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論 文
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Nonfragile Guaranteed Cost Controller Design for Uncertain Large-Scale Systems

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Abstract – In this paper, the robust non-fragile guaranteed cost control problem is studied for a class of linear large-scale systems with uncertainties and a given quadratic cost functions. The uncertainty in the system is assumed to be norm-bounded and time-varying. Also, the state-feedback gains for subsystems of the large-scale system are assumed to have norm-bounded controller gain variations. The problem is to design a state feedback control laws such that the closed-loop system is asymptotically stable and the closed-loop cost function value is not more than a specified upper bound for all admissible uncertainties and controller gain variations. Sufficient conditions for the existence of such controllers are derived based on the linear matrix inequality (LMI) approach combined with the Lyapunov method. A parameterized characterization of the robust non-fragile guaranteed cost controllers is given in terms of the feasible solutions to a certain LMI. A numerical example is given to illustrate the proposed method.

Keywords – Large-scale systems, Non-fragile guaranteed cost controller, Lyapunov method, linear matrix inequality.

1. Introduction

During A large-scale interconnected dynamical system can be usually characterized by a large number of state variables, system parametric uncertainties, and a complex interaction between subsystems [1]-[2]. In view of reliability and practical implementation, the decentralized stabilization of large-scale interconnected systems becomes a very important problem and has been studied extensively for more than two decades [3-11]. However, when controlling a real plant, it is also desirable to design a control systems which is not only stable but also guarantees an adequate level of performance. One way to address the robust performance problem is to consider a linear quadratic cost function. This approach is the so-called guaranteed cost control [12]. The approach has the advantage of providing an upper bound on a given performance index and thus the system performance degradation incurred by the uncertainties is guaranteed to be less than this bound. Recently, there have been considerable efforts to tackle the guaranteed cost controller design problem [13-17].

While the above methods yield controllers that are robust to uncertainties in the plant under control, their robustness with respect to uncertainties in the controllers themselves has not been considered. In the recent study by Keel and Bhattacharyya [18], it is shown that the controllers may be very sensitive, or fragile with respect to errors in the controller coefficients, although they are robust with respect to plant uncertainty. This raises a new issue: how to design a controller for a given plant with uncertainty such that the controller is insensitive to some amount of error with respect to its gain, i.e. the controller is non-fragile. More recently, there have been some efforts to tackle the non-fragile controller design problem [19-22]. Unfortunately, uptill now, the topic of robust non-fragile control for large-scale systems has been received little attention. Up to our knowledge, there have been few results in the literature of an investigation for the problem of the system.

In this paper, a class of uncertain large-scale systems with parametric uncertainties in the system matrices and controller gain perturbations is considered. The uncertainty is time-varying and is assumed to be norm-bounded. Using the Lyapunov functional technique combined with a linear matrix inequality (LMI) technique, we develop a robust non-fragile guaranteed cost control for this system via state feedback, which makes the closed-loop system robustly stable for all admissible uncertainties and guarantees an adequate level of performance. A stability criterion for the existence of the guaranteed cost controller is derived in terms of LMIs, and their solutions provide a parameterized representation

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of the control. The LMIs can be easily solved by various efficient convex optimization algorithms (Boyd et al. [23]). Finally, a numerical example is given to illustrate the proposed design method.

Notations: Throughout the paper, R^n denotes the n dimensional Euclidean space, and $R^{n\times m}$ is the set of all $n\times m$ real matrices. I denotes the identity matrix with appropriate dimensions. For symmetric matrices X and Y, the notation $X > Y$(respectively, $X \geq Y$) means that the matrix $X-Y$ is positive definite, (respectively, nonnegative). of delay-independent stability results. Above all, the time-delay considered in these works is constant and their results are only applicable to the systems with same delay arguments.

2. Problem Formulation

Consider a class of uncertain large-scale system composed of N interconnected subsystems described by

$$S_i:\ \dot{x}_{i(t)} = [A_i+\Delta A_{i(t)}]x_i(t)+\sum_{j\neq i}^{N}[A_{ij} + \Delta A_{ij}(t)]x_j(t)+B_iu_i(t),\quad i=1,2,\cdots,N \tag{1}$$

where $x_i(t)\in R^{n_i}$ is the state vector, and $u_i(t)\in R^{m_i}$ is the control vector. The system matrices A_i, B_i, and A_{ij} are of appropriate dimensions, and $\Delta A_i(t)$, and $\Delta A_{ij}(t)$ are real-valued matrices representing time-varying parameter uncertainties in the system.

Assume that the pair $(A_i, B_i), i=1,\cdots,N,$ is stabilizable, and the time-varying uncertainties are of the form

$$\begin{aligned}\Delta A_i(t)&=D_{ai}F_{ai}(t)E_{ai},\\ \Delta A_{ij}(t)&=D_{aij}F_{aij}(t)E_{aij},\end{aligned} \tag{2}$$

where D_{ai}, D_{aij}, E_{ai}, and E_{aij} are known constant real matrices with appropriate dimensions, and $F_{ai}(t)$, and $F_{aij}(t)$ are unknown matrix functions which are bounded as

$$F_{ai}^T(t)F_{ai}(t)\leq I,\ F_{aij}^T(t)F_{aij}(t)\leq I,\ \forall\ i,\ j\geq 0. \tag{3}$$

Associated with the each subsystem S_i is the following quadratic cost function

$$J_i=\int_0^\infty[x_i^T(t)Q_ix_i(t)+u_i^T(t)R_iu_i(t)]dt \tag{4}$$

where $Q_i\in R^{n_i\times n_i}$ and $R_i\in R^{m_i\times m_i}$ are given positive- definite matrices.

Now, although one finds the controller $u_i(t)=-K_ix_i(t)$ for each subsystems, the actual controller implemented is

$$u_i(t)=-[K_i+\Delta K_i]x_i(t),\quad i=1,2,,\cdots,N \tag{5}$$

where $K_i\in R^{m_i\times n_i}$ is the nominal controller gain to be designed and ΔK_i represents the additive gain perturbations of the form

$$\Delta K_i=H_i\Phi_i(t)E_i \tag{6}$$

with H_i and E_i being known constant matrices, and $\Phi_i(t)$ the uncertain parameter matrix satisfying

$$\Phi_i^T(t)\Phi_i(t)\leq\rho_iI,\quad \rho_i\geq 0,\quad i=1,2,,\cdots,N. \tag{7}$$

Here, the objective of this paper is to develop a procedure to design a state feedback controller $u_i(t)$ for uncertain system (1) and cost function (4), such that the resulting closed-loop subsystem given by

$$\dot{x}_i(t)= [A_i+\Delta A_i(t)-B_iK_i-B_iH_i\Phi_i(t)E_i]x_i(t) + \sum_{j\neq i}^{N}[A_{ij}+\Delta A_{ij}(t)]x_j(t) \tag{8}$$

is asymptotically stable and the closed-loop value of the cost function (4) satisfies $J_i\leq J_i^*$, where J_i^* is some specified constant.

Definition 2.1: For the uncertain large-scale discrete-time system (1) and cost function (4), if there exist a control law $u_i^*(t)$ and a positive J_i^* such that for all admissible uncertainties, the closed-loop system (8) is asymptotically stable and the closed-loop value of the cost function (4) satisfies $J_i\leq J_i^*$, then J_i^* is said to be a guaranteed cost and $u_i^*(t)$ is said to be a guaranteed cost control law of the system (1) and cost function (4).

Remark 2.1: The controller gain perturbation can result from the actuator degradations, as well as from the requirement for re-adjustment of controller gains during the controller implementation state [18-19]. These perturbations in the controller gains are modelled here as uncertain gains that are dependent on uncertain parameters. In the literature [20-21], the models of additive uncertainties and multiplicative uncertainties are used to describe the controller gain variation. The uncertainty given in (6) is a class of additive uncertainties.

Before proceeding further, we will state well known lemma.

Lemma 2.1 [23]. The linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & W(x) \end{bmatrix} > 0$$

is equivalent to

$$W(x) > 0, \quad Q(x) - S(x)W^{-1}(x)S^T(x) > 0,$$

where $Q(x) = Q^T(x)$, $W(x) = W^T(x)$ and $S(x)$ depend affinely on x.

3. Controller Design

In this section, we consider the problem of decentralized robust non-fragile guaranteed cost control for the uncertain closed-loop system described by (8) using the Lyapunov method combined with LMI technique.

Here, for simplicity, we define

$$\begin{aligned} A_{di} &= \Big(\sum_{j\neq i}^{N} A_{ij}A_{ij}^T\Big)^{1/2}, \quad D_{di} = \Big(\sum_{j\neq i}^{N} D_{aij}D_{aij}^T\Big)^{1/2}, \\ E_{di} &= \Big(\sum_{j\neq i}^{N} E_{aji}^T E_{aji}\Big)^{1/2}. \end{aligned} \tag{9}$$

Theorem 3.1: $u_i(t) = -K_i x_i(t)$ is a robust non-fragile guaranteed cost controller for each subsystems if there exist positive-definite matrix P_i and positive scalars ε_{0i} and ε_i such that for any admissible uncertain matrices $F_i(t), F_{aij}(t)$, and $\Phi_i(t)$, the following matrix inequality holds:

$$\begin{aligned} &A_i^T P_i + P_i A_i + \varepsilon_{0i} P_i D_{ai} D_{ai}^T P_i + \varepsilon_{0i}^{-1} E_{ai}^T E_{ai} \\ &- P_i B_i K_i - K_i^T B_i^T P_i + \varepsilon_i^{-1} E_i^T E_i + \varepsilon_i \rho_i P_i B_i H_i H_i^T B_i^T P_i \\ &+ P_i A_{di} A_{di}^T P_i + P_i D_{di} D_{di}^T P_i + (N-1)I \\ &+ E_{di}^T E_{di} + Q_i + K_i^T R_i K_i < 0 \quad \text{for } i = 1, 2, \cdots, N. \end{aligned} \tag{10}$$

Proof : Consider a Lyapunov function candidate

$$V = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} x_i^T(t) P_i x_i(t). \tag{11}$$

The time derivative of V is given by

$$\begin{aligned} \dot V &= \sum_{i=1}^{N} \dot x_i^T(t) P_i x_i(t) + x_i^T(t) P_i \dot x_i(t) \\ &= \sum_{i=1}^{N} 2x_i^T(t) P_i \dot x_i(t). \end{aligned} \tag{12}$$

Substituting (8) into (12), we have

$$\begin{aligned} \dot V = &\sum_{i=1}^{N}\Big\{ x_i^T(t)[A_i^T P_i + P_i A_i + 2P_i D_{ai} F_{ai}(t) E_{ai} \\ &- 2P_i B_i K_i - 2P_i B_i H_i \Phi_i(t) E_i] x_i(t) \\ &+ 2x_i^T(t) P_i \sum_{j=1, i\neq j}^{N} (A_{ij} + D_{aij} F_{aij}(t) E_{aij}) x_j(t) \Big\} \end{aligned} \tag{13}$$

Using the well-known fact that

$$U\Delta V^T + V\Delta U^T \le \varepsilon UU^T + \varepsilon^{-1} VV^T, \quad \varepsilon > 0$$

for any matrices U, V and Δ with $\Delta^T\Delta \le I$, we can eliminate the unknown factor, $F_{ai}(t), F_{aij}(t)$ and $\Phi_i(t)$, of parameter uncertainties. Then the terms on right-hand side of (13) are bounded as

$$\begin{aligned} &2x_i^T(t) P_i D_{ai} F_{ai}(t) E_{ai} x_i(t) \\ &\quad \le \varepsilon_{0i} x_i^T(t) P_i D_{ai} F_{ai}(t) F_{ai}^T(t) D_{ai}^T P_i x_i(t) \\ &\qquad + \varepsilon_{0i}^{-1} x_i^T(t) E_{ai}^T E_{ai} x_i(t) \\ &\quad \le \varepsilon_{0i} x_i^T(t) P_i D_{ai} D_{ai}^T P_i x_i(t) + \varepsilon_{0i}^{-1} x_i^T(t) E_{ai}^T E_{ai} x_i(t) \\ &- \sum_{i=1}^{N} 2x_i^T(t) P_i B_i H_i \Phi_i(t) E_i x_i(t) \\ &\quad \le \sum_{i=1}^{N}\Big(\varepsilon_i^{-1} x_i^T(t) E_i^T E_i x_i(t) \\ &\qquad + \varepsilon_i x_i^T(t) P_i B_i H_i \Phi_i(t) \Phi_i^T(t) H_i^T B_i^T P_i x_i(t) \Big) \\ &\quad \le \sum_{i=1}^{N}\Big(\varepsilon_i^{-1} x_i^T(t) E_i^T E_i x_i(t) + \varepsilon_i \rho_i x_i^T(t) P_i B_i H_i H_i^T B_i^T P_i x_i(t) \Big) \\ &\sum_{i=1}^{N} 2x_i^T(t) P_i \sum_{j\neq i}^{N} A_{ij} x_j(t) \\ &\quad \le \sum_{i=1}^{N}\Big(x_i^T(t) P_i \sum_{j\neq i} A_{ij} A_{ij}^T P_i x_i(t) + \sum_{j\neq i}^{N} x_j^T(t) x_j(t) \Big) \\ &\quad = \sum_{i=1}^{N}\Big(x_i^T(t) P_i A_{di} A_{di}^T P_i x_i(t) + (N-1) x_i^T(t) x_i(t) \Big) \\ &\sum_{i=1}^{N} 2x_i^T(t) P_i \sum_{j\neq i}^{N} D_{aij} F_{aij}(t) E_{aij} x_j(t) \\ &\quad \le \sum_{i=1}^{N}\Big(x_i^T(t) P_i \sum_{j\neq i}^{N} D_{aij} F_{aij}(t) F_{aij}^T(t) D_{aij}^T P_i x_i(t) \\ &\qquad + \sum_{j\neq i}^{N} x_j^T(t) E_{aij}^T E_{aij} x_j(t) \Big) \\ &\quad \le \sum_{i=1}^{N}\Big(x_i^T(t) P_i \sum_{j\neq i}^{N} D_{aij} D_{aij}^T P_i x_i(t) + \sum_{j\neq i}^{N} x_j^T(t) E_{aij}^T E_{aij} x_j(t) \Big) \\ &\quad = \sum_{i=1}^{N}\Big(x_i^T(t) P_i D_{di} D_{di}^T P_i x_i(t) + \sum_{j\neq i}^{N} x_j^T(t) E_{aij}^T E_{aij} x_j(t) \Big) \end{aligned}$$

where A_{di} and D_{di} are defined in (9), and ε_{0i} and ε_i are positive scalars to be chosen.

Using (14), we obtain a new bound of $\dot V$ as

$$\dot{V} \le \sum_{i=1}^{N}\Big\{x_i^T(t)[A_i^TP_i+P_iA_i+\varepsilon_{0i}P_iD_{ai}D_{ai}^TP_i + \varepsilon_{0i}^{-1}E_{ai}^TE_{ai}+\varepsilon_i^{-1}E_i^TE_i-2P_iB_iK_i+\varepsilon_i\rho_iP_iB_iH_iH_i^TB_i^TP_i + P_iA_{di}A_{di}^TP_i+P_iD_{di}D_{di}^TP_i+(N-1)I]x_i(t) + \sum_{j=1,i\neq j}^{N}x_j^T(t)E_{aij}^TE_{aij}x_j(t)\Big\}. \tag{15}$$

Note that

$$\sum_{i=1}^{N}\sum_{j=1,i\neq j}^{N}x_j^T(t)E_{aij}^TE_{aij}x_j(t) = \sum_{i=1}^{N}x_i^T(t)\Big(\sum_{j\neq i}^{N}E_{aji}^TE_{aji}\Big)x_i(t) = \sum_{i=1}^{N}x_i^T(t)E_{di}^TE_{di}x_i(t). \tag{16}$$

Then, (15) is simplified a

$$\dot{V} = \sum_{i=1}^{N}V_i \le \sum_{i=1}^{N}\Big\{x_i^T(t)[A_i^TP_i+P_iA_i+\varepsilon_{0i}P_iD_{ai}D_{ai}^TP_i + \varepsilon_{0i}^{-1}E_{ai}^TE_{ai}-2P_iB_iK_i+\varepsilon_i^{-1}E_i^TE_i + \varepsilon_i\rho_iP_iB_iH_iH_i^TB_i^TP_i+P_iA_{di}A_{di}^TP_i+P_iD_{di}D_{di}^TP_i + (N-1)I+E_{di}^TE_{di}]x_i(t)\Big\}. \tag{17}$$

Here, the matrix inequality (10) implies that

$$\dot{V} = \sum_{i=1}^{N}\dot{V}_i < -\sum_{i=1}^{N}[x_i^T(t)Q_ix_i(t)+u^T(t)R_iu(t)]<0. \tag{18}$$

Noting $Q_i > 0$ and $R_i > 0$, this implies that the system (8) is asymptotically stable by Lyapunov stability theory. Furthermore, from (18) we have

$$x_i^T(t)Q_ix_i(t)+u^T(t)R_iu(t) < \dot{V}_i < 0.$$

Integrating both sides of the above inequality from 0 to T_f leads to

$$\int_0^{\infty}[x_i^T(t)Q_ix_i(t)+u^T(t)R_iu(t)]dt < x_i^T(0)P_ix_i(0)-x_i^T(T_f)P_ix_i(T_f).$$

As the closed-loop system (8) is asymptotically stable, when $T_f\rightarrow\infty$,

$$x_i^T(T_f)P_ix_i(T_f)\rightarrow 0.$$

Hence we get

$$\int_0^{\infty}[x_i^T(t)Q_ix_i(t)+u^T(t)R_iu(t)]dt \le x_i^T(0)P_ix_i(0) \triangleq J_i^*. \tag{19}$$

■

In the following, we will show that the above sufficient condition for the existence of guaranteed cost controllers is equivalent to the feasibility of LMI.

Theorem 3.2 : For given $R_i > 0$ and $Q_i > 0$, if there exist a matrix M_i, a positive-definite matrix X_i, and positive scalars, ε_{0i} and ε_i, such that for $i=1,2,\cdots,N$, the following LMI is feasible:

$$\begin{bmatrix} \Sigma & X_iE_{ai}^T & X_iE_i^T & (N-1)^{1/2}X_i & X_iE_{di}^T & X_i & M_i^T \\ * & -\varepsilon_{0i}I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_iI & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -Q_i^{-1} & 0 \\ * & * & * & * & * & * & -R_i^{-1} \end{bmatrix} < 0 \tag{20}$$

where $X_i=P_i^{-1}$ and

$$\Sigma = X_iA_i^T+A_iX_i+\varepsilon_{0i}D_{ai}D_{ai}^T-B_iM_i-M_i^TB_i^T + \varepsilon_i\rho_iB_iH_iH_i^TB_i^T+A_{di}A_{di}^{T+}D_{di}D_{di}^T.$$

Furthermore, the state feedback control law

$$u_i(t)=-K_ix_i(t)=-M_iX_i^{-1}x_i(t) \tag{21}$$

is a non-fragile guaranteed cost control law for robust decentralized stabilization of the uncertain systems (8), and the corresponding closed-loop value of the cost function satisfies $J_i \le J_i^*$, in which J_i^* is given in (19).

Proof: By premultiplying and postmultiplying X_i onto (10), we get

$$X_iA_i^T+A_iX_i+\varepsilon_{0i}D_{ai}D_{ai}^T+\varepsilon_{0i}^{-1}X_iE_{ai}^TE_{ai}X_i - B_iK_iX_i-X_iK_i^TB_i^T+\varepsilon_i^{-1}X_iE_i^TE_iX_i+\varepsilon_i\rho_iB_iH_iH_i^TB_i^T + A_{di}A_{di}^T+D_{di}D_{di}^T+(N-1)X_i^TX_i+X_iE_{di}^TE_{di}X_i + X_iQ_iX_i+X_iK_i^TR_iK_iX_i < 0,\ i=1,2,\cdots,N. \tag{22}$$

Using change of variable, $M_i=K_iX_i$, and Lemma 2.1, the inequality (22) is equivalent to the LMI (20). This completes the proof. ■

Remark 3.1 : Since the inequality (20) is a linear matrix inequality in $X_i, M_i, \varepsilon_{0i}, \varepsilon_i$, the inequality (20) defines a convex solution set of $(X_i, M_i, \varepsilon_{0i}, \varepsilon_i)$, and therefore various efficient convex optimization algorithms can be used to check whether the LMI is feasible. Moreover, the decentralized gain matrix K_i can be calculated from the relation $M_i = K_i P_i^{-1}$ after finding the LMI solutions, $X_i (= P_i^{-1})$ and M_i from (20). In this paper, in order to solve the LMI, we utilize Matlab's LMI Control Toolbox [24], which implements state-of-the-art interior- point algorithms, which is significantly faster than classical convex optimization algorithms [23].

Theorem 3.2: presents a method of designing a state feedback guaranteed cost controller. The following theorem presents a method of selecting a controller minimizing the upper bound of the guaranteed cost (19).

Theorem 3.3: Consider the system (8) with cost function (4). If the following optimization problem

$$\begin{aligned} &\min_{X_i, M_i, \varepsilon_{0i}, \varepsilon_i, \alpha_i} \alpha_i \\ &(i)\ LMI\ (20) \\ &(ii) \begin{bmatrix} -\alpha_i & x_i^T(0) \\ x_{i(0)} & -X_i \end{bmatrix} < 0, \text{ for } i = 1,2,\cdots,N \end{aligned} \tag{23}$$

has a solution set $(\alpha_i, X_i, M_i, \varepsilon_{0i}, \varepsilon_i)$, then the control law (21) is an optimal non-fragile guaranteed cost control law which ensures the minimization of the guaranteed cost (19) for the uncertain large-scale system (8).

Proof : By Theorem 3.2, (i) in (23) is clear. Also, it follows from the Lemma 2.1 that (ii) in (23) is equivalent to $x_i^T(0) X_i^{-1} x_i(0) < \alpha_i$. So, it follows from (19) that

$$J_i^* < \alpha_i.$$

Thus, the minimization of α_i implies the minimization of the guaranteed cost for the subsystem (8). The convexity of this optimization problem ensures that a global optimum, when it exists, is reachable.

To illustrate the application of the proposed method, we present the following example.

Example 3.1: Consider a large-scale system which is composed of the following two interconnected subsystems

$$\begin{aligned} \dot{x}_1(t) &= \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.4 & 0.1 & 0.5 \\ 0.3 & 0.4 & 0.5 \end{bmatrix} x_2(t) \\ &+ \Delta A_1(t) x_1(t) + \Delta A_{12}(t) x_2(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_1(t), \end{aligned}$$

$$\begin{aligned} \dot{x}_2(t) &= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 2 & 1 & -3 \end{bmatrix} x_2(t) + \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \\ 0.5 & 1 \end{bmatrix} x_1(t) \\ &+ \Delta A_2(t) x_2(t) + \Delta A_{21}(t) x_1(t) + \begin{bmatrix} 1 & 0 \\ 1 & 0.5 \\ 0 & 1 \end{bmatrix} u_2(t) \end{aligned}$$

where

$$\Delta A_1(t) = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} \sin(t) & 0 \\ 0 & \sin(2t) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_{12}(t) = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \sin(2t) [0\ 1\ 1],$$

$$\Delta A_2(t) = \begin{bmatrix} 0 & 0 & 0.3 \\ 0.3 & 0 & 0.2 \\ 0.1 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} \sin(2t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sin(t) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta A_{21}(t) = \begin{bmatrix} 0 \\ 0.1 \\ 0.2 \end{bmatrix} \sin(t) [1\ 1],$$

and the initial condition of each subsystems are as follows:

$$\begin{aligned} x_1(0) &= [-0.7\ 0.5]^T \\ x_2(0) &= [1\ 0.5\ -1]^T. \end{aligned}$$

Also, the following additive controller uncertainties of the form (6) is considered:

$$H_1 = [1\ 1],\ E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \rho_1 = 1$$

$$H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ E_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix},\ \rho_2 = 1.$$

Associated with this system is the cost function of (4) with $Q_1 = I, Q_2 = I, R_1 = 0.2I$ and $R_2 = 0.2I$.

Here, solving the optimization problem (23) of Theorem 3.3, we find the positive solutions of the LMIs for the subsystem 1 as

$$X_1 = \begin{bmatrix} 0.5514 & 0.1383 \\ 0.1383 & 0.2599 \end{bmatrix},\quad M_1 = [5.0000\ 0.0000],$$

$$\varepsilon_{01} = 1.6277,\ \varepsilon_1 = 0.8986,\ \alpha_1 = 2.9154.$$

Similarly, the solutions for the subsystem 2 are as follows

$$X_2 = \begin{bmatrix} 0.7270 & 0.7585 & -0.6868 \\ 0.7585 & 1.1859 & -0.6482 \\ -0.6868 & -0.6482 & 1.7427 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 5.0000 & 5.0000 & -0.0000 \\ -0.0000 & 2.5000 & 5.0000 \end{bmatrix},$$

$$\varepsilon_{02} = 0.9562,\ \varepsilon_2 = 0.4873,\ \alpha_2 = 2.1258$$

Therefore, the gain matrices, K_i, of the stabilizing controller, u_i, for two subsystems are

$$K_1 = M_1 X_1^{-1} = [\,10.4650 \ \ -5.5689\,]$$

$$K_2 = M_2 X_2^{-1} = \begin{bmatrix} 12.4060 & -1.3138 & 4.4005 \\ -1.8633 & 5.6069 & 4.2204 \end{bmatrix},$$

and the optimal guaranteed costs of the uncertain closed-loop system are as follows:

$$J_1^* = \alpha_1 = 2.9154$$
$$J_2^* = \alpha_2 = 2.1258.$$

For computer simulation, the following control laws are employed:

$$u_1(t) = -(I + H_1 \Phi_1(t) E_1) K_1 x_1(t)$$

$$u_2(t) = -(I + H_2 \Phi_1(t) E_2) K_2 x_2(t).$$

where

$$\Phi_1(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix},$$

$$\Phi_2(t) = \begin{bmatrix} \cos(2t) & 0 \\ 0 & \sin(t) \end{bmatrix}.$$

The simulation results are given in Figs. 1 and 2. In the figures, one can see that the system is indeed well stabilized irrespective of uncertainties and controller gain variations.

4. Conclusion

In this paper, we have investigated the problem of non-fragile guaranteed cost control of large-scale interconnected systems under parametric uncertainties and additive controller gain variations. We have developed a state feedback controller for guaranteeing not only the robust stability of the closed-loop system but also the cost function bound constraint. Finally, a numerical example is given for illustration of controller design, and simulation result shows that the system is well stabilized in spite of controller gain variations and uncertainties.

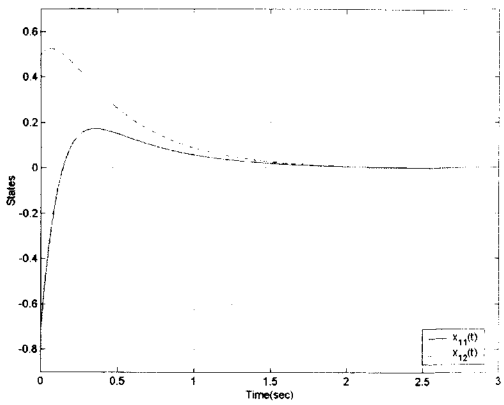

Fig. 1 State responses of subsystem 1

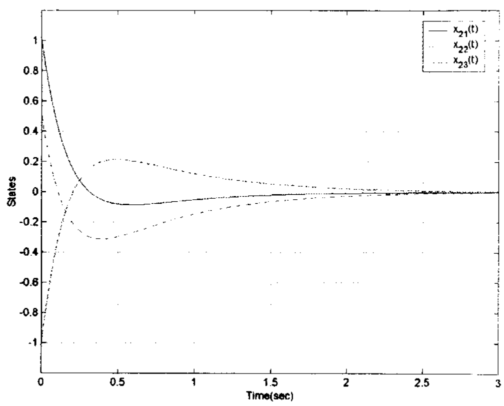

Fig. 2 State responses of subsystem 2

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