정적 부하외란이 있는 경우의 포화함수를 이용한 PID 자동동조

論 文
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PID Autotuning Based on Saturation Function Feedback with A Static Load Disturbance

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Abstract – We consider an unknown linear time invariant plant with static load disturbance. A saturation function nonlinear element is used to find the one point information in the frequency domain. We analyze an asymmetric self-oscillation caused by static load disturbance with relay feedback and saturation function feedback. We propose a new method to tune a PID controller using a saturation nonlinear feedback element in the presence of asymmetric oscillation. The proposed method does not require the knowledge of plant d.c. gain with an asymmetric oscillation in the plant output.

Key Words : PID controller, autotuning, saturation function, static load, ocsillation

1. Introduction

Automatically setting the PID controller called autotuning of PID controller received much attention because of the convenience to use in the industry. Many methods were proposed and commercialized[1]. The excellent review of the different autotuning methods can be found in [1]. One of simple methods is the use of relay feedback to identify the unknown plant[2]. The relay feedback generates the stable limit cycle in the plant output. One point frequency domain information can be obtained by the observation of the unknown plant output. In fact, one point frequency domain information called phase crossover[3] is correspond to the intersection point with negative real axis in the Nyquist plot of the unknown plant. Various PID controller design methods can be applied with phase crossover information[4,5,6,7]. Since the describing function approximation used in the relay feedback method is based on the assumption of which the plant output contains only the fundamental frequency component, it is desired that the plant output does not contain the high frequency component. However the output of relay easily excites the high frequency components, since the output of relay acts like step function. The work[8] used the saturation function instead of relay as a test signal to reduce the error due to the high frequency components. The accuracy of phase crossover estimation in the Nyquist plot of the unknown plant was improved with the use of saturation function. All of works[2,8] assumed that the plant output has a symmetric oscillation. The assumption of the symmetric oscillation enabled to the describing function approximation to find a phase crossover information. However it can be possible that there is an asymmetric oscillation caused by static load disturbance, the use of asymmetric nonlinear element, and non-zero reference input in the plant output[9,10]. A significant error in the estimation of phase crossover information can be occurred when the plant output has an asymmetric oscillation for the relay feedback case. To improve the accuracy of the phase crossover information, the static load disturbance was estimated to restore the symmetric oscillation under the assumption of knowing the plant d.c. gain. We consider an unknown linear plant with an unknown static load disturbance during the tuning of PID controller, since asymmetric oscillation and non-zero reference input is known in advance. We analyze the asymmetric oscillation using the harmonic valance approximation with an saturation function nonlinear element for the unknown plant with the static load disturbance. From the analysis, we propose a new method which does not require the knowledge of d.c. gain of plant to identify one point frequency information of the unknown plant. The identified one point frequency information by the proposed method is not necessary a

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phase crossover information. We give a PID controller design formular using the identified one point information. The performance of the proposed method demonstrated via an example.

2. The autotuning of PID controller with an asymmetric oscillation

2.1 The analysis of symmetric self-oscillation with relay and saturation function

Consider the feedback system without static load disturbance in Fig. 1 to analyze the symmetric oscillation. We need to know some information of unknown plant to design the PID controller. A relay as a nonlinear element was used to identify a phase crossover point during tuning of the PID controller[2]. After the tuning of PID controller, the PID controller is connected to the closed-loop system.

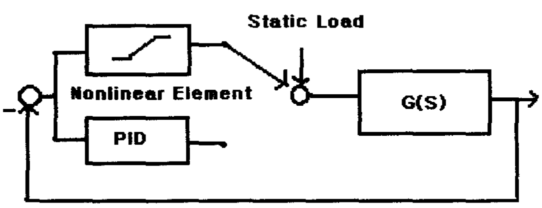

그림 1 비선형 요소를 이용한 PID 자동동조 시스템의 블록도

Fig. 1 A PID autotuning block diagram with a nonlinear element

In more detail for the identification of unknown plant, when the following harmonic equation has a non-trivial solution

$$1+G(jw)\cdot N(a)=0 \tag{1}$$

there is a symmetric self-oscillation in the plant output, where $G(jw)$ is the frequency response of the plant, a is the amplitude of fundamental frequency component of the plant output, and $N(a)=\frac{\pi a}{4d}$ is a describing function of the relay where d is magnitude of the relay. A phase crossover information of the unknown plant can be calculated using the equation (1), since a and w can be obtained from the observation of the plant output. A saturation function defined by

$$sat(x)=\begin{cases} -d & x < -\frac{d}{s} \\ sx & -\frac{d}{s} \le x \le \frac{d}{s} \\ d & x > \frac{d}{s} \end{cases} \tag{2}$$

and shown in Fig. 2 as a nonlinear element was used to improve the accuracy of the estimation of the unknown plant[8].

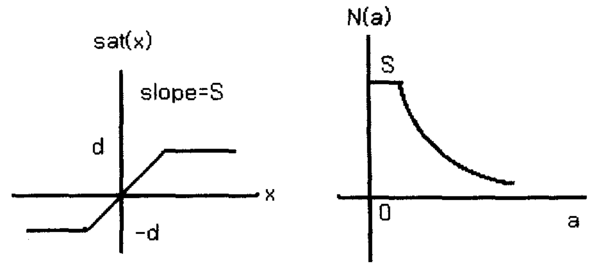

그림 2 포화 함수형 비선형 요소 및 묘사함수

Fig. 2 Saturation nonlinear element and its describing function

We can calculate the ultimate gain defined by $\frac{1}{|G(jw)|}$ from the equation (1) and the ultimate period from the observation of magnitude and period of the plant output.

2.2 The analysis of asymmetric oscillation with the relay and saturation function

It is possible that there is an asymmetric oscillation in the plant output caused by static load disturbances, the use of asymmetric relay, and non-zero constant reference input[9,10]. When there is an asymmetric oscillation caused by static load disturbance in the plant output, the output of relay and the saturation function nonlinear element is shown with $x_1-x_0 \neq x_2-x_1$ and $t_2-t_1 \neq t_5-t_4$ in the Fig. 3.

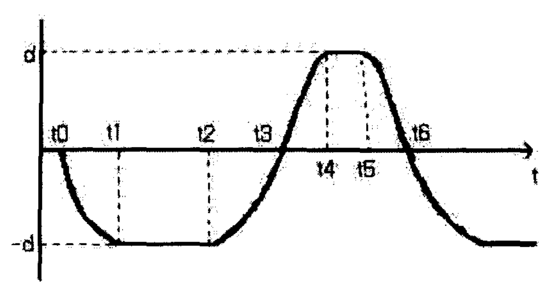

그림 3 플랜트 출력이 비대칭인 경우의 릴레이 와 포화함수의 출력.

Fig. 3 The output of relay and saturation function with asymmetric plant output

Note that when $x_1-x_0=x_2-x_1$ and $t_1-t_2=t_4-t_5$, the plant output is a symmetric one. We assume that there is an asymmetric oscillation caused by an unknown static load disturbance in the plant output. The plant output $y(t)$ and the output of nonlinear element $\psi(-y)$ can be represented by the following Fourier series.

$$y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkwt} \tag{1}$$

$$\psi(-y) = \sum_{k=-\infty}^{\infty} c_k e^{jkwt} \tag{2}$$

where a_k and c_k are Fourier series coefficients for $y(t)$ and $\psi(-y)$, respectively. Since $y(t)$ is the output of the plant, $G(s)$, and $\psi(-y)+l$ is the input of plant, the following equation should hold[11]

$$d(p)y(t) - n(p)(\psi(-y) + l) = 0 \tag{3}$$

where $p = \frac{d()}{dt}$, n(s) and d(s) are the numerator and denominators polynomials of G(s). Using the following relation,

$$d(p) \sum_{k=-\infty}^{\infty} a_k e^{jkwt} = \sum_{k=-\infty}^{\infty} d(jkw) a_k e^{jkwt}$$
$$n(p) \sum_{k=-\infty}^{\infty} c_k e^{jkwt} = \sum_{k=-\infty}^{\infty} c(jkw) c_k e^{jkwt}$$

the equation (3) can be rewritten as

$$\sum_{k=-\infty}^{\infty} [d(jkw) a_k - n(jkw)(c_k + l_k)] e^{jkwt} = 0 \tag{4}$$

where $l_k = l$ for $k=0$, and $l_k = 0$ for $k \geq 1$. Since e^{jwkt} are orthogonal for each k, the equation (4) is equivalent to

$$G(jkw)(c_k + l_k) - a_k = 0 \tag{5}$$

Then the first order harmonic balance approximation is

$$G(0)(\hat{c}_0 + l) - a_0 = 0 \tag{6}$$

$$G(jw)\hat{c}_1 - \frac{a}{2j} = 0 \tag{7}$$

where a_0 is the d.c. value of the plant output, $\hat{c}_o = w/2\pi \int_0^{2\pi/w} \psi(-(a_o + a\sin wt))dt$, $\hat{c}_1 = w/2\pi \int_0^{2\pi/w} \psi(-(a_o + a\sin wt))e^{-jwt}dt$, a and w are the amplitude and frequency of the first harmonic of the plant output, respectively. Note that c_k is a Fourier coefficient for infinite dimensional case, while $\hat{c}_k$ is Fourier coefficient for finite dimensional case which is an approximation of infinite dimensional case. Note that one can derive the equation (1) from the equation (7) with $a_0 = 0$ and the nonlinear element to be an odd function. However when the plant output has an asymmetric oscillation, a significant error was reported if the equation (1) was used to find the ultimate gain for relay feedback case[9]. The work[9] used the equation (6) with the knowledge of G(0) to estimate the static load disturbance for relay feedback case. Then the static load disturbance was cancelled using the estimation of the static load disturbance to restore the symmetric oscillation. After restoring the symmetric oscillation, the equation (1) was used to find the ultimate gain. However this method required the knowledge of the d.c. gain of unknown plant. One can use the equation (7) to find $G(jw)$. However there is a significant estimation error of the plant due to the high harmonics for relay feedback case. The error due to the high harmonics analyze in the next section. We use the saturation function to alleviate the effect of high harmonics.

2.3 The error analysis with asymmetric oscillation for relay and saturation function feedback

We consider the relay feedback and saturation function feedback case for the error analysis. The estimation error of the plant using the equation (7) comes from the high frequency components generated from the nonlinear element. We analyze the first and the second harmonics of the output of the relay and saturation function, since most of plants have a low-pass characteristics. The switching interval $x_1 - x_0$ in Fig. 3 is increased as the d.c. value of plant output a_0 increases for relay feedback case. One can verify that

$$|A_1| = (1/2\pi) \times d\sqrt{8(1-\cos(2\pi w x_1))}$$
$$|A_2| = (1/2\pi) \times d\sqrt{2(1-\cos(4\pi w x_1))}$$

where $|A_i|$, $i=1,2$, denote that the amplitude of the first and second harmonics of relay output respectively and w is the frequency of relay output. Fig. 4 shows the amplitude of first and second harmonics to illustrate the effect of the second harmonics with $d=1$, $w=2\pi/10$ [rad/sec], and $a=0.6$. Note that we assume that $a_0 < a$ to guarantee the oscillation.

The second plot of Fig. 4 is the plot of the second harmonics normalized to the first harmonics. One can verify that the results are similar to Fig. 4 for the different value of d, T, and a. Note that the amplitude of second harmonic is equal to 0 when $a_0 = 0$ which is correspond to symmetric oscillation, while the magnitude of second harmonic significantly increased as a_0 increased which is correspond to asymmetric case. In particular, the second harmonic is almost same as that of the first harmonic for large a_0. The increment of amplitude of second harmonic results in the increment of estimation error of the unknown plant. The Fourier coefficient of the first harmonic is given by

$$\begin{aligned}\hat{c}_1 = & w/2\pi\{\int_{t_0}^{t_1} -\mathrm{sl}(a_o + a\sin wt)e^{-jwt})dt \\ & + \int_{t_1}^{t_2} -d \times e^{-jwt}dt + \int_{t_2}^{t_4} -\mathrm{sl}(a_o + a\sin wt)e^{-jwt}dt \\ & + \int_{t_4}^{t_5} d \times e^{-jwt}dt + \int_{t_5}^{t_6} -\mathrm{sl}(a_o + a\sin wt)e^{-jwt}dt\}\end{aligned} \tag{8}$$

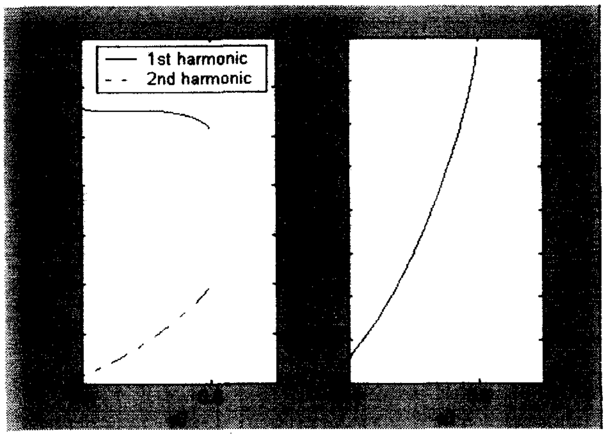

그림 4 릴레이 출력의 기본파 및 제 2 고조파 크기
Fig. 4 The plot of amplitude of the first and second harmonic for relay output

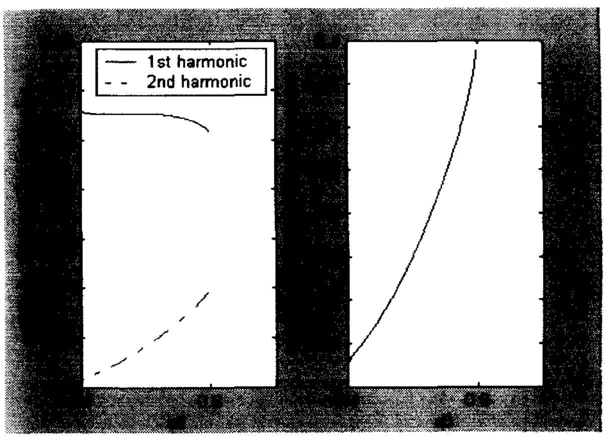

그림 5 포화함수 출력의 기본파 및 제 2 고조파 크기
Fig. 5 The plot of amplitude of the first and second harmonic for saturation function output

and the Fourier coefficient of the second harmonic is given by

$$\begin{aligned}\hat{c}_2 = & \ w/2\pi\{\int_{t_0}^{t_1} -sl(a_o+a\sin wt)e^{-j2wt}dt \\ & + \int_{t_1}^{t_2} -d\times e^{-j2wt}dt + \int_{t_2}^{t_4} -sl(a_o+a\sin wt)e^{-j2wt}dt \\ & + \int_{t_4}^{t_5} d\times e^{-j2wt}dt + \int_{t_5}^{t_6} -sl(a_o+a\sin wt)e^{-j2wt}dt\}\end{aligned}$$

for the saturation function output where t_i is defined in Fig. 3, sl and d are the slope and magnitude of the saturation function, respectively. Fig. 5 shows the amplitude of the first and second harmonic with the slope of saturation function=1 and same a, w for relay feedback case. Comparing with the relay feedback case, the normalized magnitude of the second harmonics is significantly decreased. The decrement of the second harmonics results in improving the accuracy of estimation of unknown plant. Note that it can be verified that the normalized magnitude of the second harmonics is increased as the slop of the saturation function is increased for the same a_0.

2.4 The tuning of PID controller using saturation function feedback without knowledge of plant d.c. gain

Since a saturation function can alleviate the high harmonics as we analyze the previous section, we will use the saturation function feedback as a test signal to identify one point information of the plant in the Nyquist plot. We will show that one point information of the plant in the Nyquist plot can be found by solving the equation (7). We can calculate $\hat{c}_1$ using the equation (8) when the saturation function is given. Suppose that $\hat{c}_1 = a + j\beta$ in the equation (8). After substituting the $\hat{c}_1$ into the equation (7),

$$Re[G(jw)]a - Im[G(jw)]\beta + j(Im[G(jw)]a + Re[G(jw)]\beta + a/2) = 0$$
$$\Rightarrow Re[G(jw)]a - Im[G(jw)]\beta = 0 \text{ and}$$
$$Im[G(jw)]a + Re[G(jw)]\beta + a/2 = 0$$

Thus

$$\begin{aligned} Re[G(jw)] &= (\beta/a)(a/2)\frac{1}{a+\beta^2/a} \\ Im[G(jw)] &= -\frac{a}{2(a+\beta^2/a)} \end{aligned} \tag{9}$$

where $Re[\cdot]$ and $Im[\cdot]$ denote the real part of argument and the imaginary part of argument, respectively. One point frequency information founded by the equation (9) is not necessary phase crossover information, since $Im[G(jw)] \neq 0$ in general. The equation (9) does not require the plant d.c. gain to find $G(jw)$. Note that it is possible that there is no oscillation when the magnitude of static load greater than that of saturation function. In this case, the magnitude of saturation function should be increased to sustain an oscillation[9]. Once we know one point information in the Nyquist plot of the unknown plant, we can design the PID controller[1] using frequency domain design scheme. Suppose that we get $G(jw) = r_p e^{j(\pi+\phi_p)}$ from the equation (9). Suppose that the desired phase margin is ϕ_m for the closed-loop system and the structure of PID controller is given by $G_c(s) = k(1 + sT_d + \frac{1}{sT_i})$ where k, T_d, and T_i are proportional gain, differential time, and integral time, respectively. It is shown in the Appendix A that k, T_d, and T_i satisfied the phase margin specification is given by

$$\begin{aligned} k &= \frac{\cos(\phi_m-\phi_p)}{r_p} \\ T_d &= (1/2w_1)\{\tan(\phi_m-\phi_p)+\sqrt{4a+\tan^2(\phi_m-\phi_p)}\} \\ T_i &= \frac{T_d}{a} \end{aligned} \tag{10}$$

where the typical value of $a=0.25$ and w_1 is the period of the plant output. Note that it is possible that the static load disturbance disappear or change the its magnitude by the nature of disturbance after the tuning phase. The static load disturbance might result in the presence of a steady state offset. However this will not cause the problem, since our controller contains an integrator.

3. Example

Consider the plant given by $G(s)=10/(5s^3+11s^2+7s+1)$. Suppose that static load disturbance is equal to 0.9. The Nyquist plot of $G(s)$ is shown in Fig. 6. We simulate three cases for comparison purpose. The first one uses the relay with the magnitude of relay=1 and static disturbance=0.9[2]. The second one uses the relay with the same magnitude and the perfect cancellation of static load disturbance which is correspond to the methodproposed by [9]. The third one uses the saturation

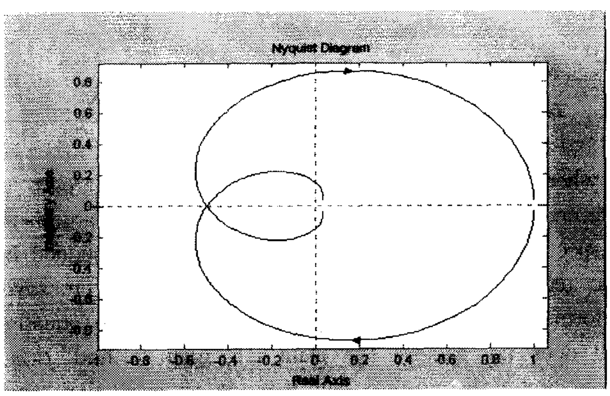

그림 6 $G(s)$ = $10/(5s^3+11s^2+7s+1)$의 Nyquist 선도
Fig. 6 The Nyquist plot of $G(s)$ = $10/(5s^3+11s^2+7s+1)$

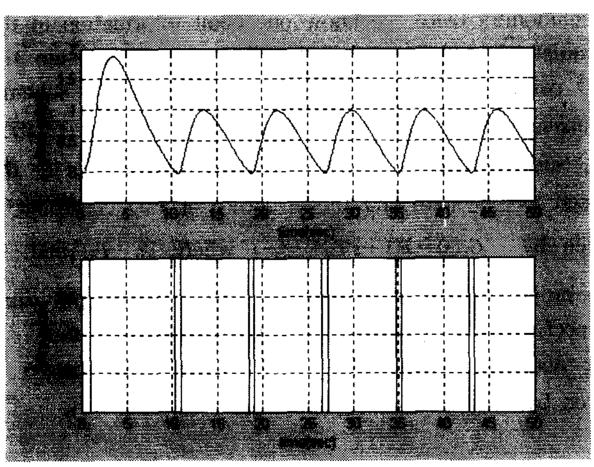

그림 7 $l=0.9$ 인 경우의 플랜트 출력 및 릴레이 출력.
Fig. 7 The plot of plant output and the output of relay with $l=0.9$.

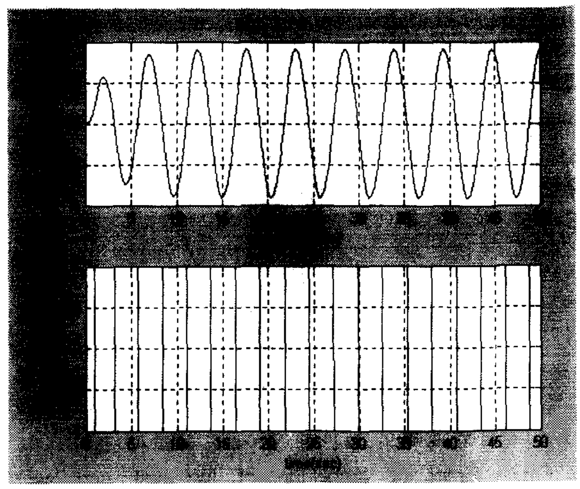

그림 8 경우 2의 플랜트 출력 및 릴레이 출력
Fig. 8 The plant output and relay output for case 2

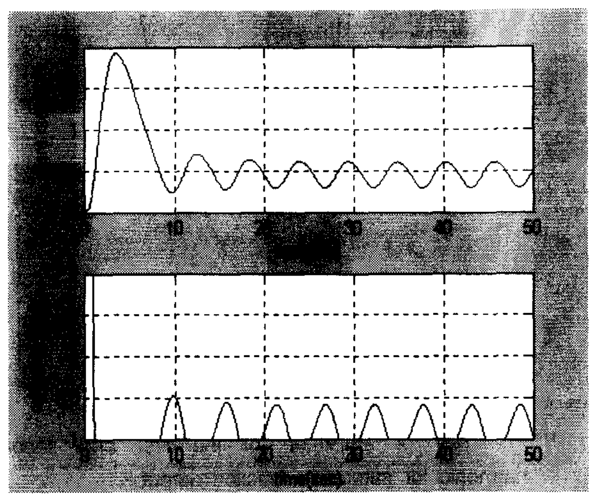

그림 9 경우 3의 플랜트 출력 및 포화함수 출력
Fig. 9 The plant output and saturation function output for case 3

function nonlinear element with $d=1$, $S1=2$, and static load disturbance=0.9 which is correspond to the method proposed in this paper. Fig. 7 to Fig. 9 show that plant output relay and/or saturation function output during the tuning period for three cases. Fig. 7 shows the plant output and relay output for the first case.

One can observe that $a\approx0.5$, and the period=8.7[sec] from the plant output. One can verify that $G(j0.72)=-0.4$ using the describing function of relay. Since the real one is $G(j1.18)=-0.7$ in the Fig. 6. A significant error can be observed. The Fig. 8 shows the plant output and the relay output for the case 2. One can observe that $a=0.91$ and $w=1.16$ from the plant output. One can calculate $G(j1.16)=-0.71$. The accuracy of one point estimation of unknown plant is improved. The plant output and the output of saturation function are shown for case 3 in the

Fig. 9. We calculate $\hat{c}_1 \approx 0.01 + j0.1$ using the equation (8) from the plant observation and $G(j1.14) = -0.76 - j0.06$ using the equation (9). Since the real one $G(j1.14) = -0.75 - j0.04$, the accuracy of one point estimation of unknown plant is similar to case 2. However we do not use the $G(0)$ in the estimation of one point information in the Nyquist plot. We design a PID controller according to the equation (10) with a desired phase margin= 50° for three cases. The designed PID parameters and phase margins are $k=1.6$, $T_d=1.9$, $T_i=7.6$, phase margin= 39° for case 1, $k=0.9$, $T_d=1.2$, $T_i=4.7$, phase margin= 51° for case 2, $k=0.9$, $T_d=1.1$, $T_i=4.3$, phase margin= 48° for case 3. The design results are expected one, since the more accurate estimation of unknown plant is resulted in the better controller design result. Comparing with case 1 which do not use G(0) in the estimation of unknown plant, the design result of our method(case 3) is significantly improved in the design accuracy.

The accuracy of design result of case 3 is similar to case 2 which uses the assumption of known G(0), even though we do not use the $G(0)$ in the estimation of one point information in the Nyquist plot.

4. Conclusion

We consider the asymmetric oscillation caused by a static load disturbance in the plant output. The asymmetric oscillation with a saturation nonlinear element was analyzed by using a first order harmonic balance equation. From the analysis, we propose a new method which does not require a knowledge of the plant d.c. gain to find one point information on the Nyquist plot of unknown plant. Comparing with the work[2] which does not require the knowledge of the plant d.c. gain, the proposed method significantly improved the accuracy of the estimation of the unknown plant in the Nyquist plot. We demonstrate that the accuracy of estimation of unknown plant using the proposed method is similar to the work[9] which requires the knowledge of the plant d.c. gain. As a result of accurate estimation of unknown plant, we demonstrate that PID controller design results using the proposed method is much better than that of the work[2] and is similar to those of the work[9].

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Appendix A

The $G_c(jw_1)$ can be rewritten as $G_c(jw_1) = r_c e^{j\phi_c}$ where $r_c = k\sqrt{1+(w_1 T_d - 1/w_1 T_i)^2}$ and $\phi_c = \tan^{-1}(w_1 T_d - \frac{1}{w_1 T_i})$. Since the desired phase margin is ϕ_m, the following equation should hold

$$G(jw_1)G_c(jw_1) = e^{j(\pi+\phi_m)}$$

$$\Rightarrow r_p r_c e^{j(\pi+\phi_p+\phi_c)} = e^{j(\pi+\phi_m)}$$

$$\Rightarrow \phi_c = \phi_m - \phi_p \text{ and } r_c = \frac{1}{r_p}$$

$$\Rightarrow w_1 T_d - \frac{1}{w_1 T_i} = \tan(\phi_m - \phi_p)$$

Using $r_c = k\sqrt{1+(w_1 T_d - 1/w_1 T_i)^2}$, $r_c = \frac{1}{r_p}$, and

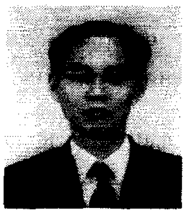
$$w_1 T_d - \frac{1}{w_1 T_i} = \tan(\phi_m - \phi_p),$$

$$k = \frac{\cos(\phi_m - \phi_p)}{r_p}$$

Using $w_1 T_d - \frac{1}{w_1 T_i} = \tan(\phi_m - \phi_p)$ and $T_i = \frac{T_d}{\alpha}$,

$$T_d = \left(\frac{1}{2w_1}\right)\{\tan(\phi_m - \phi_p) + \sqrt{4\alpha + \tan^2(\phi_m - \phi_p)}\}$$

저 자 소 개



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