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제어기의 이득 섭동을 갖는 이산 시간지연 대규모 시스템을 위한 강인 비약성 제어기

(Decentralized Stabilization for Uncertain Discrete-Time Large-Scale Systems with Delays in Interconnections and Controller Gain Perturbations)

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요 약

본 논문에서는, 섭동과 제어기 이득 섭동을 갖는 이산 대규모 시간지연 시스템의 강인 비약성 제어기 설계에 관하여 논한다. 리아프노프 해석법을 의거하여 선형행렬 부등식으로 표현되는 주어진 시스템의 강인 안정화를 꾀하는 상태 궤환 제어기의 존재를 보장하는 조건 식을 구한다. 이 조건 식의 해로부터 각 부 시스템에서의 제어기의 이득 및 제어기의 비약성 지수도 얻을 수 있다. 제시된 선형행렬 부등식은 잘 알려진 최적화 기법으로 쉽게 풀 수 있으며, 예제를 통하여 제어기 설계 방법을 보인다.

Abstract

This paper considers the problems of robust decentralized control for uncertain discrete-time large-scale systems with delays in interconnections and state feedback gain perturbations. Based on the Lyapunov method, the state feedback control design for robust stability is given in terms of solutions to a linear matrix inequality (LMI), and the measure of non-fragility in controller is presented. The solutions of the LMI can be easily obtained using efficient convex optimization techniques. A numerical example is included to illustrate the design procedures.

Keyword : Large-scale system, delay, controller gain perturbation, nonfragile control, linear matrix inequality (LMI).

I. Introduction

With the enlargement of dimension of a control system, analysis and control for the system becomes very complicated. It is standard to divide such systems into a number of interconnected subsystems. In general, a large-scale interconnected dynamical system can be usually characterized by a large number of state variables, system parametric uncertainties, and a complex interaction between subsystems (Siljak[17], Mahmoud et al.[14]). During the last decade, the problem of decentralized stabilization of large-scale systems has received considerable attention, because there are a large number of large scale interconnected dynamical systems in many practical control problems, e.g. transportation sys-

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tems, power systems, communication systems, economic systems, social systems, and so on (Chen et al.[2], Chen[3], Geromel and Yamakami[8]).

Time-delays, due to the information transmission between subsystems, naturally exist in large-scale systems and the existence of the delay is frequently a source of instability. Therefore, the stabilization problem of the large-scale system with time-delay in subsystem interconnections has been investigated by many researchers (Hu[10], Lee and Radovic[13], Trinh and Aldeen[18], Yan et al.[19], Mahmoud and Bingulac[15], Oucheriah[16]).

Although all the methods in the literature yield controllers that are robust with regard to system uncertainties, their robustness with regard to controller uncertainty has not been considered. Recently, the controller robustness subjected to controller gain variations has been discussed (Keel and Bhattacharyya[11], Dorato[5]). This raises a new issue: how to design a controller for a given plant with uncertainty such that the controller is insensitive to some amount of error with regard to its gain, i.e. the controller is non-fragile. Recently, there have been some efforts to tackle the non-fragile controller design problem (Corrado and Haddad[4], Dorato et al.[6], Famularo et al.[7], Haddad and Corrado[9], Kim and Park[12]). However, there has been few works considering non-fragile controller design methods of discrete-time large-scale systems with delay.

This paper is concerned with the design problem of robust non-fragile decentralized controller for discrete-delay large-scale systems with parametric uncertainties and controller gain variation. A stability criterion for robust stability of the system is derived in terms of LMI using the Lyapunov method. Moreover, the measure of non-fragility in controller can be calculated by solving the LMI. The controller parameters which satisfy the above LMIs can be easily found by various efficient convex optimization algorithms[1].

Notations : Throughout the paper, R^n denotes the n dimensional Euclidean space, $R^{n\times m}$ is the set of all $n\times m$ real matrices, I is the identity matrix with appropriate dimensions, and block diag{ · } denotes a block diagonal matrix. $*$ denotes the symmetric part. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X-Y$ is positive definite, (respectively, non negative).

Ⅱ. Problem Formulation

Consider a class of uncertain discrete-delay large-scale system composed N interconnected subsystems described by

$$S_i:\ x_i(k+1) = (A_i + A_i(k))x_i(k) + \sum_{j\neq i}^{N}(A_{ij} + \Delta A_{ij}(k))x_j(k-h_j) + (B_i + \Delta B_i(k))u_i(k),\quad i=1,2,\cdots,N \quad (1)$$

where $x_i(k)\in R^{n_i}$ is the state vector, $u_i(k)\in R^{m_i}$ is the control vector, and the time-delays, h_j, are the positive constants. The system matrices A_i, B_i, and A_{ij} are of appropriate dimensions, and $\Delta A_i(k)$, $\Delta B_i(k)$ and $\Delta A_{ij}(k)$ are real-valued matrices representing time-varying parameter uncertainties in the system.

Assume that the pair (A_i, B_i), $i=1,\cdots,N$, is stabilizable, and assume that the time-varying uncertainties are of the form

$$\Delta A_i(k) = D_{ai}F_{ai}(k)E_{ai},\quad \Delta B_i(k) = D_{bi}F_{bi}(k)E_{bi},\quad \Delta A_{ij}(k) = D_{aij}F_{aij}(k)E_{aij}, \quad (2)$$

where D_{ai}, D_{bi}, D_{aij}, E_{ai}, E_{bi}, and E_{aij} are known constant real matrices with appropriate dimensions, and $F_{ai}(k)$, $F_{bi}(k)$, and $F_{aij}(k)$ are unknown matrix functions which are bounded as

$$F_{ai}^T(k)F_{ai}(k)\leq I,\quad F_{bi}^T(k)F_{bi}(k)\leq I,\quad F_{aij}^T(k)F_{aij}(k)\leq I,\quad \forall i,j\geq 0. \quad (3)$$

Now, although one finds the controller $u_i(k) = -K_i x_i(k)$ for each subsystems, the actual controller implemented is

$$u_i(k) = -[K_i + \Delta K_i]x_i(k) \qquad (4)$$

where $K_i \in R^{m_i \times n_i}$ is the nominal controller gain to be designed and ΔK_i represents the multiplicative gain perturbations of the form

$$\Delta K_i = \delta_i \Phi_i(k) K_i \qquad (5)$$

with δ_i is an uncertain real parameter and $\Phi_i(k)$ is assumed to be bounded as

$$\Phi_i^T(k)\Phi_i(k) \le I \qquad (6)$$

Here, the value of δ_i indicates the measure of non-fragility against controller gain variations.

Remark 1. The structure of the uncertainties being of the form given in (2) and (5) has been widely used in many papers dealing with uncertain dynamic control systems. The controller gain perturbation can result from the actuator degradations, as well as from the requirement for re-adjustment of controller gains during the controller implementation state (Porato[5], Keel and Bhatacharyya[11]). These perturbations in the controller gains are modeled here as uncertain gains that are dependent on uncertain parameters. In the literature (Corrado and Haddad[4], Famularo et al.[7], Haddad and Corrado[9]), the models of additive uncertainties and multiplicative uncertainties are used to describe the controller gain variation. The uncertainty given in (5) is a class of multiplicative uncertainties.

With the control law (4), the resulting closed-loop subsystem becomes

$$\begin{aligned} x_i(k+1) = & [A_i + \Delta A_i(k) - (B_i + \Delta B_i(k)) \cdot \\ & (I + \delta_i \Phi_i(k))K_i]x_i(k) + \\ & \sum_{j \ne i}^{N} [A_{ij} + \Delta A_{ij}(k)]x_j(k-h_j). \end{aligned} \qquad (7)$$

From (7) we can write the overall system in the following way:

$$\begin{aligned} X(k+1) = & [A + \Delta A - (B + \Delta B)(K + \delta\Phi(k)K)]X(k) \\ & + (A_D + \Delta A_D)X_d(k) \end{aligned} \qquad (8)$$

where

$$\begin{aligned}
X(k) &\triangleq [x_1^T(k)x_2^T(k)\cdots x_N^T(k)]^T \\
X_d(k) &\triangleq [x_1^T(k-h_1)x_2^T(k-h_2)\cdots x_N^T(k-h_N)]^T \\
A &\triangleq \text{block diag } (A_1, A_2, \cdots, A_N) \\
\Delta A &\triangleq \text{block diag } D_a F_a(k) E_a \\
D_a &\triangleq \text{block diag } (D_{a1}, D_{a2}, \cdots, D_{aN}) \\
F_a(k) &\triangleq \text{block diag } (F_{a1}(k), F_{a2}(k), \cdots, F_{aN}(k)) \\
E_a &\triangleq \text{block diag } (E_{a1}, E_{a2}, \cdots, E_{aN}) \\
B &\triangleq \text{block diag } (B_1, B_2, \cdots, B_N) \\
\Delta B &\triangleq \text{block diag } D_b F_b(k) E_b \\
D_b &\triangleq \text{block diag } (D_{b1}, D_{b2}, \cdots, D_{bN}) \\
F_b(k) &\triangleq \text{block diag } (F_{b1}(k), F_{b2}(k), \cdots, F_{bN}(k)) \\
E_b &\triangleq \text{block diag } (E_{b1}, E_{b2}, \cdots, E_{bN}) \\
\delta &\triangleq \text{block diag } (\delta_1 I, \delta_2 I, \cdots, \delta_N I) \\
\Phi(k) &\triangleq \text{block diag } (\Phi_1(k), \Phi_2(k), \cdots, \Phi_N(k)) \\
K &\triangleq \text{block diag } (K_1, K_2, \cdots, K_N) \\
A_D &\triangleq \text{block matrix with elements } A_{ij} (i \ne j) \\
\Delta A_D &\triangleq \sum_{i=1}^{N} D_{di} F_{di}(k) E_{di} \\
D_{di} &\triangleq \text{block matrix with elements } D_{di}(r, j) \\
D_{di}(r, j) &= \begin{cases} D_{aij}, & r = i, i \ne j \\ 0, & \text{otherwise} \end{cases} \\
F_{di}(k) &\triangleq \text{block diag with diagonal elements } F_{di}(j, j) \\
F_{di}(j, j) &= \begin{cases} F_{aij}, & j \ne i, j = 1, \cdots, N \\ 0, & \text{otherwise} \end{cases} \\
E_{di} &\triangleq \text{block diag with diagonal elements } E_{di}(j, j) \\
E_{di}(j, j) &= \begin{cases} E_{aij}, & j \ne i, j = 1, \cdots, N \\ 0, & \text{otherwise.} \end{cases}
\end{aligned} \qquad (9)$$

Then, the problem addressed in this paper is that of finding a robust stabilizing decentralized state feedback controllers of the form

$$U(k) = -(I + \delta\Phi(k))KX(k), \qquad (10)$$

so that the closed-loop system (8) is asymptotically stabilized with non-fragility delta δ.

Ⅲ. Robust Non-fragile Stabilization

In this section, we consider the problem of decentralized robust stabilization of the uncertain closed-loop system described by (8) using the Lyapunov method with LMI technique.

Before proceeding further, we will state well known lemma.

Lemma 1[1]. The linear matrix inequality

$$\begin{bmatrix} H(x) & W(x) \\ W(x)^T & R(x) \end{bmatrix} > 0,$$

is equivalent to

$$R(x)>0,\ H(x)-W(x)R(x)^{-1}W(x)^T>0,$$

where $H(x)=H(x)^T, R(x)=R(x)^T$ and $W(x)$ depend affinely on x.

Then, we have following theorem for robust stability of system (8).

Theorem 1 : The closed-loop system (8) is asymptotically stable, if there exist positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and ε_4, a block diagonal matrix M and positive definite block diagonal matrices δ, Q and S satisfying the following LMI :

$$\left[\begin{array}{ccccc} \Sigma & B\delta & 0 & AQ-BM & 0 \\ * & -I & \delta^T E_b^T & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 \\ * & * & * & -Q & Q \\ * & * & * & * & -S \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{array}\right.$$
$$\left.\begin{array}{cccc} 0 & 0 & 0 & A_D S E_D^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ M^T E_b^T & Q E_a^T & M^T & 0 \\ 0 & 0 & 0 & 0 \\ -\varepsilon_4 I & 0 & 0 & 0 \\ * & -\varepsilon_1 I & 0 & 0 \\ 0 & 0 & -I & 0 \\ * & * & * & -\varepsilon_2 I + E_D S E_D^T \end{array}\right] < 0 \qquad (11)$$

where

$$\Sigma = -Q + \varepsilon_1 D_a D_a^T + \varepsilon_2 D_D D_D^T + \varepsilon_3 D_b D_b^T + \varepsilon_4 D_b D_b^T + A_D S A_D^T$$

and

$$D_D = \left(\sum_{i=1}^{N} D_{di} D_{di}^T\right)^{1/2},\quad E_D = \left(\sum_{i=1}^{N} E_{di}^T E_{di}\right)^{1/2}. \qquad (12)$$

Then, the feedback gain K of the controller (10) is

$$K = MQ^{-1}, \qquad (13)$$

and the nonfragility of the controller (13) is δ.

Proof : Consider a Lyapunov function

$$V(X(k)) = X^T(k)PX(k) + \sum_{i=k-h}^{k-1} X^T(i)RX(i) \qquad (14)$$

where P and R are the positive-definite matrices.

The difference of V is given by

$$\begin{aligned} \Delta V_k &= V(X(k+1)) - V(X(k)) \\ &= X^T(k+1)PX(k+1) + \sum_{i=k+1-h}^{k} X^T(i)RX(i) \\ &\quad - X^T(k)PX(k) - \sum_{i=k-h}^{k-1} X^T(i)RX(i) \\ &= X^T(k)\{[A+\Delta A-(B+\Delta B)K-(B+\Delta B) \\ &\quad \delta\Phi(k)K]^T \cdot P[A+\Delta A-(B+\Delta B)K-(B+\Delta B) \\ &\quad +2X^T(k)[A+\Delta A-(B+\Delta B)K-(B+\Delta B) \\ &\quad \delta\Phi(k)K]^T\ \delta\Phi(k)K] - P + R\}X(k) \cdot P[A_D+\Delta A_D] \\ &\quad X_d(k) + X_d^T(k)[(A_D+\Delta A_D)^T P(A_D+\Delta A_D) - R] \\ &\quad X_d(k) \\ &\equiv \overline{X}^T(k) M \overline{X}(k) \end{aligned} \qquad (15)$$

where

$$\overline{X}(k) = [X^T(k)\ X_d^T(k)]^T$$

and

$$M = \begin{bmatrix} \begin{pmatrix} [A+\Delta A-(B+\Delta B)K \\ -(B+\Delta B)\delta\Phi(k)K]^T \\ \cdot P[A+\Delta A-(B+\Delta B)K \\ -(B+\Delta B)\delta\Phi(k)K] \\ -P+R \end{pmatrix} & \begin{pmatrix} [A+\Delta A-(B+\Delta B)K \\ -(B+\Delta B)\delta\Phi(k)K]^T \\ \cdot P[A_D+\Delta A_D] \end{pmatrix} \\ * & (A_D+\Delta A_D)^T P(A_D+\Delta A_D) - R \end{bmatrix}$$

Hence, ΔV_k is negative if the matrix M is negative definite. By Lemma 1 (Schur Complements), the fact that $M<0$ is equivalent to

$$W_0 \equiv \begin{bmatrix} -P^{-1} & A+\Delta A-BK-B\delta\Phi(k)K & A_D+\Delta A_D \\ * & -(P-R) & 0 \\ * & * & -R \end{bmatrix}$$

$$= \begin{bmatrix} -P^{-1} & \begin{pmatrix} A-BK \\ +D_aF_a(k)E_a \\ -B\delta\Phi(k)K \end{pmatrix} & A_D+\sum_{i=1}^{N} D_{di}F_{di}(k)E_{di} \\ * & -(P-R) & 0 \\ * & * & -R \end{bmatrix} < 0. \quad (16)$$

Using the known fact that

$$U\Delta V^T + V\Delta U^T \le \varepsilon UU^T + \varepsilon^{-1}VV^T,\ \ \varepsilon > 0 \quad (17)$$

for any matrices U, V and Δ with $\Delta^T\Delta \le I$, we can eliminate the unknown factor , $F_a(k)$, $F_{di}(k)$ and $\Phi(k)$, in (16). Then we have

$$W_0 \le W_1 \equiv \begin{bmatrix} \begin{pmatrix} -P^{-1}+\varepsilon_1 D_aD_a^T \\ +(B+\Delta B)\delta \\ \cdot\delta(B+\Delta B)^T \\ +\varepsilon_2\sum_{i=1}^{N} D_{di}D_{di}^T \end{pmatrix} & A-(B+\Delta B)K & A_D \\ * & \begin{pmatrix} -(P-R) \\ +\varepsilon_1^{-1}E_a^TE_a \\ +K^TK \end{pmatrix} & 0 \\ * & * & -R+\varepsilon_2^{-1}\sum_{i=1}^{N} E_{di}^TE_{di} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{pmatrix} -P^{-1}+\varepsilon_1 D_aD_a^T \\ +\varepsilon_2 D_DD_D^T \\ +(B+\Delta B)\delta \\ \cdot\delta(B+\Delta B)^T \end{pmatrix} & A-(B+\Delta B)K & A_D \\ * & \begin{pmatrix} -(P-R) \\ +\varepsilon_1^{-1}E_a^TE_a \\ +K^TK \end{pmatrix} & 0 \\ * & * & -R+\varepsilon_2^{-1}E_D^TE_D \end{bmatrix} < 0 \quad (18)$$

where ε_1 and ε_2 are positive scalar to be chosen later, and D_D and E_D are defined in (12).

Using Lemma 1, the inequality (18) is equivalent to

$$W_2 = \begin{bmatrix} -P^{-1}+\varepsilon_1 D_aD_a^T+\varepsilon_2 D_DD_D^T & (B+\Delta B)\delta & A-(B+\Delta B)K & 0 & 0 & A_D & 0 \\ * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & -P+R & E_a^T & K^T & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -R & E_D^T \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0. \quad (19)$$

Again, using the fact (17), we have

$$W_2 \le W_3 = \begin{bmatrix} W(1,1) & B\delta & A-BK & 0 & 0 & A_D & 0 \\ * & -I+\varepsilon_3^{-1}\delta^TE_b^TE_b\delta & 0 & 0 & 0 & 0 & 0 \\ * & * & \begin{pmatrix} -P+R \\ +\varepsilon_4^{-1}K^TE_b^TE_bK \end{pmatrix} & E_a^T & K^T & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -R & E_D^T \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix}$$

$$\le \begin{bmatrix} W(1,1) & B\delta & 0 & A-BK & 0 & 0 & 0 & A_D & 0 \\ * & -I & \delta^TE_b^T & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P+R & K^TE_b^T & E_a^T & K^T & 0 & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -R & E_D^T \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0 \quad (20)$$

where $W(1,1) = -P^{-1}+\varepsilon_1 D_aD_a^T+\varepsilon_2 D_DD_D^T+\varepsilon_3 D_bD_b^T+\varepsilon_4 D_bD_b^T$, and ε_3 and ε_4 are the positive scalars to be chosen later.

Again, using Lemma 1, the inequality (20) is also equivalent to

$$\begin{bmatrix} W(1,1) & B\delta & 0 & A-BK & 0 & 0 & 0 & 0 \\ * & -I & \delta^TE_b^T & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P+R & K^TE_b^T & E_a^T & K^T & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} + \begin{bmatrix} A_D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_D \end{bmatrix} R^{-1} \begin{bmatrix} A_D^T & 0 & 0 & 0 & 0 & 0 & 0 & E_D^T \end{bmatrix}$$

$$
= \begin{bmatrix} W(1,1)+A_DR^{-1}A_D^T & B\delta & 0 & A-BK & 0 & 0 & 0 & A_DR^{-1}E_D^T \\ * & -I & \delta^TE_b^T & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P+R & K^TE_b^T & E_a^T & K^T & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I+E_DR^{-1}E_D^T \end{bmatrix} < 0 \qquad (21)
$$

Pre- and post-multiply inequality (21) by T^T and T, where $T= block\ diag\{I,I,I,P^{-1},I,I,I,I\}$, we obtain

$$
\begin{bmatrix} W(1,1)+A_DR^{-1}A_D^T & B\delta & 0 & AP^{-1}-BKP^{-1} & 0 & 0 & 0 & A_DR^{-1}E_D^T \\ * & -I & \delta^TE_b^T & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P^{-1}+P^{-1}RP^{-1} & P^{-1}K^TE_b^T & P^{-1}E_a^T & P^{-1}K^T & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I+E_DR^{-1}E_D^T \end{bmatrix} < 0. \qquad (22)
$$

Using some change of variables, $M=KP^{-1}$, $S=R^{-1}$ and $Q=P^{-1}$, the inequality (22) is changed to

$$
\begin{bmatrix} \Sigma & B\delta & 0 & AQ-BM & 0 & 0 & 0 & A_DSE_D^T \\ * & -I & \delta^TE_b^T & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q+QS^{-1}Q & M^TE_b^T & QE_a^T & M^T & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I+E_DSE_D^T \end{bmatrix} < 0 \qquad (23)
$$

where Σ is defined in (12).

By Lemma 1, the inequality (23) is equivalent to the LMI (11). This completes the proof. ■

Remark 2 : The measure of non-fragility in each controller for subsystems, δ_i, can be obtained from $\delta(=diag(\delta_1 I, \delta_2 I, \cdots, \delta_N I))$, after finding the LMI solutions of (11). Note that there is a trade-off between robustness of the system and non-fragility of controller. That is, when the value of measure of non-fragility is increased, the system has the poor response.

Remark 3 : When the controller gain variations of the large-scale systems (1) are of the following multiplicative form[4, 7, 9]:

$$\Delta K_i = H_i \Phi_i(k) E_i K_i, \quad \Phi_i^T(k)\Phi_i(k) \le I$$

with H_i and E_i being known constant matrices, and $\Phi(k)$ the uncertain parameter matrix, by defining $H= block\ diag(H_1, \cdots, H_N)$ and $E= block\ diag(E_1, \cdots E_N)$ the stability criterion of the closed- loop system with the control law (10) is identical to the LMI (11) except that the (1,2)th entry and (4,8)th entry of $\Sigma(\cdot)$ given in (11) are changed as BH^T and M^TE^T, respectively. The proof is trivial and omitted. ■

Remark 4 : In order to solve the LMI (11) given in Theorem 1, we can utilize Matlab's LMI Control Toolbox[20], which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms[1].

Numerical Example : Consider a large-scale system which is composed of the following three inter-connected subsystems

$$
\begin{aligned}
x_1(k+1) = & \left(\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0.4\cos(k) \\ 0.2\sin(k) & 0 \end{bmatrix} \right) x_1(k) \\
& + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(k) \\
& + \left(\begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.05\cos(k) \end{bmatrix} \right) x_2(k-h_2) \\
& + \left(\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} + \begin{bmatrix} 0.04\cos(k) & 0 \\ 0 & 0.04\sin(k) \end{bmatrix} \right) x_3(k-h_3)
\end{aligned}
$$

$$
\begin{aligned}
x_2(k+1) &= \left(\begin{bmatrix} 0 & 1 \\ 0.5 & -0.5 \end{bmatrix} + \begin{bmatrix} 0 & 0.09\cos(k) \\ 0.09\sin(k) & 0 \end{bmatrix}\right) x_2(k) \\
&+ \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_2(k) \\
&+ \left(\begin{bmatrix} 0 & 0.05 \\ 0.05 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0.04\cos(k) \\ 0 & 0 \end{bmatrix}\right) x_1(k-h_1) \\
&+ \left(\begin{bmatrix} 0.09 & 0 \\ 0 & 0.09 \end{bmatrix} + \begin{bmatrix} 0.05\cos(k) & 0 \\ 0 & 0.05\sin(k) \end{bmatrix}\right) x_3(k-h_3) \\
x_3(k+1) &= \left(\begin{bmatrix} -0,5 & 0.5 \\ 0 & 1.2 \end{bmatrix} + \begin{bmatrix} 0.1\cos(k) & 0 \\ 0 & 0,2\sin(k) \end{bmatrix}\right) x_3(k) \\
&+ \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_3(k) \\
&+ \left(\begin{bmatrix} 0 & 0.1 \\ 0.02 & 0.1 \end{bmatrix} + \begin{bmatrix} 0 & 0.04\cos(k) \\ 0.04\sin(k) & 0 \end{bmatrix}\right) x_1(k-h_1) \\
&+ \left(\begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix} + \begin{bmatrix} 0 & 0.04\cos(k) \\ 0.04\sin(k) & 0 \end{bmatrix}\right) x_2(k-h_2)
\end{aligned} \tag{24}
$$

where

$$x_i(k) = [x_{i1}^T(k) \quad x_{i2}^T(k)]^T,$$

$$u_i(k) = [u_{i1}^T(k) \quad u_{i2}^T(k)]^T, \quad i = 1,2,3$$

and the time-delays and initial conditions are

$$h_1 = 3, \quad h_2 = 5, \quad h_3 = 10,$$

$$x_1(k) = [1 \quad -0.5]^T, \quad x_2(k) = [0.5 \quad 1]^T,$$

$$x_3(k) = [-1 \quad 0.5]^T \quad \text{for} \quad -h_3 \le k \le 0$$

The above system is of the form of system (7) with

$$
\begin{aligned}
&D_{a1} = \begin{bmatrix} 0.6325 & 0 \\ 0 & 0.447 \end{bmatrix}, \ D_{a2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
&D_{a3} = \begin{bmatrix} 0.3162 & 0 \\ 0 & 0.4472 \end{bmatrix}, E_{a1} = \begin{bmatrix} 0 & 0.6325 \\ 0.4472 & 0 \end{bmatrix}, \\
&E_{a2} = \begin{bmatrix} 0 & 0.3 \\ 0.3 & 0 \end{bmatrix}, \ E_{a3} = \begin{bmatrix} 0.3162 & 0 \\ 0 & 0.4472 \end{bmatrix}, \\
&D_{a12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.2236 \end{bmatrix}, \ D_{a13} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
&D_{a21} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}, D_{a23} = \begin{bmatrix} 0.2236 & 0 \\ 0 & 0.2236 \end{bmatrix}, \\
&D_{a31} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \ D_{a32} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
&E_{a12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.2236 \end{bmatrix}, \ E_{a13} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
&E_{a21} = \begin{bmatrix} 0 & 0 \\ 0 & 0.2 \end{bmatrix}, \ E_{a23} = \begin{bmatrix} 0.2236 & 0 \\ 0 & 0.2236 \end{bmatrix}, \\
&E_{a31} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \ E_{a32} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \\
&D_{b1} = D_{b2} = D_{b3} = 0I, \\
&E_{b1} = E_{b2} = E_{b3} = 0I.
\end{aligned}
$$

Solving the LMI (11), we can obtain the solutions as

$$
Q = \begin{bmatrix}
1.2159 & 0.0118 & 0 & 0 & 0 & 0 \\
0.0118 & 0.2675 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.0667 & 0.3353 & 0 & 0 \\
0 & 0 & 0.3353 & 0.2362 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0594 & -0.0882 \\
0 & 0 & 0 & 0 & -0.0882 & 0.2810
\end{bmatrix}
$$

$$
S = \begin{bmatrix}
3.259 & 0.1314 & 0 & 0 & 0 & 0 \\
0.1314 & 1.6926 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9443 & -0.8275 & 0 & 0 \\
0 & 0 & -0.8275 & 2.0377 & 0 & 0 \\
0 & 0 & 0 & 0 & 2.5128 & 0.3920 \\
0 & 0 & 0 & 0 & 0.3393 & 1.1948
\end{bmatrix},
$$

$$
M = \begin{bmatrix}
0.0114 & 0.2586 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.3724 & -0.0246 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.166 & 0.2933
\end{bmatrix},
$$

$$
\delta = \begin{bmatrix}
0.1897 & 0 & 0 \\
0 & 0.1976 & 0 \\
0 & 0 & 0.1859
\end{bmatrix}
$$

$$\varepsilon_1 = 0.4228, \quad \varepsilon_2 = 0.4374, \quad \varepsilon_3 = 131.8999, \quad \varepsilon_4 = 131.8999.$$

Since $K = MQ^{-1} = diag(K1, K2, K3)$, the gain matrices, K_i, of the stabilizing controller, $u_i(k)$, for three subsystems are

$$
\begin{aligned}
K_1 &= [0.0000 \quad 0.9655], \\
K_2 &= [0.6898 \quad -1.0836], \\
K_3 &= [-0.0717 \quad 1.0213].
\end{aligned} \tag{25}
$$

The obtained robust decentralized controller guarantees the asymptotic stability of the closed-loop system in spite of each controller gain variations of the subsystem 1, 2 and 3 within 18.97%, 19.76%, and 18.59%, respectively.

For numerical simulation, the following control laws are employed:

$$
\begin{aligned}
u_1(k) &= -(1 + 0.1897\sin(k))K_1 x_1(k) \\
u_2(k) &= -(1 + 0.1976\sin(k))K_2 x_2(k) \\
u_3(k) &= -(1 + 0.1859\sin(k))K_3 x_3(k).
\end{aligned}
$$

The simulation results are in Figs. 1, 2, 3 and 4. In the figures, one can see that the system is well stabilized irrespective of uncertainties and controller gain perturbations.

On the other hand, to briefly show the fragility of a controller designed without thinking over controller gain variations, consider a stabilizing controller obtained by classical pole-placement approach of the system :

$$K_1 = [0.06 \quad 0.5],$$
$$K_2 = [0.92 \quad -1.72],$$
$$K_3 = [-0.2824 \quad 0.5824].$$

so that the closed-loop poles (eigenvalues of $A_i - B_iK_i$) of each subsystem are $\{0.2, 0.3\}$, $\{0.2, 0.1\}$, and $\{0.3, 0.1\}$, respectively. For the controller (25) with 18.97%, 19.76%, and 18.59% gain variations, respectively, the simulation result are given in Fig. 5. From this, one can see that the system has poor response due to the controller gain variations.

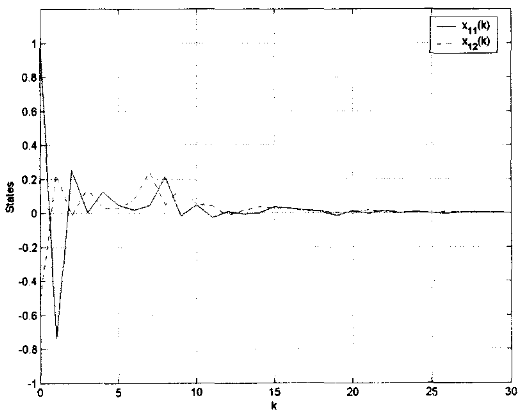

그림 1. 부 시스템1의 상태 응답
Fig. 1. State responses of subsystem 1.

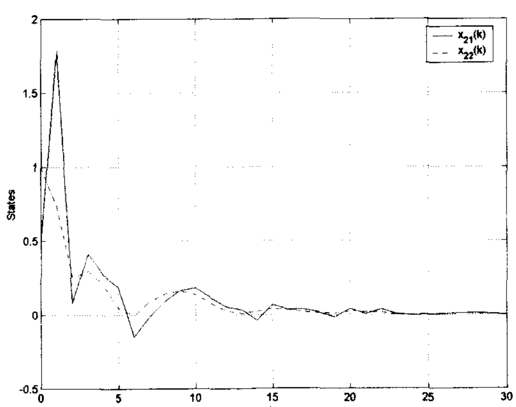

그림 2. 부 시스템2의 상태 응답
Fig. 2. State responses of subsystem 2.

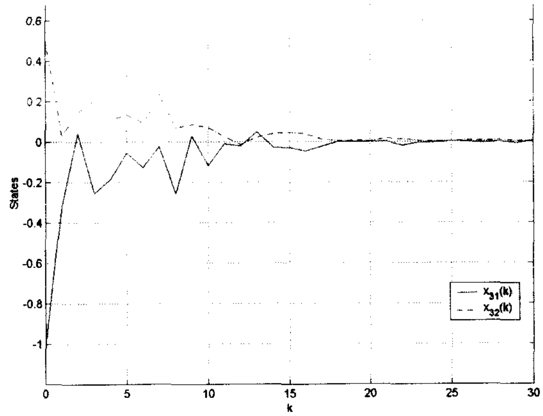

그림 3. 부 시스템3의 상태 응답
Fig. 3. State responses of subsystem 3

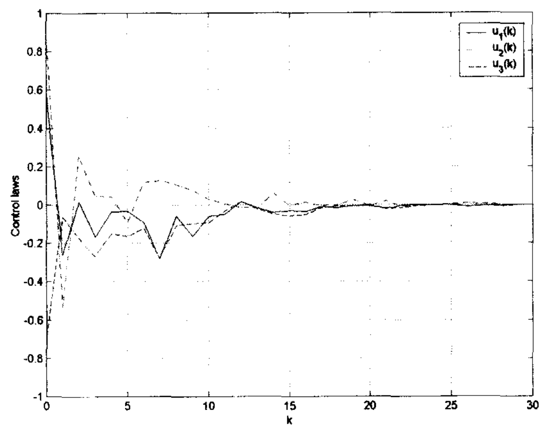

그림 4. 부 시스템 1, 2, 3의 제어입력
Fig. 4. Control inputs for subsystem 1, 2, and 3.

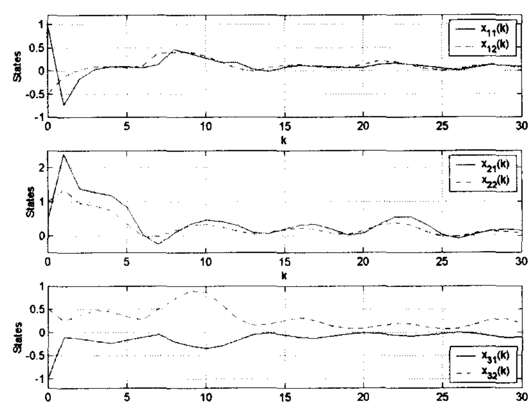

그림 5. 비약성을 고려하지 않은 제어기를 갖는 시스템의 응답
Fig. 5. State responses of subsystems with conventional controller.

Ⅳ. Conclusion

In this paper, we have investigated the problem of robust decentralized non-fragile control of uncertain discrete-time large-scale systems with delays and controller gain perturbations. Using the Lyapunov method, a stability criterion for robust stability of the system is derived in terms of LMI. Furthermore, the measure of non-fragility in the controller is presented. Finally, a numerical example is given for illustration of controller design, and simulation result shows that the system is well stabilized in spite of controller gain variations and uncertainties.

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