Accurate Formulas for Frequency-Dependent Resistance and Inductance Per Unit Length of On-Chip Interconnects on Lossy Silicon Substrate

H. Ymeri, B. Nauwelaers, K. Maex, D. De Roest, S. Vandenberghe, and M. Stucchi

Abstract— A new closed-form expressions to calculate frequency-dependent distributed inductance and the associated distributed series resistance of single interconnect on a lossy silicon substrate (CMOS technology) are presented. The proposed analytic model for series impedance is based on a self-consistent field method and the vector magnetic potential equation. It is shown that the calculated frequency-dependent distributed inductance and the associated resistance are in good agreement with the results obtained from rigorous full wave solutions and CAD-oriented equivalent-circuit modeling approach.

Index Terms **— On-chip interconnects, MIS microstrip, self-consistent method, quasi-magnetostatic procedure, skin effect.**

I. INTRODUCTION

In the past few years, we have witnessed a spectacular development in the complexity and speed of operation of IC circuits. With increasing chip areas and clock rates the performance of VLSI circuits based on MIS (metal-insulator-semiconductor) technology depends more and more upon the properties of interconnections. To optimize electrical properties of IC interconnects, the estimation of the transmission line parameters requires an accurate and frequency-dependent model which includes the effects of semiconductor substrates at high frequencies.

Transmission interconnect lines on MIS structures have been investigated for many years. In [1], [2] Hasegawa et. al. presented an analysis of microstrip line on a Si-SiO_2 system using parallel-plate waveguide model. In [3] , the new model is developed to represent fin line and wide line interconnect behaviour over a 20 GHz frequency range and includes the substrate conductance effects. In [4] , propagation properties of multilayer coplanar lines on different types of silicon substrates are investigated. In [5,6], quasi-analytical analysis of broadband properties of multiconductor transmission lines on semiconducting substrates is done, and the calculated results for line parameters as function of frequency are discussed. Numerous electromagnetic approaches have been published which contains results of numerical full-wave or quasi-TEM analyses [7 - 13]. We can mention, the method based upon the classical mode-matching procedure [7], the spectral-domain analysis method [8-10], and the finite element method [11] have been investigated for this structure. It has been well established that the resistivity-frequency plane can be divided into main three regions: the dissipative dielectric region, the slow-wave region, and the skin-effect region. Recently quasi-TEM analysis on coplanar structure has made the incorporation of metallic conductor losses in the analysis possible and has

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provided a physical basis for the construction of equivalent circuits [12]. In recent works [14 - 18] the authors succeeded to describe the frequency-dependent transmission line parameters of single and coupled interconnects on general lossy silicon substrate in a unified way. The developed formulae describe the self and mutaula admittance and impedance behaviour over the whole frequency range (i.e. also in the transition region between the skin effect, slow-wave and quasi-TEM dielectric modes). In [19], the CAD-oriented equivalent-circuit modeling procedure based on a quasi-stationary spectral domain approach which takes into account the skin effect in the silicon semiconducting substrate is presented.

The silicon interconnects represent an important class of lossy and dispersive transmission line structures and, therefore, should be described by frequency-dependent admitance and impedance per unit length Y(ω) and Z(ω). For more accurate parameter calculations and practical application in advanced VLSI technology, the following specific electrical properties of transmission lines on semiconductor substrate, as used in common VLSI chips, have to be considered (that are different from microwave transmission line): first, the interconnects in IC 's have line widths of order of micrometers. With decreasing line width, the effects of the stripline metal loss increase. Second, in most IC interconnect structures the return lines (grounded) are placed in the same plane as the signal line and distances between signal and grounded lines are often very large, the structure cannot treated as coplanar line. Third, for practical calculations of the parameters of the given structure full-wave procedure developed in classical electromagnetic theory and microwaves are very complicated and generaly time consuming.

In this paper accurate closed-form expressions for the frequency-dependent distributed inductance and the associated distributed series resistance of single on-chip interconnects on conductive silicon substrate are developed using a self-consistent field method and the vector magnetic potential equations. The new approach and the accuracy of the results is applied to a microstrip line on a lossy silicon substrate.

II. Modeling Approach

The new modeling procedure is described for a single on-chip interconnect on a lossy silicon substrate with permittivity ε_{si} and conductivity σ, as illustrated in Fig. 1. For substrate with low resistivity, the time-varying magnetic vector potential in the substrate give rise to frequency-dependent z-oriented eddy currents in the silicon substrate. To develope an expression for series impedance of single interconnect on lossy silicon substrate (see Fig. 1), the quasi-TEM wave is considered, and the magnetic vector potential **A** determinated as

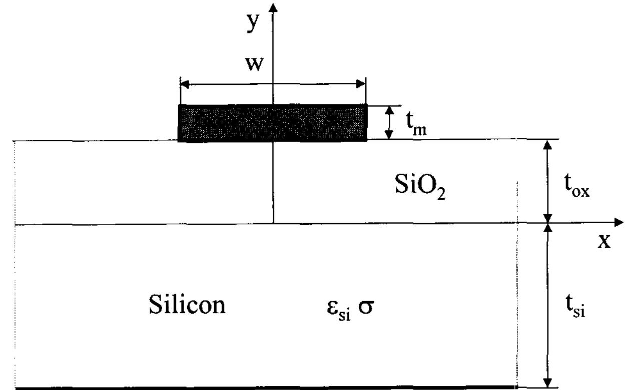

Fig. 1. Single interconnect line on lossy silicon substrate

$$\mathbf{H} = \frac{1}{\mu} rot\ \mathbf{A} \tag{1}$$

has only a z-component and satisfies the magnetic vector potential equation

$$\nabla^2 A_i - j\omega\mu_i\sigma_i A_i = -\mu J_z(x,y) \tag{2}$$

where $J_z(x,y)$ is unknown current density on the interconnect conductor strip. Results obtained from the full-wave analysis [1] have shown that the influence of the finite substrate thickness t_{si} can be neglected for practical dimensions (i.e. $t_{si} >> w$, $t_{si} >> t_m$, and $t_{si} >> t_{ox}$). The silicon substrate is therefore assumed to be infinitely thick in the following derivation.

The current along the microstrip is surface one and can be written as

$$J_z(x,y) = \begin{cases} J_1(x)\delta(y-t_{ox}), & for \quad -w/2 \le x \le w/2 \\ J_2(x)\delta(y-t_{ox}-t_m), & for \quad -w/2 \le x \le w/2 \\ J_3(y)\delta(x-w/2), & for \quad t_{ox} \le y \le t_{ox}+t_m \\ J_4(y)\delta(x+w/2), & for \quad t_{ox} \le y \le t_{ox}+t_m \end{cases} \tag{3}$$

where $J_k(l)$ ($k = 1,\ldots,4$) are unknown distribution functions. The self-consistent field method [20, 21] consists of using a boundary condition on the perfectly conducting microstrip surface

$$\mathbf{n}_k \bullet \mathbf{H}_t = \mathbf{J}_k(l) \quad (4)$$

where $\mathbf{n}_k$ is the unit vector normal from the surface and $\mathbf{H}_t$ tangential magnetic field.

A general solution of eq. (2) may be looked for in the form [21]

$$A_1(x,y) = \int_0^\infty [C_{11}(\lambda)e^{\lambda y} + C_{12}(\lambda)e^{-\lambda y}]\cos(\lambda x)d\lambda, \quad for \quad y \geq 0 \quad (5a)$$

$$A_2(x,y) = \int_0^\infty [C_2(\lambda)e^{my}\cos(\lambda x)d\lambda, \quad for \quad y \leq 0 \quad (5b)$$

where $m = (\lambda^2 + j\omega\mu(\sigma + j\omega\varepsilon_{si}))^{1/2}$.

The integration coefficients may be determined by imposing, at the interface silicon-silicon oxide ($y = 0$), the continuity of the tangential components of the magnetic field and of the normal component of the magnetic flux density.

The proposed method consists of using J_0 as initial constant value of the current density. After a long mathematical manipulations [20], the following closed form expression for the series impedance per unit length of microstrip line on lossy silicon substrate is obtained:

$$Z'_s = \frac{1}{\sigma_m w t_m} + j\omega\mu\left\{\frac{1}{2\pi}\log\left(\frac{1+\gamma_{si}t_{ox}}{\gamma_{si}t_{ox}}\right) + \right.$$
$$\frac{1+\frac{t_m}{4t_{ox}}}{\frac{w+t_m}{t_{ox}}} + \frac{1/\pi}{\left(\frac{w+t_m}{t_{ox}}\right)^2}\left[\frac{1}{2}\left(\frac{t_m^2}{t_{ox}^2} - \frac{w^2}{t_{ox}^2}\right)a\tan\frac{t_m}{w}\right.$$
$$+\frac{1}{4}\left(4 - \frac{w^2}{t_{ox}^2}\right)a\tan\frac{2t_{ox}}{w} +$$
$$\frac{1}{4}\left(\frac{w^2}{t_{ox}^2} - 4\left(1+\frac{t_m}{t_{ox}}\right)^2\right)a\tan\frac{2\left(1+\frac{t_m}{t_{ox}}\right)}{\frac{w}{t_{ox}}} + \left(1+\left(1+\frac{t_m}{t_{ox}}\right)^2\right)\log 2$$
$$+\left(1+\frac{t_m}{t_{ox}}\right)^2\log\left(1+\frac{t_m}{t_{ox}}\right) - \frac{1}{2}\left(\frac{w^2}{t_{ox}^2}\log\frac{w}{t_{ox}} + \frac{t_m^2}{t_{ox}^2}\log\frac{t_m}{t_{ox}}\right) -$$
$$\frac{w}{2t_{ox}}\log\left(4+\frac{w^2}{t_{ox}^2}\right) - \frac{wt_m}{2t_{ox}^2}\log\frac{w^2+t_m^2}{t_{ox}^2}$$
$$+\frac{w}{2t_{ox}}\left(1+\frac{t_m}{t_{ox}}\right)\log\left(\frac{w^2}{t_{ox}^2} + 4\left(1+\frac{t_m}{t_{ox}}\right)^2\right) +$$
$$\left.\left.\frac{1}{4}\left(\frac{w^2}{t_{ox}^2} - \left(2+\frac{t_m}{t_{ox}}\right)^2\right)\log\left(\frac{w^2}{t_{ox}^2} + \left(2+\frac{t_m}{t_{ox}}\right)^2\right)\right]\right\} \quad (6)$$

where γ_{si} is the propagation constant in the lossy silicon substrate given by

$$\gamma_{si} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon_{rsi}\varepsilon_0)} \quad (7)$$

and σ_m, σ, and ε_{rsi} are the microstrip conductivity, and conductivity and relative permittivity of silicon substrate, respectively.

III. Results

The proposed modeling procedure has been applied to various on-chip interconnects. In order to demonstrate the accuracy and efficiency of the analytic model, the frequency-dependent distributed series inductance and associated series resistance of a microstrip on a heavily doped CMOS substrate (resistivity $\rho_{si} = 0.01$ Ω-cm) with a 2 μm oxide layer have been computed using the proposed closed-form expressions, and compared with the solutions obtained by full-wave simulator and CAD-oriented circuit modeling approach, respectively.

The width of the microstrip is 4 μm, thickness 1 μm and the thickness of the silicon substrate is 500 μm. Fig. 2 shows the variation in the distributed resistance, R(ω), as a function of frequency. Similarly, Fig. 3 shows the change in the distributed inductance, L(ω), as a function of frequency. It is observed that the values of the inductance and resistance per unit length, calculated from the new formulas, are found to be in good agreement with those of [6, 19] (full-wave quasi-TEM technique and CAD-oriented circuit modeling approach). The agreement is excellent over the entire frequency range of 0.2 - 10 GHz. The maximum deviations

observed between the new analytic formulas and the EM simulation results of Figs. 2 and 3 correspond to relative error less than 4 %. As expected, the lossy silicon substrate has a significant impact on the frequency-dependent characteristics of the microstrip interconnect and must be attributed to the skin effect in the substrate (the skin effect in the conductor metal plays only a minor role and can be neglected).

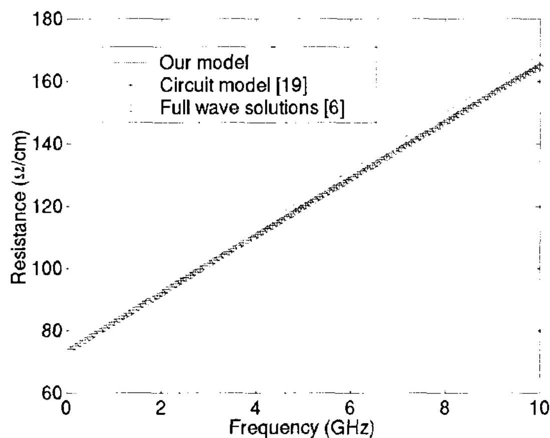

Fig. 2. Distributed series resistance per unit length of a single interconnect on silicon substrate as the function of frequency

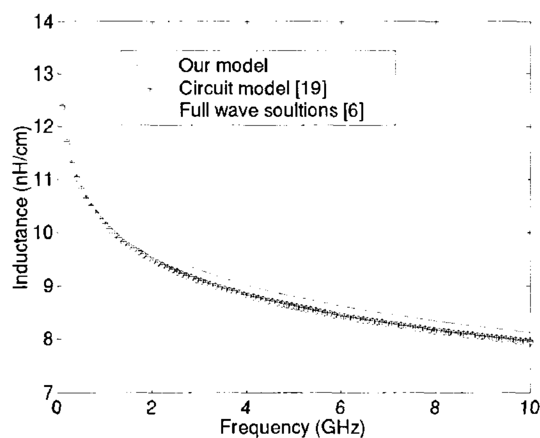

Fig. 3. Distributed series inductance per unit length of a single interconnect on silicon substrate as the function of frequency

IV. Conclusion

A simple and accurate frequency-dependent formulas are proposed to calculate the distributed inductance and associated series resistance of single interconnect on a lossy silicon substrate over the entire frequency range. The results obtained by using these closed-form expressions were compared with results obtained from electromagnetic simulations as weell as CAD-oriented circuit modeling approach. Their simple form enables the application in the design and validation phase of RF and microwave integrated circuits in CMOS technology.

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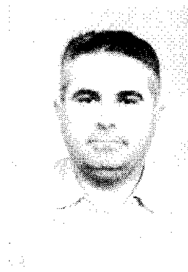

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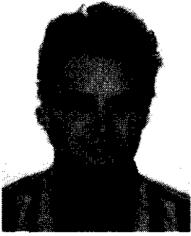

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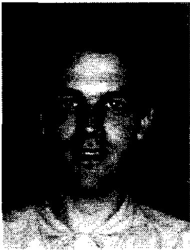

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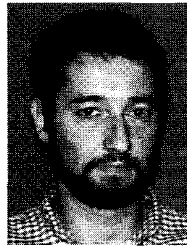

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