Variable Structure Controller for Linear Time-Varying Sampled-Data Systems with Disturbances

論 文
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Abstract – In this paper, a discrete-time variable structure controller for linear time-varying sampled-data systems with disturbances is proposed. The proposed method guarantees that the system state is globally uniformly ultimately bounded (G.U.U.B.), and the ultimate bound is shown to be the order of T^2, $O(T^2)$, where T is a sampling period.

Key Words : Variable Structure System, Sliding Mode Control, Robust Control, Linear Time-Varying Sampled-Data Systems.

1. Introduction

It has been known that the continuous-time Variable Structure Control (VSC) has a robust and invariant property to parameter uncertainties and external disturbances [1]-[3]. The actual control systems, however, have been implemented in the discrete-time domain in general. Furthermore, it is well known that a system with the controller designed in the continuous-time domain may become unstable after the sampling [4]. Thus, the research of a variable structure control scheme in the discrete-time domain is inevitable.

Recently, therefore, variable structure control schemes in the discrete-time domain have attracted the attention [5]-[10]. Generally speaking, lots of previous works of discrete-time variable structure control have been designed for the linear *time-varying* discrete-time plant with *no external disturbances* and/or parameter uncertainties:

$$x_{k+1} = A_k x_k + Bu_k.$$

Another works has been studied for the *time-invariant* system *with uncertainty*[11].

Thus, in this paper, a linear *time-varying* plant *with disturbances and parameter uncertainties* is considered. Even the plant under the control is in the continuous-time domain, an actual digital control system is constructed with A/D and D/A converters and a digital computer. Hence, a sampled-data system is considered in this paper.

It is guaranteed that the system state is globally uniformly ultimately bounded (G.U.U.B.) under the existence of parameter uncertainties and external disturbances. It is also shown that the ultimate bound of the state is the order $O(T^2)$, where T is the sampling period. Moreover, an inter-sample value is also shown to be the order $O(T^2)$.

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2. Problem Formulation

Consider a continuous-time linear time-varying plant with the parameter uncertainties and the external disturbances of the following form:

$$\begin{aligned} \dot{x}(t) &= \overline{A}(t)x(t) + Bu(t) + m(t) \\ &= (A(t) + \Delta A(t))x(t) + Bu(t) + m(t) \\ &= A(t)x(t) + Bu(t) + \zeta(t) \end{aligned} \tag{1}$$

where $x(\cdot) \in R^n$ is the state vector, $u(\cdot)$ is the scalar input, $A(\cdot) \in R^{n\times n}$ is the nominal part of a linear time-varying system matrix $\overline{A}(\cdot) \in R^{n\times n}$, and it is assumed that there exists a positive constant α such that $\|A(\cdot)\| \le \alpha$. B is the input matrix with the appropriate dimension, $m(\cdot) \in R^n$ is the vector of external disturbances, and $\zeta(\cdot) = \Delta A(\cdot)x(\cdot) + m(\cdot) \in R^n$ is the vector of time-varying parameter uncertainties and external disturbances. It is assumed that the following assumption holds for $\zeta(\cdot)$.

Assumption 1 *$\zeta_i(\cdot)$ is a smooth function satisfying following conditions:*

$$\zeta_i \in C^1[0, \infty),\ \zeta_i,\ \dot{\zeta}_i \in L^\infty,$$

and there exists a positive constant ξ_i such that

$$|\zeta_i(\cdot)| \le \xi_i < \infty,$$

where $i=1,2,\cdots,n$.

It is also assumed that the control input applied to the plant shows a piecewise constant signal, i.e.,

$$u(t)=u_k \quad \text{if} \quad kT \le t < (k+1)T.$$

Then, using the Euler's method, the discrete-time system of the plant (1) can be obtained as follows:

$$\begin{aligned} x_{k+1} &= x_k + T\cdot \dot{x}(kT) + T\cdot h_k \\ &= x_k + T\{A_k x_k + Bu_k + \zeta_k\} + T\cdot h_k \qquad (2) \\ &= x_k + T\{A_k x_k + Bu_k + d_k\} \qquad (3) \end{aligned}$$

where T is the sampling period, x_k represents the k-th sampled-data of $x(t)$, i.e., $x_k \equiv x(kT)$, h_k is the higher-order terms of Euler's approximation errors of the order of T, i.e., $h_{i_k}=O(T)$, $d_k=\zeta_k+h_k$, and $k=1,2,\cdots$.

Remark 1 *From the approximation error of the Euler's method, it is clear that there exists a vector* $H \in R^n$ *such that*

$$\max_{0\le\tau\le T}|x_i(kT+\tau)-\{x_i(kT)+\tau\cdot \dot{x}_i(kT)\}| \le H_i T^2, \qquad (4)$$

where $k=1,2,\cdots,$ *and* $i=1,2,\cdots,n$. *Thus, for the vector* h_k *in Eq. (2), the following inequality can be guaranteed.*

$$|h_{i_k}| \le H_i T, \qquad (5)$$

where $i=1,2,\cdots,n,$ *and* $k=1,2,\cdots$.

Remark 2 *There exists a positive constant* D_i *such that*

$$|d_{i_{k+1}} - d_{i_k}| \le (\xi_i+2H_i)T = D_i T = O(T), \qquad (6)$$

where $D_i=\xi_i+2H_i$, $i=1,2,\cdots,n,$ *and* $k=1,2,\cdots$.

3. Main Results

Let the sliding surface as

$$s_k = Cx_k$$

where $C^T \in R^n$ is assumed to be designed such that CB is nonsingular and the sliding dynamics, $s_k \equiv 0$, is globally uniformly asymptotically stable. Then, the following theorem can be derived for the closed-loop system.

Theorem 1 *For the discrete-time linear time varying system (3) with the proposed controller (7), it is guaranteed that the sliding surface function,* s_k, *is globally uniformly ultimately bounded (G.U.U.B.):*

$$\begin{aligned} u_k &= u_{k-1} + v_k \\ &= u_{k-1} + (CB)^{-1}\Big[-C\{A_k x_k - A_{k-1}x_{k-1}\} \qquad (7) \\ &\quad -\frac{1}{T}\{s_k - s_{k-1}\} - K_1 s_k - K_2 T\, \mathrm{sgn}(s_k)\Big] \end{aligned}$$

where $0 < K_1 < \frac{1}{T}$, $K_2 > |CD|$, *and*

$$v_k = -(CB)^{-1}\Big[C\{A_k x_k - A_{k-1}x_{k-1}\} \frac{1}{T}\{s_k - s_{k-1}\} + K_1 s_k + K_2 T\, \mathrm{sgn}(s_k)\Big].$$

Proof : From Eq. (3) and $s_k = Cx_k$, the past input signal u_{k-1} can be derived as

$$u_{k-1} = (CB)^{-1}\Big[-CA_{k-1}x_{k-1} - Cd_{k-1} + \frac{1}{T}\{s_k - s_{k-1}\}\Big].$$

Thus, s_{k+1} can be derived as

$$\begin{aligned} s_{k+1} &= Cx_{k+1} \\ &= C[x_k + T\{A_k x_k + Bu_k + d_k\}] \\ &= s_k + T[CA_k x_k + CBu_k + Cd_k] \\ &= s_k + T[C\{A_k x_k - A_{k-1}x_{k-1}\} \\ &\quad + \frac{1}{T}\{s_k - s_{k-1}\} + C\{d_k - d_{k-1}\} \qquad (8) \\ &\quad + (CB)v_k] \\ &= s_k + T[C\{d_k - d_{k-1}\} \\ &\quad - K_1 s_k - K_2 T\, \mathrm{sgn}(s_k)] \\ &= (1-K_1 T)s_k \\ &\quad + T[C\{d_k - d_{k-1}\} - K_2 T\, \mathrm{sgn}(s_k)]. \end{aligned}$$

Since $0 < K_1 < \frac{1}{T}$ and $K_2 > |CD|$, it is clear that

$$|s_k| \le T^2\{|CD| + |K_2|\}, \quad k \ge k_h \qquad (9)$$

where k_h is the time instant when the system state hits the sliding surface $s_k=0$ for the first time.

■

The proposed control system (7) used the past input signal, u_{k-1}, since the overall unknown term, d_k, can be observed from u_{k-1} although it cannot be measured directly.

Remark 3 *In the continuous-time domain, one of the reaching conditions is*

$$\dot{s}(t) = -K_1 s(t) - K_2 T \operatorname{sgn}(s(t)).$$

By applying the Euler's approximation, in the discrete-time domain, the above equation can be written as

$$\frac{s_{k+1} - s_k}{T} = -K_1 s_k - K_2 T \operatorname{sgn}(s_k).$$

Clearly, it can be rewritten as

$$\begin{aligned} s_{k+1} &= s_k + T[-K_1 s_k - K_2 T \operatorname{sgn}(s_k)] \\ &= (1 - K_1 T) s_k - T^2 K_2 \operatorname{sgn}(s_k). \end{aligned}$$

Thus, it is easily known that Eq. (8) is one of the discrete-time version of several kind of reaching conditions.

Remark 4 *From Eqs. (6), and (9), it is clear that the ultimate bound of* s_k *can be made very small by shortening the sampling period* T, *i.e., by increasing the sampling frequency since its ultimate bound is the order of* T^2, $O(T^2)$.

$$|s_k| \le T^2\{|CD| + |K_2|\} = O(T^2) \quad (10)$$

where $k \ge k_h$.

All the results obtained so far is guaranteed at every sampling instants, kT, $k = 1, 2, \cdots$. For example, the result obtained in Remark 4 is valid only at every sampling instants. Therefore, for the inter-sample data, the following corollary is derived.

Corollary 1 *The inter-sample data of the sliding surface function,* $s(kT+\tau)$, *is also of the order of* T^2, *i.e.,* $s(kT+\tau) = O(T^2)$, *where* $k \ge k_h$ *and* $0 \le \tau \le T$.

Proof: Applying Euler's method to the original plant (1), the following equation can be obtained for the inter-sample data.

$$x(kT+\tau) = x_k + \tau\{A_k x_k + B u_k + \zeta_k\} + O_1(\tau^2) \quad (11)$$

where the magnitude of $O_1(\tau^2)$ is bounded as follows:

$$|O_{1i}(\tau^2)| \le H_i T^2, \quad \forall\, 0 \le \tau \le T. \quad (12)$$

And the Eq. (11) implies that

$$s(kT+\tau) = s_k + O_2(\tau^2) + \tau C\{A_k x_k + B u_k + \zeta_k\}. \quad (13)$$

For $O_2(\tau^2)$, from Eq. (12), it is clear that

$$|O_{2i}(\tau^2)| \le |CH| T^2, \quad \forall\, 0 \le \tau \le T. \quad (14)$$

When $\tau = T$, Eq. (13) can be rewritten as

$$C\{A_k x_k + B u_k + \zeta_k\} = \frac{1}{T}\{s_{k+1} - s_k - O_2(T^2)\}. \quad (15)$$

Substituting Eq. (15) into Eq. (13), the following inequality can be obtained using Eqs. (10) and (14).

$$\begin{aligned} &|s(kT+\tau)| \\ &= \left| s_k + O_2(\tau^2) + \frac{\tau}{T}\{s_{k+1} - s_k - O_2(T^2)\} \right| \\ &\le \left(1 - \frac{\tau}{T}\right)|s_k| + \frac{\tau}{T}|s_{k+1}| + |O_2(\tau^2)| \quad (16) \\ &\quad + \frac{\tau}{T}|O_2(T^2)| \\ &\le 2T^2\{|CD| + |K_2| + |CH|\} = O(T^2) \end{aligned}$$

where $k \ge k_h$ and $0 \le \tau \le T$.

■

Remark 5 *From Theorem 1 and Corollary 1, it has been shown that* s_k *is ultimately bounded and the ultimate bound is the order of* T^2. *Moreover, the inter-sample data of* s_k *is also ultimately bounded, and its bound is also* $O(T^2)$, *too. Thus, it can be said that* $s(t)$ *is ultimately bounded as can be seen in Eq. (16), i.e.,*

$$|s(t)| \le 2T^2\{|CD| + |K_2| + |CH|\} = O(T^2) \quad (17)$$

where $t \ge k_h T$.

Corollary 2 *The system state,* x_k, *is globally uniformly ultimately bounded.*

Proof: Since C is chosen such that $s_k \equiv 0$ is asymptotically stable, it is obvious.

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Remark 6 *Consider a second-order systems with a canonical form of*

$$\dot{x}_1(t) = x_2(t),$$

where the sliding surface is defined as

$$s(t) = Cx(t) = c_1x_1(t)+c_2x_2(t),$$

where $c_1 > 0$ *and* $c_2 = 1$. *Although the actual discrete-time control system uses the discrete-time sliding surface designed using sampled data as*

$$s_k = Cx_k = c_1x_{1_k}+c_2x_{2_k} = c_1x_{1_k}+x_{2_k},$$

the ultimate bound of $x_1(t)$ *can be found as follows:*

$$\lim_{t\to\infty}|x_1(t)| \le \frac{2T^2\{|CD|+|K_2|+|CH|\}}{c_1} \qquad (18)$$

Proof: Since it is so clear from Eq. (17), the proof is omitted.

■

4. Simulation Results

Consider the following continuous-time system:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & a_{22}(t)\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)+\begin{bmatrix}0\\ \zeta_2(t)\end{bmatrix} \qquad (19)$$

where $a_{22}(t) = -1 + 0.1\cos(0.05\pi t)$, and $\zeta_2(t) = 0.5\cos(0.1\pi t) + 0.5\sin(0.05\pi t)$. The sampling period was set by 0.1 second. The sliding surface $s(t)$ was designed by

$$s(t) = c_1x_1(t) + c_2x_2(t),$$

where $c_1 = 2$, and $c_2 = 1$. The control gain K_1 and K_2 was set by $K_1 = 5$ and $K_2 = 1$.

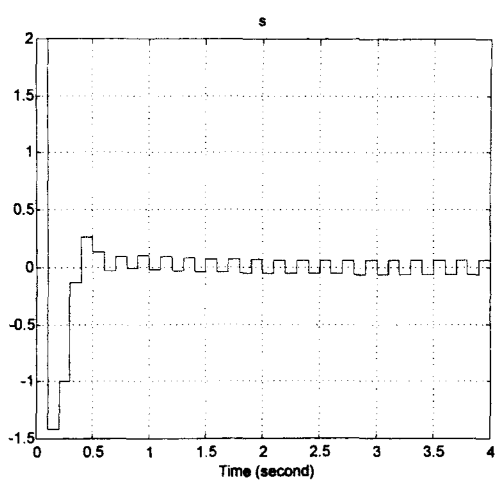

Fig. 1 Sliding Sufrace (cl=2, K1=5, T=0.1)

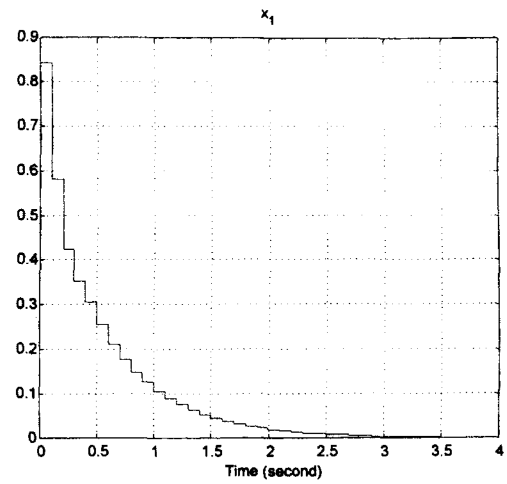

Fig. 2 System State (cl=2, K1=5, T=0.1)

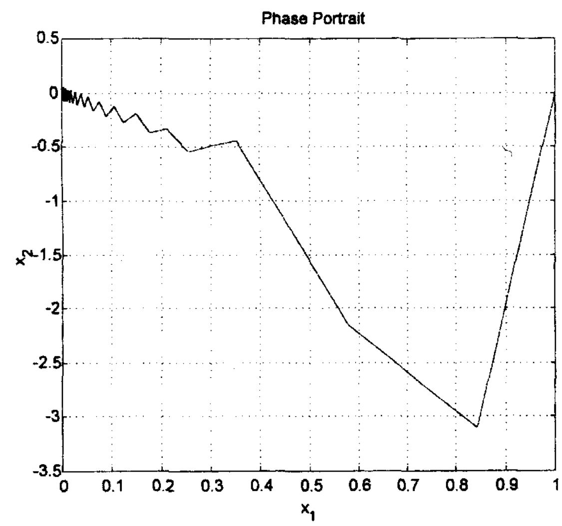

Fig. 3 Phase Portrait (cl=2, K1=5, T=0.1)

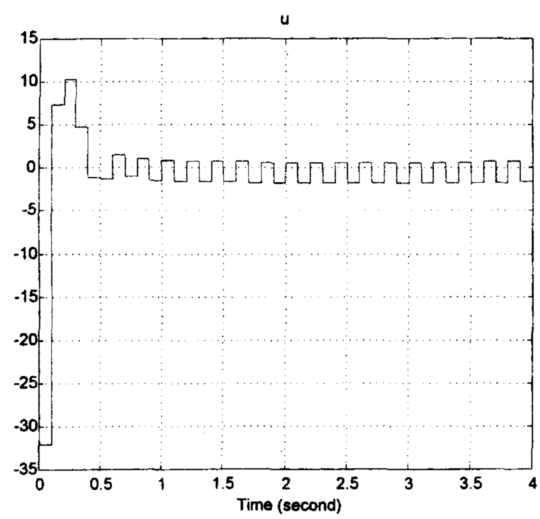

Fig. 4 Control Input (cl=2, K1=5, T=0.1)

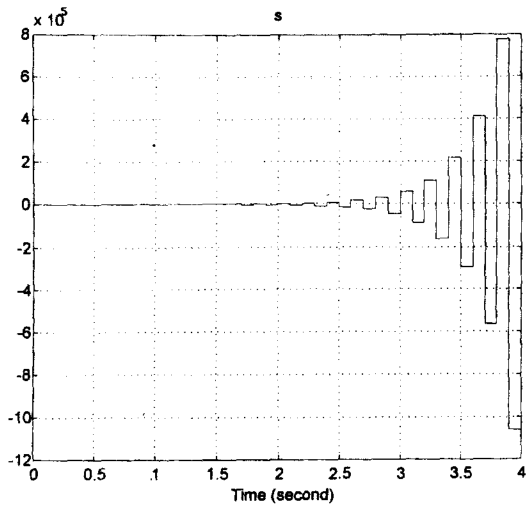

Fig. 5 Sliding Sufrace (cl=2, K1=5, T=0.1)

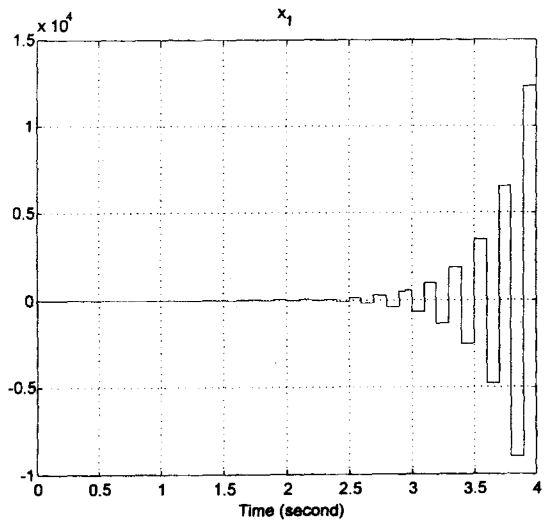

Fig. 6 Sliding Sufrace (cl=2, K1=5, T=0.1)

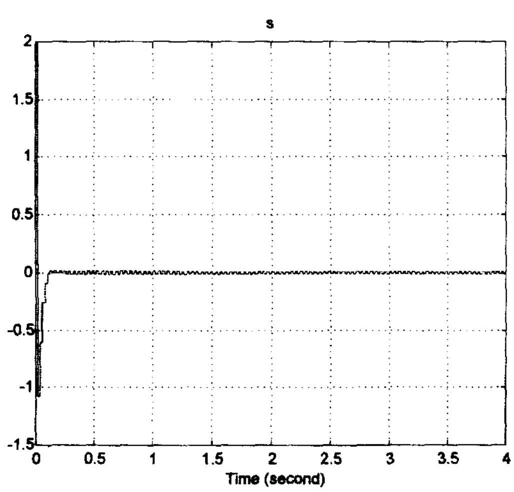

Fig. 7 Sliding Sufrace (cl=2, K1=5, T=0.1)

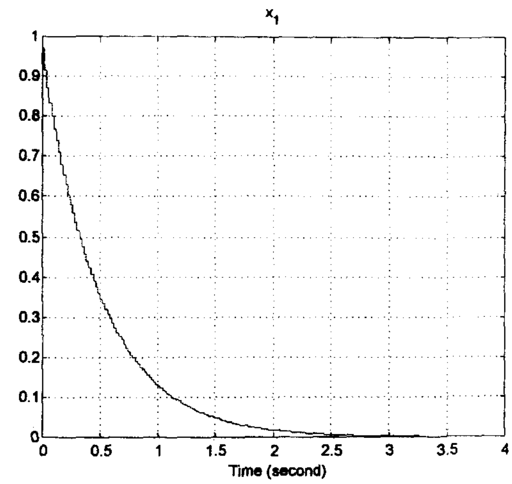

Fig. 8 System State (cl=2, K1=5, T=0.1)

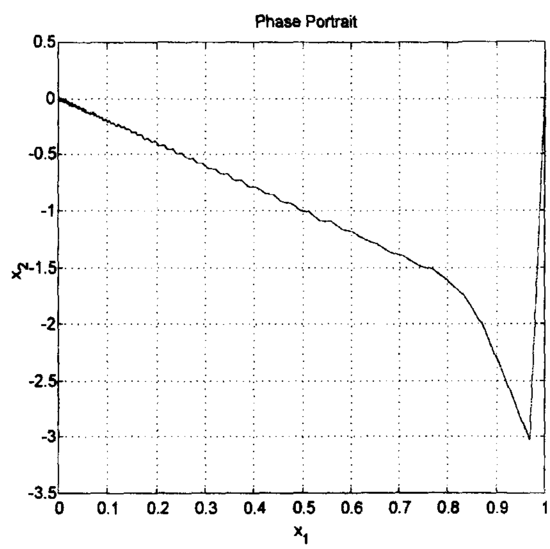

Fig. 9 System State (cl=2, K1=5, T=0.1)

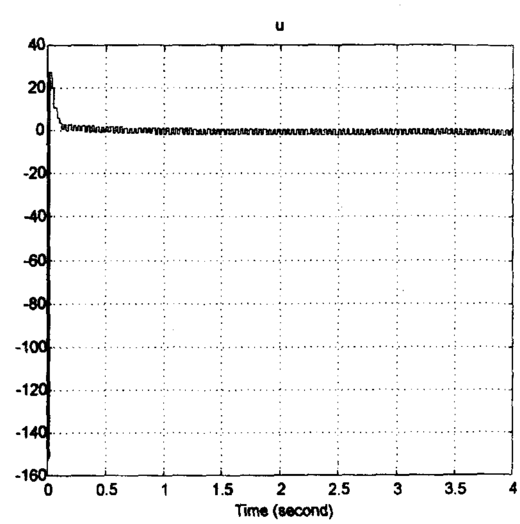

Fig. 10 System State (cl=2, K1=5, T=0.1)

As shown in Figs. 1 and 2, s_k and x_{1k} are ultimately bounded under the existence of the time-varying uncertainty and disturbance. Figure 3 shows the phase portrait, and the control input profile of the proposed controller is shown in Fig. 4.

When the control gain K_1 was set by 25, i.e., $K_1 = 25 > \frac{1}{T} = 10$, the system state of the closed-loop system diverged as can be seen in Figs. 5 and 6.

For the same control gain, i.e., $K_1 = 25$, it was shown that the overall system can be stabilized by shortening the sampling period. Figs. 7 ~ 10 show the results when the sampling period was set by 0.02 second. In this case, it is clear that $K_1 = 25 < \frac{1}{T} = 50$. Figs. 7 and 8 show the ultimate boundedness of s_k and x_{1k}. As can bee seen in Figs. 2 and 8, the profile of x_{1k} shows very similar one since it come from the same structure of the sliding surface, i.e., $c_1 = 2$, and $c_2 = 1$ in both cases.

The phase portrait can be seen in Fig. 9, and Figure 10 shows the control input signal.

5. Conclusions

In this paper, a discrete-time variable structure controller for linear time-varying sampled-data systems with disturbances has been presented. It has been shown that the system state is globally uniformly ultimately bounded (G.U.U.B.) with the order of $O(T^2)$ under the existence of external disturbances and parameter uncertainties. In addition, an inter-sample value has been shown to be $O(T^2)$. Thus, it has been known that the ultimate bound of the system state can be made very small with the order of T^2 by increasing the sampling frequency.

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