A Hybrid Coordinate Partitioning Method in Mechanical Systems Containing Singular Configurations

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Key words *: Hybrid Coordinate Partitioning, LU Decomposition, QR Decomposition*

Abstract

In multibody dynamics, DAE(Differential Algebraic Equations) that combine differential equations of motion and kinematic constraint equations should be solved. To solve these equations, either coordinate partitioning method or constraint stabilization method is commonly used. The most typical coordinate partitioning methods are LU decomposition, QR decomposition, and SVD(singular value decomposition). The objective of this research is to suggest a hybrid coordinate partitioning method in the dynamic analysis of multibody systems containing singular configurations. Two coordinate partitioning methods, i.e. LU decomposition and QR decomposition for constrained multibody systems, are combined for a new hybrid coordinate partitioning method. The proposed hybrid method reduces the simulation time while keeping accuracy of the solution.

1. Introduction

Multibody dynamics, which investigates the motion of systems consisting of several bodies, is extensively applied for the analysis of spacecraft, mechanical systems and vehicle dynamic systems. These days, the mechanisms are required to operate at higher speeds, but the weight of components is reducing. Therefore, the deformation of elements must be considered in order to analyze the motion of a system accurately. To represent the elastic deformation, nodal coordinates or modal coordinates are used. Modal coordinates[1], based on finite element method, are widely used in multibody dynamics due to a smaller number of coordinates to express the deformation of a flexible body. Modal coordinate formulation is more efficient than nodal coordinates when high frequency components are removed.

Either Cartesian coordinate system or joint coordinate system is used in deriving the equations of motion for multibody systems. Although the former is convenient to derive the equations of motion, it is inefficient because a larger number of coordinates and constraint equations are involved in the formulation. On the other hand, the latter can generates equations

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of motion with a minimal number of coordinates, and thus efficiency can be obtained. But it may be less general than Cartesian coordinates in formulating constraint equations

To solve the equations of motion with an integration routine like a predictor-corrector method, a procedure that checks and guarantees constraint equations should be included in dynamic analysis algorithms. The coordinate partitioning method, which divides coordinates into independent and dependent coordinates and modifies the values of dependent coordinates to satisfy the constraint equations, is widely used for this purpose.

Typical coordinate partitioning methods are as follows; LU decomposition used by Wehage[2], QR decomposition used by Kim[3], SVD(Singular Value Decomposition) used by Mani[4], Gram-Schmidt method used by Liang[5]. Both the QR and the SVD are introduced as examples of tangent plane parametrization methods[6] and tangent space parametrization is also used to form an index one system of DAEs[7]. The projected invariants formulation is known to be equivalent to the tangent plane parametrization[8]. The projection methods[9] are applied to the DAEs, and several partitioning methods are explained in reference[10].

Although many methods have been studied for the efficiency and stability, no results, combining the two methods, have been published so far. Therefore, this paper aims to compare two methods and to propose a hybrid decomposition method for a multibody system. LU coordinate partitioning and QR partitioning are compared and a new hybrid coordinate partitioning method combining the advantages of both LU and QR methods is suggested. To verify the suggested method, a fourbar example is demonstrated.

2. Reduced System of Differential Equation of Motion

2.1 Equations of Motion for a Multibody System

The coordinates of a flexible multibody system consisting of n coordinates for rigid body motion and k modal coordinates for elastic deformation are written as:

$$q = [\, q_1, q_2, \cdots\cdots, q_n, a_1, a_2, \cdots\cdots, a_k] \tag{1}$$

The m independent constraint equations in the system is defined as:

$$\Phi(q,t) = [\, \Phi_1(q,t), \cdots\cdots, \Phi_m(q,t)] = 0 \tag{2}$$

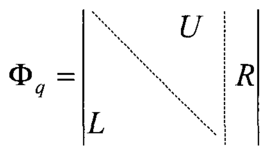
The DAEs(Differential Algebraic Equations) for a constrained flexible multibody system are written as:

$$\begin{bmatrix} M & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} F \\ -2\,\Phi_{qt}\dot{q} - (\Phi_q \dot{q})_q - \Phi_{tt} \end{bmatrix} \tag{3}$$

where M is the mass matrix for a system, Φ_q is the Jacobian matrix of the constraint equations, and λ is a vector of Lagrange multipliers. In this research, the DE[12] routine including predictor-corrector method is used for the integration.

2.2 Generalized Coordinate Partitioning with LU

Decomposing the Jacobian matrix using the LU method, the matrix can be constructed when rows of the Jacobian matrix are independent of each other.



When the generalized coordinates q of the system are partitioned into independent coordinates v and dependent coordinates u , dependent coordinates u can be calculated from the constraint equations via the Newton-Raphson method.

Velocity vector $\dot{q}$ can also be partitioned into independent velocity vector $\dot{v}$ and dependent velocity vector $\dot{u}$. Velocity relation can be found by solving the linear equation obtained from differentiation of constraint equations with respect to time. If position and velocity are known, acceleration and Lagrange multiplier can be obtained from equation (3). Integrating $\dot{v}$ and $\ddot{v}$ at this time with a predictor-corrector method, then allows the independent coordinate v and the independent velocity $\dot{v}$ to be obtained for the next time. This LU method has computational efficiency because it performs integration only for independent coordinates.

2.3 State Space with QR

Let the Jacobian matrix be $m \times n$ matrix with a full row rank, then it can be partitioned into an orthogonal matrix Q and an upper triangle matrix R.

$$\Phi_q^T = QR \tag{4}$$

The matrix Q is also partitioned as:

$$Q = [\, Q_1 \; Q_2 \,] \tag{5}$$

where Q_1 forms an orthonormal basis for the row space of the Jacobian matrix. The generalized velocity vector is then written as:

$$\dot{q} = [\, \tilde{Q}_1 \; \tilde{Q}_2 \,] \begin{bmatrix} b \\ \dot{z} \end{bmatrix} \tag{6}$$

Where $\tilde{Q}_1 = Q_1 (\Phi_q Q_1)^{-1}$, $\tilde{Q}_2 = Q_2 - \tilde{Q}_1 \Phi_q Q_2$ and $b = -\Phi_t(q, t)$. The $\dot{z}$ is a free variable parallel to the tangent plane of the Jacobian matrix. Q_1 spans the row space of the Jacobian matrix and vector $\tilde{Q}_1$ is the updated nullspace of the Jacobian matrix.

The time derivative of equation (6) produces the following acceleration relation

$$\ddot{q} = [\, \tilde{Q}_1 \; \tilde{Q}_2 \,] \begin{bmatrix} a \\ \ddot{z} \end{bmatrix} \tag{7}$$

where $a = -\Phi_q \dot{q} - \Phi_t$ Independent acceleration vector $\ddot{z}$ and Lagrange multiplier λ of a system can be written as:

$$\ddot{z} = M_{11}^{-1} \tilde{Q}_2^T (g - M \tilde{Q}_1 a) \tag{8}$$

$$\lambda = \tilde{Q}_1^T (M \ddot{q} - F) \tag{9}$$

where $M_{11} = \tilde{Q}_2^T M \tilde{Q}_2$.

Since this QR method performs integration along the tangent plane of the constrained surface at each time, the violation of constraint equation is much less than LU method. But it sometimes requires a longer simulation time due to the calculation of tangent plane frequency.

3. Suggestion of a Hybrid Coordinate Partitioning Method

When the LU decomposition scheme is employed for a system involving singular points, the mass matrix in the equations of motion becomes ill conditioned and coordinate partitioning is carried out continuously to find the proper independent coordinates. If proper independent coordinates are not chosen, the simulation may stop at that time. Although proper coordinates are chosen, the simulation results might contain a large numerical error which would damage the efficiency. In the QR method that traces the constrained nullspace at each time, the property of convergence at singular points is much better than the LU method. QR method, however, is undesirable in terms of efficiency since it requires more complicated evaluations, such as inverse of Jacobian matrix and calculation of nullspace each time.

The method suggested in this paper is to use the LU method for overall integration intervals except around singular points, in which the QR method has to be employed to retain the advantage of high accuracy around the singular point. Then, the efficiency of the LU method and the accuracy of the QR method can be combined. When the Cartesian coordinates are employed in the formulation, a large number of coupled equations must be solved for acceleration analysis. In this case, a sparse matrix code is used.

The criteria of choosing efficient coordinate partitioning methods are employed as follows. Firstly, use the QR method which gives a better numerical stability than other methods when the selection of independent coordinates is continuously changed during the short period or the total energy of system changes suddenly. Secondly, use the LU method when the maximum step size is maintained more than 5 times in the QR method. In general, the QR method preserves stability due to independent coordinates along its tangential direction but falls behind the LU method in terms of efficiency. The bigger the size of the Jacobian matrix, the worse the efficiency is. With a proper combination of desirable features for the two methods, efficiency and stability can be retained at the same time.

The following fourteen steps briefly outline the proposed algorithm, and the flow chart of the method is shown in Fig.1.

Step 1 : Check the initial conditions whether they satisfy the constraint equations.

Step 2 : Evaluate the Jacobian matrix and partition coordinate q into independent coordinates v and dependent coordinates u with the LU method.

Step 3 : Solve the equations of motion with the LU method until the following criteria suggests new independent coordinates.

1) integration step size decreases in the corrector step

2) sustains more than 5 times the same integration step size. If independent coordinates need to be changed, go back to step 2. Check the following criteria to switch QR method.

3) abrupt change of the total energy of the system.

If one of the above criteria works, then go to step 8.

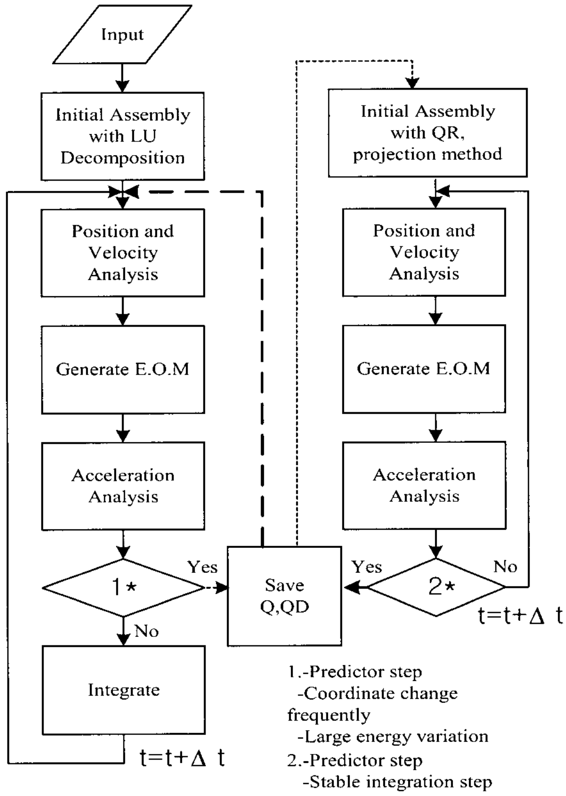


Fig.1. Flow chart of the proposed algorithm

Step 4 : Evaluate dependent velocity vector using independent velocity vector.

Step 5 : Evaluate acceleration and Lagrange multiplier from equation (3).

Step 6 :Using the explicit/implicit Adams PECE algorithms, predict v and $\dot{v}$ for the next time step.

Step 7 : If the end is reached, then stop the simulation. If not, return to step 3.

Step 8 : Execute QR coordinate partitioning and evaluate Q_2, $\tilde{Q}_1$, $\tilde{Q}_2$ and the order of Jacobian matrix.

Step 9 : Fix the independent coordinate z and evaluate the position with the Newton-Raphson method.

Step 10 : Evaluate the velocity of a generalized coordinate.

Step 11 : Evaluate the independent acceleration $\ddot{z}$ from equation(8). Using $\ddot{z}$, calculate $\ddot{q}$ from equation and evaluate Lagrange multiplier λ from equation.

Step 12 : Check whether to decompose the Jacobian matrix again according to the value $\alpha = \frac{\dot{z}^T \dot{z}}{\dot{q}^T \dot{q}}$ for the QR coordinate partitioning method. If decomposition is newly performed, restart the integration using the current values as the initial conditions.

Step 13 : Switch to the LU method if the system seems to be stable. In this paper, when the integration is continuously performed more than 5 times with prescribed maximum integration step size, the system is judged as stable.

Step 14 : Perform the integration for $\dot{q}$ and $\dot{z}$ and return to step 9 and proceed integration to final time.

4. Numerical Example

This paper suggests a new hybrid coordinate partitioning method, which is efficient and accurate for a multibody system involving singular points. This new method is based on the comparison of advantages and disadvantages of two different coordinate partitioning methods, i.e. LU and QR. For the purpose of comparison, a four-bar linkage shown in Fig.2, which is initially rotated 10°from the horizontal plane, is chosen as an example.

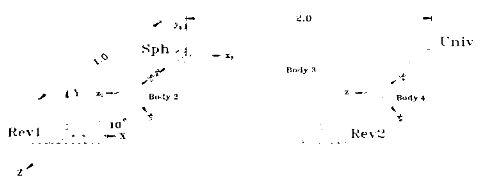


Fig.2. Schematic diagram of 4-bar linkage

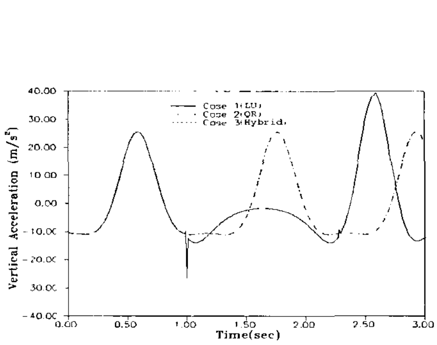


Fig.3 Vertical acceleration of the coupler

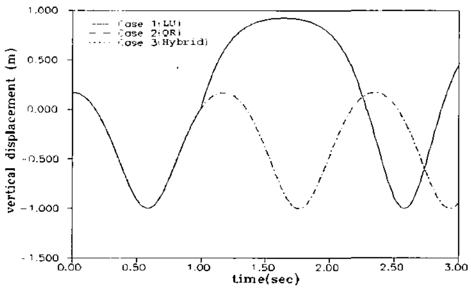


Fig.4 Vertical displacement of the coupler

When this mechanism is moving due to the gravity, it meets a singular configuration at the horizontal position. This mechanism reaches the first singular point at 0.18 second from the initial state and the second singular point at 1.0 second.

The hybrid coordinate partitioning method presented in this paper is verified by analyzing a rigid four-bar linkage. The maximum allowable integration step size and solution tolerance are assigned as 0.01 and 0.0001 respectively. Fig.3 illustrates vertical acceleration of the coupler. The LU method was employed for case 1 and the QR method was employed for case 2 and both the LU and QR methods were simultaneously used for case 3. As shown in Fig.3, case1(LU method) shows an incorrect result, and a discontinuity appears in acceleration. The results for case 2 and case 3 are almost identical.

Fig.4 shows vertical displacement of the coupler, case 2 and case 3 give the correct results. At this point, a hybrid coordinate partitioning method suggested in this paper works well without any divergence in solution.

As shown in Fig.5 the integration step size for each case, the LU method shows a sudden decrease in integration step size at singular points. The QR method shows frequent decrease in integration step size during the simulation. For the hybrid method, however, decrement in step size is less frequent than the QR method. Fig.6 illustrates total energy of the system. In the LU method, a tremendous change occurs in the total energy around the singular points.

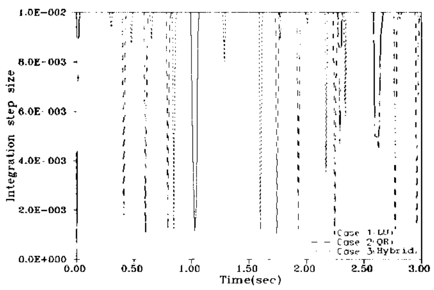


Fig.5 Comparison of the integration step size

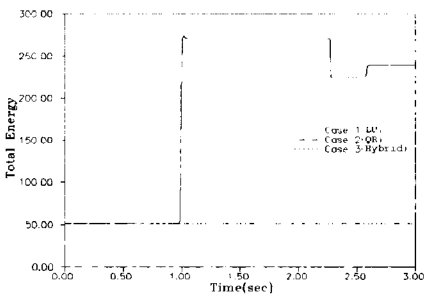


Fig.6 Total energy of the system

Table 1 illustrates CPU times used for each simulation. The simulation is carried out on a SGI-INDIGO2 machine equipped with a R10000 CPU.

As shown in Table 1, the LU method is efficient so far as the accuracy of solution is guaranteed. For this kind of example involving singular points, the hybrid method can possess efficiency and accuracy.

Table 1 Comparison of CPU times

	LU Method	QR Method	Hybrid Method
CPU time [seconds]	2.307	9.366	6.803

5. Conclusions

The advantages and disadvantages of the LU method and the QR method are compared with a four-bar linkage involving singular points. A hybrid partitioning method, which retains both desirable features of two methods, is suggested in this paper.

The usage and strong points of the proposed method was demonstrated via a rigid body system. From these observations, the following conclusions are obtained.

1) The LU method is efficient so far as the accuracy of solution is guaranteed.
2) The QR method is proved to be stable. But, it takes a longer simulation time.
3) The hybrid partitioning method, which combines both advantages of the LU and the QR methods, is suggested and implemented.
4) Applying to systems involving singular points, advantages of the hybrid coordinate partitioning method is proved via a numerical example.

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