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Hybrid Type Vibration Power Flow Analysis Method Using SEA Parameters

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Abstract

This paper proposes a hybrid method for vibration analysis in the medium to high frequency ranges using Power Flow Analysis (PFA) algorithm and Statistical Energy Analysis (SEA) coupling concepts. The main part of the developed method is the application of coupling loss factor (CLF) suggested in SEA to the power transmission, reflection coefficients in PFA boundary conditions. The developed hybrid method shows very promising results with regard to the applications for the various damping loss factors in wide frequency ranges. And also this paper presents the applied results of Power Flow Finite Element Method (PFFEM) by forming the new joint element matrix with CLF to analyze the various plate structures in shape. The analytical results of automobile, complex plate structures show good agreement with those of PFFEM using the PFA coefficients.

Keywords: Power flow analysis (PFA), Power flow finite element method (PFFEM), Statistical energy analysis (SEA), Damping loss factor, Coupling loss factor, Joint element matrix

I. Introduction

Power Flow Analysis (PFA) is understood to be one of the reliable methods in medium to high frequency ranges, and has remarkable advantages compared to other vibration analysis tools. But there are some limitations in related information for the various structural elements. This paper presents the algorithm for the use of coupling loss factor (CLF) in PFA boundary conditions, and numerically analyzes it using practical examples to evaluate its validity. Formulation using CLF in PFA boundary condition is developed to cover the flexural, longitudinal and shear waves in plates, and is proven to be valid by numerically analyzing the coupled plate structures and at an arbitrary angle. In addition, this paper has developed new joint element matrix using CLF to extend the application area into complex plate structures by Power Flow Finite Element Method (PFFEM), and analyzes numerically the open box-type structures to evaluate its validity. Finally, the energy density and intensity of automotive structures has been successfully predicted by PFFEM.

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II. PFA Boundary Condition Using SEA Parameters

The vibrational response of homogeneous isotropic two-dimensional structural component, such as plate, can be expressed by the linear superposition of plane wave components, with the displacement w at the position $x=(x_1, x_2)$ as,

$$w(x) = \int_0^{2\pi} A(\theta) e^{-ikn(\theta)\cdot x + i\omega t} \, d\theta. \quad (1)$$

Here, w is vibration frequency, k is the wavenumber, $A(\theta)$ is the complex amplitude of the wave with heading θ, and $n(\theta) = (\cos\theta, \sin\theta)$. The kinetic energy density $T(\theta)$ associated with equation (1) under the conditions such as uncorrelated wave components or local spatial averaging may be written as,

$$T(x) = (\rho\omega^2/4) \int_0^{2\pi} f(\theta) \, e^{-(\omega\eta/c_g)n(\theta)\cdot x} d\theta \quad (2)$$

where $f(\theta)$is the energy in the wave with heading angle, θ, ρ is the mass of component per unit area, η is the loss factor and c_g is the group velocity. The total energy density $e(x)$ will approximately be $2T(x)$ in reverberant wave field. The intensity of an individual plane wave in the propagation direction is equal to c_g times the energy density; if it is assumed that the various wave components are statistically independent, then the relation of the total intensity and the gradient of the total energy density can be written in the form below;

$$I = -(c_g^2/\omega\eta) \nabla e. \quad (3)$$

In considering the two two-dimensional structural components which are coupled along a common edge, the total energy flow P_{12} per unit length at some point on the coupling boundary may be expressed in the form

$$P_{12} = I_1 \cdot n_1 = I_2 \cdot n_1 \quad (4)$$

where n_1 represents the outward pointing normal for component 1, and I_1 and I_2 are the wave intensities in the two components. The energy flow may also be expressed in terms of the wave transmission coefficients of the boundary. For instance, the energy which flows from component 1 to component 2 per unit length of the boundary can be expressed in terms of the wave transmission coefficient $\tau_{12}(\theta)$ in case $\eta = 0$ and $e(x) =$ constant as,

$$P_{12} = (c_{g1}/2\pi) \, e_1(x) \times \int_\theta \tau_{12}(\theta)(\cos\theta \sin\theta) \cdot n_1 d\theta. \quad (5)$$

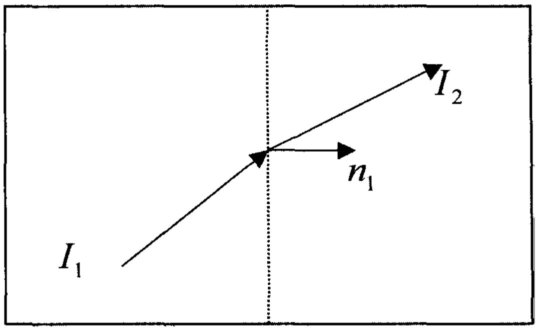

Figure 1. Structural intensities at a boundary between two components.

The integral over θ yields twice the diffuse wave field transmission coefficient $\langle \tau_{12} \rangle$, and thus equation (5) may be written in the form

$$p_{12} = (\omega S_1 / L)\eta_{12} \; e_1(x) \quad (6)$$

$$\eta_{12} = c_{g1} L \langle \tau_{12} \rangle \; / \; (\omega \pi S_1) \quad (7)$$

where S_1 is the surface area of component 1, L is the length of the boundary and η_{12} is the coupling loss factor(CLF) known for a wide range of structural junctions in SEA. The total energy flow P_{12} is given by $p_{12} - p_{21}$, where P_{21} is given by the equivalent of equations (6) and (7). Equations (3) and (4) and (6) thus lead to the following boundary conditions :

$$\begin{aligned} &-(c_{g1}^2/\omega\eta_1) \nabla e_1 \cdot n_1 \\ &= -(c_{g2}^2/\omega\eta_2) \nabla e_2 \cdot n_1 \\ &= (\omega/L)[S_1 \eta_{12} e_1 - S_2 \eta_{21} e_2]. \end{aligned} \quad (8)$$

The expanded boundary conditions including in-plane wave as well as flexural wave may be written, using the same algorithm, as

$$\begin{aligned} &-(c_{g1m}^2/\omega\eta_{1m}) \nabla e_{1m} \cdot n_1 \\ &= \sum_{n=f,l,s} (\omega/L)[S_{1m} \eta_{12mn} e_{1m} - S_2 \eta_{21nm} e_{2n}] \end{aligned} \quad (9)$$

$$\begin{aligned} &-(c_{g2m}^2/\omega\eta_{2m}) \nabla e_{2m} \cdot n_1 \\ &= \sum_{n=f,l,s} (\omega/L)[S_{1m} \eta_{12mn} e_{1m} - S_2 \eta_{21nm} e_{2n}] \end{aligned} \quad (10)$$

where the range of an index m is f, l, s (flexural, longitudinal and shear, respectively).

III. PFFEM Joint Element Matrix Using SEA Parameters

For vibrational analysis of complex plate structure using this algorithm, the joint element matrix of Power Flow Finite Element Method (PFFEM) are to be modified so as to include SEA parameter, coupling loss factor. The PFA energy governing differential equation for propagating waves (i.e. flexural, longitudinal and shear wave) is given by

$$-\frac{c_g^2}{\eta\omega}\nabla^2\langle e\rangle+\eta\omega\langle e\rangle=\Pi_{in}, \tag{11}$$

where ω is the angular frequency, c_g is the group velocity, η is the internal loss factor of structure, $\langle e\rangle$ is time- and space-averaged energy density and Π is input power to structure. To implement equation (12) numerically with the finite element method, the weak form variational statement by using the gradient and divergence theorems, can be written as,

$$\int_D\left\{\frac{c_g^2}{\eta\omega}\nabla\langle e\rangle\cdot\nabla v+\eta\omega\langle e\rangle v\right\}dD=\int_D\Pi_{in}v\,dD+\int_\Gamma v(-n)\cdot\langle I\rangle d\Gamma \tag{12}$$

where n is a unit vector normal to the element boundary D and v is the test function. For a Galerkin weighted residual scheme, the approximation are made as,

$$\langle e\rangle=\sum_i e_i\,\phi_i \quad\text{and}\quad v=\sum_i\phi_i \tag{13-14}$$

Substituting equations (14) and (15) into equation (13) yields the following matrix equation.

$$[K^{(e)}]\{e^{(e)}\}=\{F^{(e)}\}+\{Q^{(e)}\} \tag{15}$$

Here, $K_{ij}^{(e)}$ is a term in the coefficient matrix which contains both stiffness- and mass-matrix terms, $F_i^{(e)}$ is the input power and $Q_i^{(e)}$ is power flow of which the positive direction is defined as a vector into its element. The global matrix equation can be represented by assembling the element matrix equation (16) as,

$$[K]\{e\}=\{F\}+\{Q\} \tag{16}$$

In the matrix equation (17), it is necessary to insert joint element matrix which represents the relation between energy density and power flow so as to solve linear matrix equation. In the existing researches, joint element matrix was derived from the concept of power transmission and reflection. However, for application of algorithm using SEA parameters in boundary condition, new joint element matrix including SEA parameters must be formulated. The new derived joint element matrix using concepts in equation (8), (9), (10) and (13) is given by equation (18).

$$[J]=-\frac{\omega}{L}\begin{bmatrix} S_1\left(\sum_{j=f,l,s}\eta_{fj12}\right) & -S_2\eta_{ff21} & 0 & -S_2\eta_{lf21} & 0 & -S_2\eta_{sf21} \\ -S_1\eta_{ff12} & S_2\left(\sum_{j=f,l,s}\eta_{fj21}\right) & -S_1\eta_{fl12} & 0 & -S_1\eta_{fs12} & 0 \\ 0 & -S_2\eta_{fl21} & S_1\left(\sum_{j=f,l,s}\eta_{lj12}\right) & -S_2\eta_{ll21} & 0 & -S_2\eta_{sl21} \\ -S_1\eta_{fl12} & 0 & -S_1\eta_{ll12} & S_2\left(\sum_{j=f,l,s}\eta_{lj21}\right) & -S_1\eta_{ls12} & 0 \\ 0 & -S_2\eta_{fs21} & 0 & -S_2\eta_{ls21} & S_1\left(\sum_{j=f,l,s}\eta_{sj12}\right) & -S_2\eta_{ss21} \\ -S_1\eta_{fs12} & 0 & -S_1\eta_{ls12} & 0 & -S_1\eta_{ss12} & S_2\left(\sum_{j=f,l,s}\eta_{sj21}\right) \end{bmatrix} \tag{17}$$

where η_{mn12} is the coupling loss factor in case where m-type wave in component 1 transmits n-type wave in component 2. Also, the relation between power flow matrix $\{Q\}$ and energy density matrix $\{e\}$ is given by

$$\{Q\}=[J]+\{e\} \tag{18}$$

IV. Computational Examples

4.1. PFA of Plate Structures Coupled at an Arbitrary Angle Using CLF

In the previous part, formulation using CLF in boundary condition of PFA is developed to cover the flexural, longitudinal and shear waves in plates. To evaluate its validity, computations are performed for the thin plate coupled structure simply-supported along the edges and with coupling angle of 90°.

Computation results obtained using hybrid and classical methods for the case of 3000Hz input frequency are shown in Figures 3 and 4, and show good agreement. As internal loss factor decreases, the result of each method shows better agreement. These results guarantee that assumption in developing hybrid boundary condition is valid.

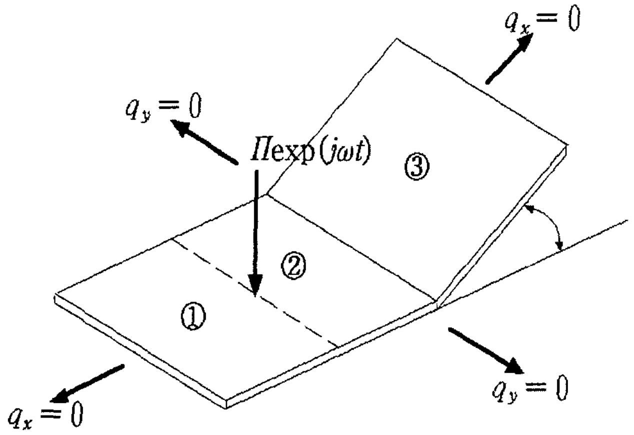

Figure 2. Thin plate structures coupled at an arbitrary angle.

4.2. PFFEM of Open Box-type Plate Structure

To extend the application area of hybrid method into complex structures, the joint element matrix of PFFEM using CLF in SEA is newly derived. The computation has been performed initially to the case of a point loaded open-box type plate structures with Young's modulus $E=2\times10^{11}\ N/m^2$, mass density $\rho = 7800 kg/m^3$, Poisson ratio $\nu=0.28$, damping loss factor $\eta = 0.02$, frequency of 5000 Hz and thickness of 3 mm. The plates are taken to have a length of 2 m in the x_1-direction and a width of 2 m in the x_2-direction. Figures 5, 6, 7 and 8 show the results of flexural and in-plane (longitudinal and shear) energy density in structures. These results show that energy level and energy distribution each method agree well in spite of the high η value of 0.02.

4.3. PFFEM of Automotive Model

The validity of PFFEM using hybrid concepts was

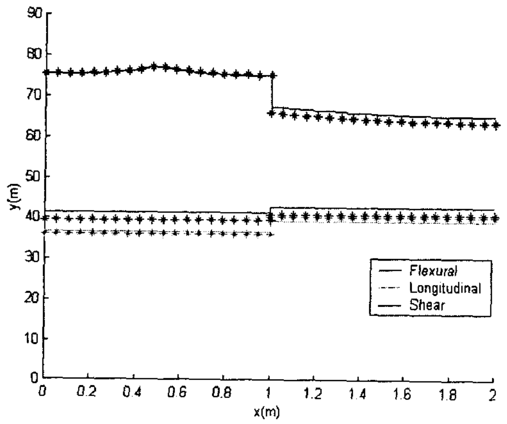

Figure 3. Comparison of energy density level (dB) (3000 Hz, $\eta=0.02$) ('****': new hybrid method, '—': classical method).

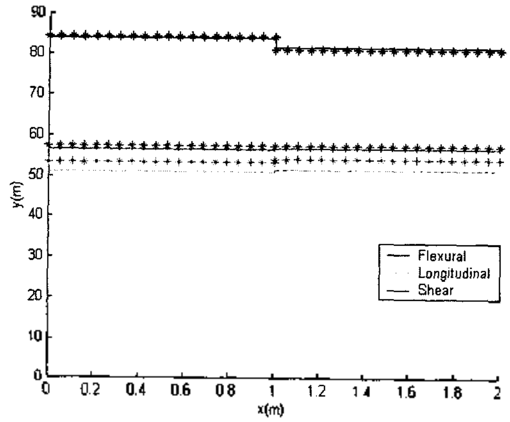

Figure 4. Comparison of energy density level (dB) (3000 Hz, $\eta=0.002$) ('****': new hybrid method, '—': classical method).

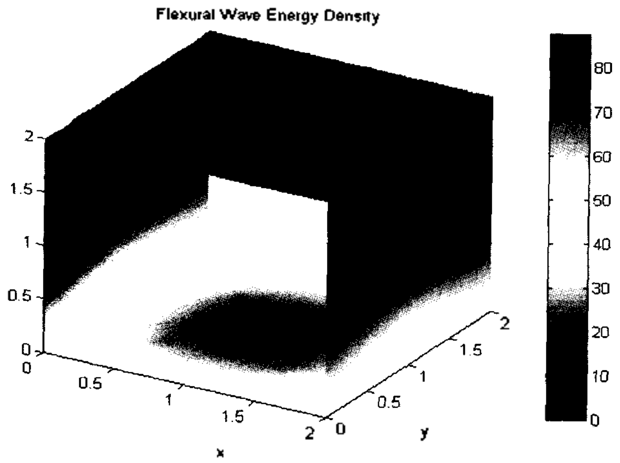

Figure 5. Flexural energy density (Classical PFFEM).

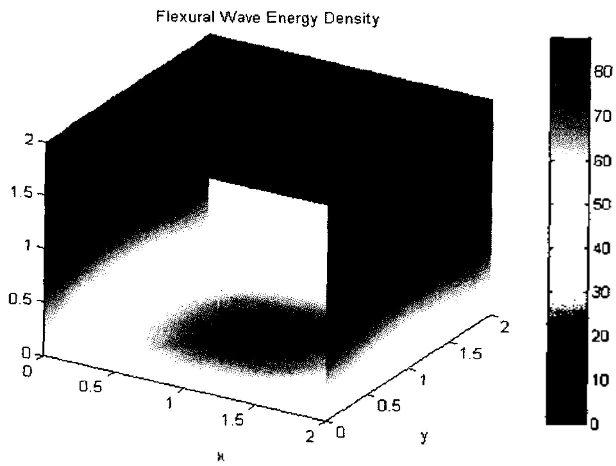

Figure 6. Flexural energy density (Hybrid PFFEM).

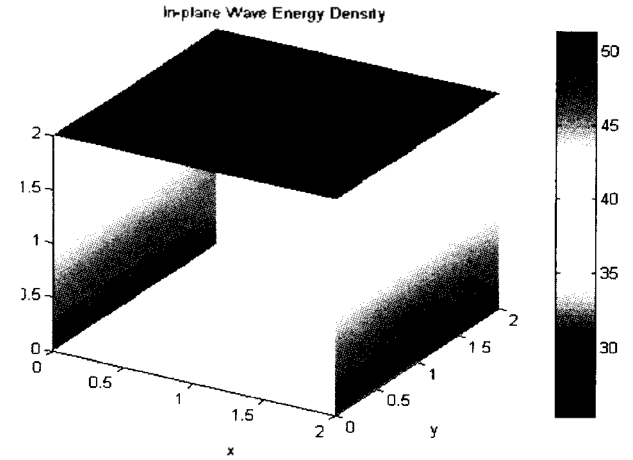

Figure 7. In-plane energy density (Classical PFFEM).

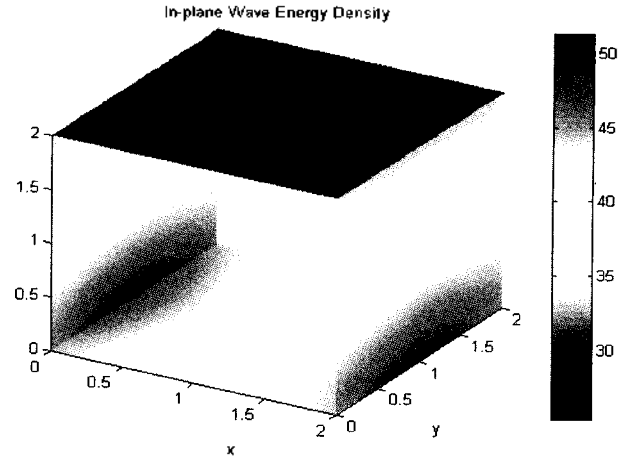

Figure 8. In-plane energy density (Hybrid PFFEM).

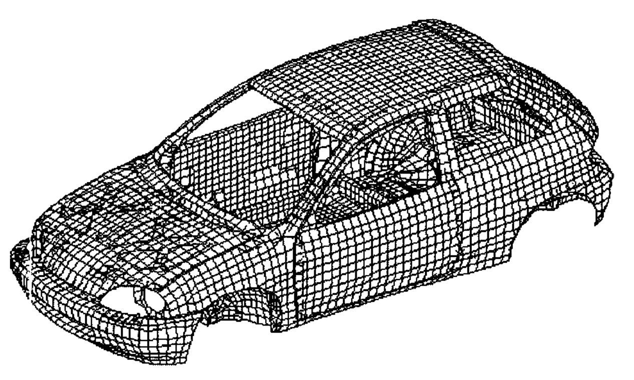

Figure 9. Automobile FE model.

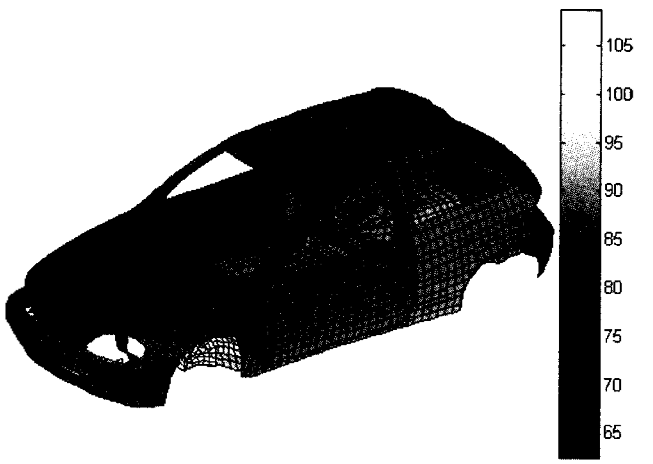

Figure 10. Flexural energy density of automobile (dB).

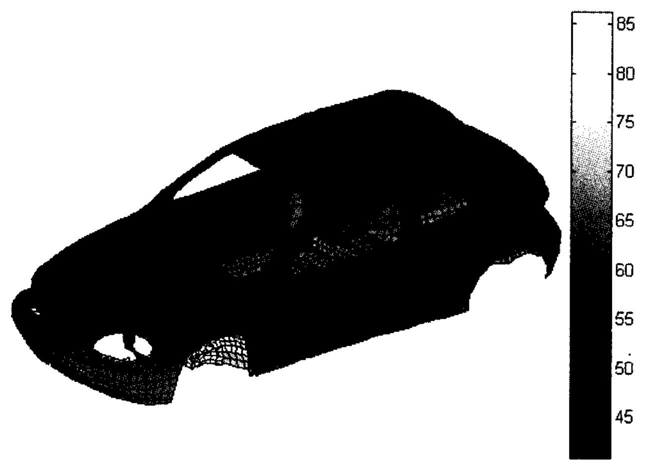

Figure 11. In-plane energy density of automobile (dB).

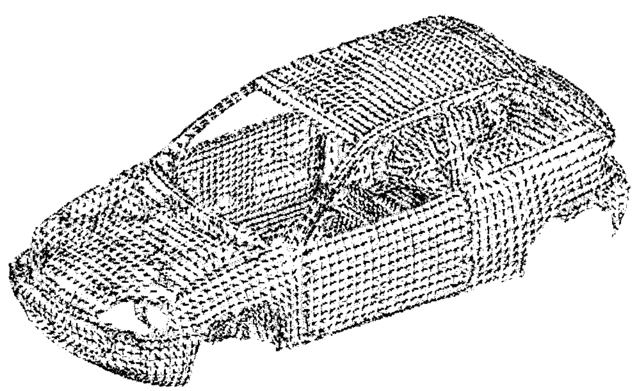

Figure 12. Total intensity distribution of automobile (dB).

shown in the previous part. Here, the applicable region of PFFEM using hybrid concepts is extended to more complicated models.

Figure 9 shows FE model of automobile which consists of 10971 elements of different materials. The excitations on this model are point loadings with the value of $P=1.3\ W$ at two engine frames, $P=0.7\ W$ at the position of transmission and $P=0.5\ W$ at the position at the rear part. The computation is performed for 2000 Hz. In analytic process, the entire 6771 joint elements were created and coupling loss factor (CLF) of SEA was used here. For the automotive model, the results of hybrid PFFEM have been successively predicted in medium to high frequency ranges. The following figures show the computational results of various properties.

V. Conclusions

The hybrid formulation using SEA parameter in boundary condition of PFA has been developed to apply the plentiful junction's information of SEA to PFA. The validity of this new algorithm has been proven through a numerical analysis of the plate structures coupled at an arbitrary angle. The computational results show good agreement with those of PFA. In addition, new joint element matrix using CLF has been developed to extend the application area into complex plate structures by PFFEM. The energy density and intensity of automobile have been successfully predicted by PFFEM using CLF of SEA.

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[Profile]

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Suk-Yoon Hong received the B.E. degree in naval architecture from Seoul National University, Seoul, Korea in 1982. He has been working as an engineer at HYUNDAI heavy industries Co., Ltd. from 1982 to 1987, before he received the Ph.D. graduate program in acoustics from Pennsylvania State University, U.S.A. in 1992. He had been working as NVH team leader Automotive tech. lab. at Institute for Advanced Engineering in Korea from 1992 to 1997. From 1997, he is a faculty member at Seoul National University and is a professor of the Department of Naval Architecture and Ocean Engineering. He is interested in vibration control, radiation noise analysis and structural acoustics using power flow analysis.