$\ddot{d}^{cp} = \frac{d\dot{d}^{cp}}{dt} = \dot{v}(t)\frac{\partial w_3}{\partial x} + v^2(t)\frac{\partial^2 w_3}{\partial x^2} + 2v(t)\frac{\partial^2 w_3}{\partial x \partial t} + \frac{\partial^2 w_3}{\partial t^2} \quad (37c)$$

where $\dot{v}(t)$ is the traveling acceleration of the vehicle in the x direction.

It is worth noting that the kinematic coupling term $\delta^2 w_3/\delta x \delta t$ in (37c) arises due to the spatial and time dependent characteristics of the vertical displacement of the bridge deck at the contact point A. In this paper, the direct treatment of the kinematic coupling term is carried out through the application of the assumed element displacement functions. The global vertical DOF $w_3(\boldsymbol{x}, t)$ of the bridge deck can be approximated by using the assumed displacement functions for the plate elements as

$$w_3(x,t) \approx \sum_{a=1}^{n_{en}} N_a(x,y)\, w_{3a}^e(t) \quad (38)$$

where w_{3a}^e is the finite element approximation of the global vertical DOF at node a of plate element e containing the contact point.

Using (39), the vehicle DOFs at the contact point A can be written as

$$d^{cp} = B^T(x)\underline{w}_3(t) \quad (39a)$$

$$\dot{d}^{cp} = v(t)B^T_{,x}(x)\underline{w}_3(t) + B^T(x)\underline{\dot{w}}_3(t) \quad (39b)$$

$$\ddot{d}^{cp} = [\dot{v}(t)B_{,x}(x) + v^2(t)B_{,xx}(x)]^T\underline{w}_3(t) + 2v(t)B^T_{,x}(x)\underline{\dot{w}}_3(t) + B^T(x)\underline{\ddot{w}}_3(t) \quad (39c)$$

where the components of the vector B are the plate element shape functions associated with element nodes, and can be written as

$$B = (N_1, N_2, \dots, N_a, \dots, N_{n_{en}})^T \quad (40a)$$

$\boldsymbol{B}_{,x}$ and $\boldsymbol{B}_{,xx}$ are the vectors whose components are the first and the second derivatives with respect to x, respectively, of the element shape functions associated with element nodes, i.e.,

$$B_{,x} = (N_{1,x}, N_{2,x}, \dots, N_{a,x}, \dots, N_{n_{en},x})^T \quad (40b)$$

$$B_{,xx} = (N_{1,xx}, N_{2,xx}, \dots, N_{a,xx}, \dots, N_{n_{en},xx})^T \quad (40c)$$

The vectors of vertical DOFs of the plate element, and the corresponding velocities and accelerations, can be written respectively as

$$\underline{w}_3(t) = (w_{31}^e(t), w_{32}^e(t), .., w_{3a}^e(t), .., w_{3n_{en}}^e(t))^T \quad (41a)$$

$$\underline{\dot{w}}_3(t) = (\dot{w}_{31}^e(t), \dot{w}_{32}^e(t), \dots, \dot{w}_{3a}^e(t), \dots, \dot{w}_{3n_{en}}^e(t))^T \quad (41b)$$

$$\underline{\ddot{w}}_3(t) = (\ddot{w}_{31}^e(t), \ddot{w}_{32}^e(t), \dots, \ddot{w}_{3a}^e(t), \dots, \ddot{w}_{3n_{en}}^e(t))^T \quad (41c)$$

Solution of Vehicle DOFs $\boldsymbol{d}^r$, $\boldsymbol{v}^r$, **and** $\boldsymbol{a}^r$

From (30), at iteration i-1, the matrix equation of vehicle motion corresponding to the vehicle DOFs at t_{n+1}, excluding those at the contact point, can be written as

$$m^r a^r_{n+1} + c^r v^r_{n+1} + k^r d^r_{n+1} = f^{mr}_{n+1} \tag{42}$$

in which $\boldsymbol{m}^r$, $\boldsymbol{c}^r$, and $\boldsymbol{k}^r$ are the partitioned mass, damping, and stiffness matrices corresponding to $\boldsymbol{a}^r_{n+1}$, $\boldsymbol{v}^r_{n+1}$, and $\boldsymbol{d}^r_{n+1}$, respectively, given in (7) through (8). $\boldsymbol{f}^{mr}_{n+1}$ is a modified form of $\boldsymbol{f}^r_{n+1}$. For example, for the 1DMM-2 vehicle model, $\boldsymbol{f}^{mr}_{n+1}$ can be written as

$$f^{mr}_{n+1} = -m_1 g + c_1 v^{cp}_{n+1} + k_1 d^{cp}_{n+1} \tag{43}$$

Finite-difference approximation of $\boldsymbol{d}^r_{n+1}$ and $\boldsymbol{v}^r_{n+1}$ in (42), based on the Newmakr method can be written as

$$\begin{aligned} d^r_{n+1} &= d^r_n + \triangle t v^r_n + \frac{1}{2} \triangle t^2 \\ &\quad [(1-2\beta) a^r_n + 2\beta a^r_{n+1}] \qquad (44a) \\ &= \overline{d}^r_{n+1} + \triangle t^2 \beta a^r_{n+1} \end{aligned}$$

$$\begin{aligned} v^r_{n+1} &= v^r_n + \triangle t[(1-\gamma) a^r_n + \gamma a^r_{n+1}] \\ &= \overline{v}^r_{n+1} + \triangle t \gamma a^r_{n+1} \end{aligned} \tag{44b}$$

whrere $\overline{\boldsymbol{d}}^r_{n+1}$ and $\overline{\boldsymbol{v}}^r_{n+1}$ are described in terms of the previously determined quantities at t_n. It should be noted that (42) has the same form as (28) which corresponds to the equation of system motion. However, the characteristics of these two equations are considerably different. In (28), the force vector is dependent on the structural DOFs which are to be determined. This renders (28) to be nonlinear. On the other hand, as indicated in (43), force vector $\boldsymbol{f}^{mr}_{n+1}$ in (42) can be determined once the vehicle DOFs at the contact point are computed from structural DOFs. Hence, the solution of (42) can be accurately determined within the current time step by employing a single predictor-corrector scheme.

After direct substitition of (44), (42) can be written, in terms of primary unknown $\boldsymbol{a}^r_{n+1}$ only, as

$$m^* a^r_{n+1} = f^* \tag{45}$$

where

$$m^* = m^r + \triangle t \gamma c^r + \triangle t^2 \beta k^r \tag{46a}$$

$$f^* = f^{mr}_{n+1} - c^r \overline{v}^r_{n+1} - k^r \overline{d}^r_{n+1} \tag{46b}$$

$$\overline{v}^r_{n+1} = v^r_n + \triangle t(1-\gamma) a^r_n,$$

$$\overline{d}^r_{n+1} = d^r_n + \triangle t v^r_n + \frac{1}{2} \triangle t^2 (1-2\beta) a^r_n \tag{46c}$$

in which $\boldsymbol{m}^*$ and $\boldsymbol{f}^*$ are referred to as the vehicle effective mass matrix and force vector, respectively. $\overline{\boldsymbol{v}}^r_{n+1}$ and $\overline{\boldsymbol{d}}^r_{n+1}$ are the predicted values of $\boldsymbol{v}^r_{n+1}$ and $\boldsymbol{d}^r_{n+1}$, respectively. (45) can now be solved for $\boldsymbol{a}^r_{n+1}$ as

$$a^r_{n+1} = [m*]^{-1} f* \tag{47}$$

Then, $\boldsymbol{v}^r_{n+1}$ and $\boldsymbol{d}^r_{n+1}$ can be corrected in accordance with (44) as

$$\begin{aligned} \overline{v}^r_{n+1} &= \overline{v}^r_{n+1} + \triangle t \gamma a^r_{n+1}, \\ d^r_{n+1} &= \overline{d}^r_{n+1} + \triangle t^2 \beta a^r_{n+1} \end{aligned} \tag{48}$$

At this stage, based on the values of $\boldsymbol{a}^{cp}_{n+1}$, $\boldsymbol{v}^{cp}_{n+1}$, and $\boldsymbol{d}^{cp}_{n+1}$, $\boldsymbol{a}^r_{n+1}$, $\boldsymbol{v}^r_{n+1}$ and

d^r_{n+1} are determined. Then, on the basis of vehicle DOFs, the interaction force can be computed as discussed in the following section.

VEHICLE-SUPERSTRUCTURE INTERACTION FORCE

The magnitude of the moving force, produced by the MF model in Fig. 2a, can be computed using (5). The determination of the interaction force due to the other vehicle models in Fig. 2b through 2e is described below.

1DMM-1 Model

By substituting (39c) into (6), the interaction force induced by the 1DMM-1 model in Fig. 2b can be written as (Yener, Chompooming and Yi, 1993)

$$f^{cp} = mg + m[\, C_1^T \underline{w}_3 + C_2^T \underline{\dot{w}}_3 + C_3^T \underline{\ddot{w}}_3 \quad (49)$$

where

$$C_1 = \dot{v}(t) B,_x + v^2(t) B,_{xx},$$
$$C_2 = 2v(t) B,_x, \quad C_3 = B \quad (50)$$

It is noted that C_1, C_2, and C_3 in (50) are evaluated at the contact point A at time t (see Fig. 2b).

It is obvious from (49) that the interaction force is a function of the mass, velocity and acceleration of the moving vehicle. Furthermore, the nodal DOFs of the superstructure, and their rate of change with respect to space coordinates are shown to influence the interaction force.

1DMM-2 Model

Considering the equation of motion in (7) for the 1DMM-2 model, the interaction force can be written as (Yener, Chompooming and Yi, 1993a)

$$f^{cp} = k_1(d^{cp} - d_1^r) + c_1(\dot{d}^{cp} - \dot{d}_1^r) \quad (51)$$

Substituting the expressions for vertical translation d^{cp} and Velocity $\dot{d}^{cp}$ of the contact point, respectively from (39a) and (39b), into (51) gives

$$f^{cp} = C_0 + C_1^T \underline{w}_3 + C_2^T \underline{\dot{w}}_3 \quad (52)$$

where

$$C_0 = -k_1 d_1^r - c_1 \dot{d}_1^r,$$
$$C_1 = k_1 B + v(t) c_1 B,_x, \quad C_2 = c_1 B \quad (53)$$

It is noted that C_0, C_1, and C_2 in (53) are evaluated at the contact point A at time t (see Fig. 2c). For this vehicle model, the interaction force is also a function of the vertical displacement, and the corresponding velocity, of the sprung mass m.

1DMM-d Model

Comparing the equations of motion in (7) and (8) indicates that the interaction force due to the 1DMM-3 model has the same form as that derived for the 1DMM-2 model. Therefore, the interaction force for this vehicle model can be computed by using (52). It is noted, however, that for the present vehicle model, DOFs d_1^r and $\dot{d}_1^r$ of mass m_1 depend also on masses m_1 and m_2, DOFs d_2^r, $\dot{d}_2^r$, and $\ddot{d}_2^r$, spring constant k_2, and damping constant c_2 of the vehicle suspension model.

2DMM Model

On the basis of the equation of motion in (8) for the 2DMM model, the interaction forces can be written as (Yener and Chompooming 1991b)

$$f_1^{cp} = k_3(d_1^{cp} - d_1^r) + c_3(\dot{d}_1^{cp} - \dot{d}_1^r) \quad (54a)$$

$$f_2^{cp} = k_4(d_2^{cp} - d_2^r) + c_4(\dot{d}_2^{cp} - \dot{d}_2^r) \tag{54b}$$

Substituting approximate expressions for d_1^{cp} similar to (39a), and $\dot{d}_1^{cp}$ similar to (39b) into (54a) results in

$$f_1^{cp} = C_0 + C_1^T \underline{w}_3^A + C_2^T \underline{\dot{w}}_3^A \tag{55}$$

where

$$\begin{aligned} C_0 &= -k_3 d_1^r - c_3 \dot{d}_1^r, \\ C_1 &= k_3 B(x_A) = v(t) c_3 B,_x(x_A), \\ C_2 &= c_3 B(x_A) \end{aligned} \tag{56}$$

with $\underline{\boldsymbol{w}}_3^A$ representing the vector of vertical nodal displacements of the element containing point A, and $\boldsymbol{x}_A$ the space coordinates of contact point A. In a similar manner, (54b) can be written as

$$f_2^{cp} = C_3 + C_4^T \underline{w}_3^B + C_5^T \underline{\dot{w}}_3^B \tag{57}$$

where

$$\begin{aligned} C_3 &= -k_4 d_2^r - c_4 \dot{d}_2^r, \\ C_4 &= k_4 B(x_B) + v(t) c_4 B,_x(x_B), \\ C_5 &= c_4 B(x_B) \end{aligned} \tag{58}$$

It is noted that C_0, $\boldsymbol{C}_1$, and $\boldsymbol{C}_2$ in (55) are evaluated at the contact point A at time t. Similarly, C_3, $\boldsymbol{C}_4$, and $\boldsymbol{C}_5$ in (57) are evaluated at contact point B at time t. Similar to the 1DMM-3 model, the interaction forces f_1^{cp} and f_2^{cp} of the current vehicle model are implicit functions of masses m_1 and m_2, the rigid mass m_0 and its DOFs d_i^r, $\dot{d}_i^r$, and $\ddot{d}_i^r$, where i=3 and 4, and the mass moment of inertia I_0. In addition, the interaction forces are expressed implicitly in terms of spring and damping constants of the suspension model; k_1, k_2, c_1, and c_2.

It should be noted that the interaction forces computed in (49), (52), (55), and (57) are considered as concentrated loads applied on a plate element. Therefore, equivalent nodal forces in the global z direction due to these interaction forces, which constitute the force vector in (28), need to be calculated in combination with the own weight of the bridge, and an equivalent wind and earthquake loading if desired.

SUMMARY AND CONCLUSIONS

In this paper, a general finite element formulation and solution methodology is presented for accurate dynamic analysis of vehicle-bridge interaction problems. The mathematical formulation takes into account the parameters which significantly affect the dynamic behavior of bridge structures. These parameters include such bridge superstructure characteristics as mass, stiffness and damping properties, and natural frequencies. Furthermore, such vehicle characteristics as axle spacing, speed, acceleration, and traveling path are also considered. The time-varying vehicle load, which is referred to as the interaction force, is determined by taking into account the bouncing of the vehicle on its suspension and by considering the traversing vehicle and the vibrating bridge as an integrated system. The Newmark method, in connection with a multi predictor-corrector scheme, is used to obtain discrete time-histories of bridge dynamic response.

ACKNOWLEDGMENT

The author wish to thank the financial support of SunMoon University made by research fund in year 2002.

REFERENCES

Fleming, J. F., and Romualdi, P. (1961). "Dynamic Response of Highway Bridges." J. Struct. Engrg., ASCE, 87(7), 31-61.

Fox, L., and Mayers, D. F. (1987). *Numerical Solution of Ordinary Differential Equations*. Chapman and Hall, New York.

Huang, T. (1976). "Vibrations of Bridges." Shock Vib. Dig., 8(3), 61-76.

Hughes, T. J. R. (1987). The Finite Element Method: *Linear Static and Dynamic Finite Element Analysis*. Prentice-Hall, New Jersey.

Newmark, N. M. (1959). "A Method of Computation for Structural Dynamics." J. Engrg. Mech. Div., ASCE, 85(3), 67-94.

Ting, E. C., and Yener, M. (1983). "Vehicle-Structure Interactions in Bridge Dynamics." Shock Vib. Dig., 15(12), 3-9.

Yener, M., and Chompooming, K. (1991a). "An Overview on Vehicle-Bridge Interaction Problems and Optimum Design of Bridge System." Structural Engineering and Mechanics Division Report No. CEE-SEMD-91-09, Civil and Envi. Engrg. Dept., Utah State Univ.

Yener, M., and Chompooming, K. (1991b). "Formulation of Dynamic Vehicle-Bridge Interaction Systems." Report No. CEE-SEMD-91-11, Civil and Envi. Engrg. Dept., Utah State Univ.

Yener, M., Chompooming, K. and YI, G.S(1993). "Numerical Method of Lines for Analysis of Vehicle-Bridge Dynamic Interaction." CEE-SEMD-93-8, Civil and Envi. Engrg. Dept., Utah State Univ.

Yener, M., Chompooming, K.and Yi, G.S. (1993a). "Numerical Implementation of Dynamic Vehicle-Bridge Interaction Problems." CEE-SEMD-93-10, Civil and Envi. Engrg. Dept., Utah State Univ.

APPENDIX NOTATION

The following symbols are used in this paper:

$\boldsymbol{D}, \dot{\boldsymbol{D}}, \ddot{\boldsymbol{D}}$ = structure nodal displacement, velocity, and acceleration vectors;

$\boldsymbol{D}^e$ = element nodal displacement vector;

$\boldsymbol{D}_{n+1}, \boldsymbol{V}_{n+1}, \boldsymbol{A}_{n+1}$ = structure nodal displacement, velocity, and acceleration vectors at time step n+1;

$D^{cp}_{n+1}, \boldsymbol{V}^{cp}_{n+1}, \boldsymbol{A}^{cp}_{n+1}$ = nodal displacements, velocities, and accelerations of the plate element containing the contact point at time step n+1;

$\boldsymbol{d}, \dot{\boldsymbol{d}}, \ddot{\boldsymbol{d}}$ = vehicle displacement, velocity, and acceleration vectors;

$\boldsymbol{d}^{cp}, \boldsymbol{v}^{cp}, \boldsymbol{a}^{cp}$ = vehicle displacement, velocity, and acceleration vectors at the contact point;

$\boldsymbol{d}^r, \boldsymbol{v}^r, \boldsymbol{a}^r$ = vehicle displacement, velocity, and acceleration vectors excluding DOFs at the contact point;

$\overline{\boldsymbol{d}}^r_{n+1}, \overline{\boldsymbol{v}}^r_{n+1}$ = vectors of the predicted values of $\boldsymbol{d}^r$ and $\boldsymbol{v}^r$ at time step n+1;

$\boldsymbol{F}$ = structure force vector;

$\boldsymbol{F}^{\in}_{n+1}$ = interaction force vector at time step n+1;

$\boldsymbol{F}^{sp}_{n+1}$ = specified structure external force vector at time step n+1;

f = vehicle force vector;

f^{cp} = vehicle force at the contact point;

$\boldsymbol{f}^{mr}$ = modified $\boldsymbol{f}^r$;

$\boldsymbol{f}^r$ = vehicle force vector associated with $\boldsymbol{d}^r$;

$\boldsymbol{f}^*$ = vehicle effective force vector;

$\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ = structure mass, damping, and stiffness matrices;

$\boldsymbol{M}^e, \boldsymbol{C}^e, \boldsymbol{k}^e$ = element mass, damping, and stiffness matrices;

$\boldsymbol{M}^*$ = structure effective mass matrix;

$\boldsymbol{m}, \boldsymbol{c}, \boldsymbol{k}$ = vehicle mass, damping, and stiffness matrices;

$\boldsymbol{m}^*$ = vehicle effective mass matrix;

N_a = element displacement shape function associated with node a;

$\boldsymbol{u}_0$, $\dot{\boldsymbol{u}}_0$ = vectors of specified values of nodal displacements and velocities;

w, $\dot{w}$, $\ddot{w}$ = vertical displacement, velocity, and acceleration of the superstructure at the contact point;

w_3 = vertical displacement of the bridge deck at the contact point;

$\underline{\boldsymbol{w}}_3$, $\dot{\underline{\boldsymbol{w}}}_3$, $\ddot{\underline{\boldsymbol{w}}}_3$ = vector of nodal vertical displacements, velocities, and accelerations of the plate element containing the contact point;

w^e_{ij} = translation of node j along direction i of element e;

β, γ = parameters governing stability and accuracy for time integration;

$\Delta\boldsymbol{D}$, $\Delta\boldsymbol{V}$, $\Delta\boldsymbol{A}$ = vectors of corrected values of $\boldsymbol{D}$, $\boldsymbol{V}$, $\boldsymbol{A}$;

$\Delta\boldsymbol{F}$ = residual force vector;

Δt = time step increment; and

θ^e_{ij} = rotation of node j about axis i of element e.