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Modal Radiation Efficiency of Swaged Panels

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Abstract

Swaging technique is frequently used to stiffen thin panels for reducing the vibration levels of the machine or vehicle structure. Because the internal constraints imposed by swages can distort the mode shapes of panels, they affect the sound radiation characteristics. In this paper, the radiated sound field generated by the idealized and baffled finite swaged panel is studied, in which the curved swage section is modeled as an incomplete cylindrical shell. The modal radiation efficiencies are predicted using the transfer matrix concept and compared with those of flat panels. It is observed that the radiation efficiencies of the swaged vibrational modes can increase slightly for frequencies below the critical frequency, while increase of radiation efficiency depends on the mode shapes and other related structural parameters.

Keywords: *Swaged panel, Transfer matrix, Modal radiation efficiency*

I. Introduction

Stiffened panels are extensively used for body structures of automobiles, air-crafts, submarines, *etc.*, to reduce the vibration level. Because the radiated sound power is directly proportional to the time- and space-averaged, squared velocity, the reduction of velocity amplitude would be beneficial in the viewpoint of noise reduction. The 'stiffening' can be achieved by the addition of stiffeners or by artificial modification of structure itself which is called 'swaging'. However, the internal constraints imposed by stiffeners or swages distort the natural mode shapes of panels and, consequently, affect the sound radiation characteristics.

For many decades, considerable interests have been focused on the analysis of the stiffening effect[1-3] and the variation of the sound radiation characteristics of panels caused by additive stiffeners[4,5]. Lin[1] calculated the natural frequencies and corresponding mode shapes of skin-stringer panels, typically used for body structures of aircrafts, using the finite difference method. A series of papers on this topic[2,3] focused on the prediction of the increase of natural frequencies and change of natural modes. Maidanik[4] investigated the radiation resistance of the ribbed panel assuming that it was constructed as the multi-panel structure and the vibration field was a diffuse one. Evseev[5] utilized the spatial Fourier transform in the analysis of the sound radiation and transmission of an infinite plate with periodic inhomogenieties, assuming the line force excitations. On the other hand, the swage, due to their geometrical nature, can not be treated as a line discontinuity like an additive stiffener, and that makes the analysis complicated. The effect of swaging has been treated by a limited number of works. Pawlowski[6] suggested a correction method of swaging in a qualitative manner, stating that the swaging

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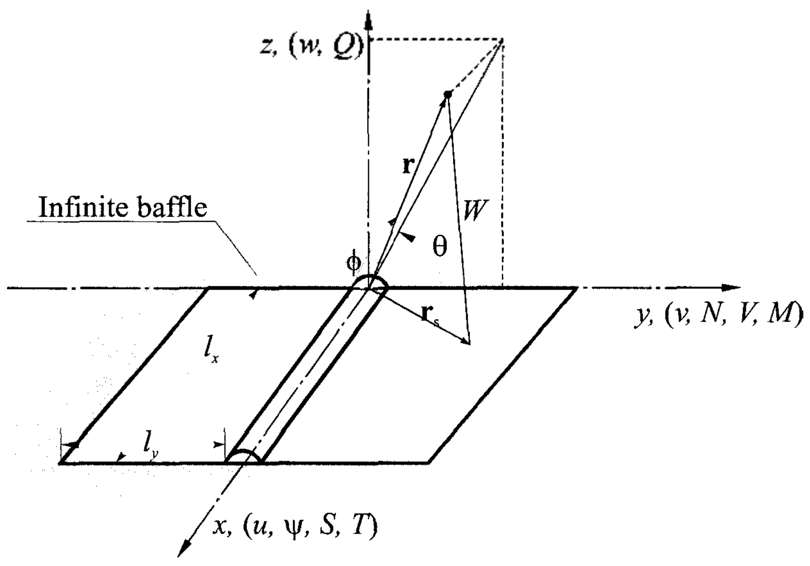
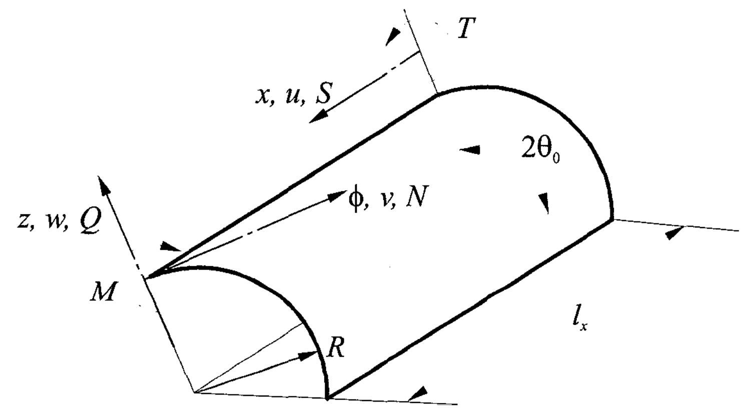


(a) Total structural system



(b) Incomplete cylindrical shell model

Figure 1. Simplified geometrical model of the swaged panel.

technique has a benefit of increasing the structural stiffness without giving rise to the increase of the total weight of structure: the floor panel of the automobile is an example[5].

In this paper, the radiated sound field generated by the idealized and baffled finite swaged panels is studied, in which the swage is modeled as an incomplete cylindrical shell[7]. Figure 1 shows the configuration and the sign convention adopted for the present analysis.

II. Analysis

2.1 Natural Mode Shape

For the analysis of natural vibrations of swaged panels, the transfer matrix method is utilized. The transfer matrix method relates state variables that contribute to the vibrational power such as deflection, rotation angle, bending moment and shear force at a point in a structure to those at another point. This method has been successfully applied to the analysis of the vibrations of multiply constructed structures, in particular, the periodic structures [2,3]. However, the one-dimensional nature of transfer matrices is the principal difficulty in applying this technique to general problems. In order to analyze the swaged panel structure, this problem should be somehow sorted out. Consider a simply supported panel of which the edges are parallel to the y-axis, then it is possible that the state variable, z, can be expanded by Fourier series as

$$z=\sum_{n=1}^{\infty} z_n \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} \left(\frac{n\pi x}{l_x}\right) e^{-i\omega t}, \tag{1}$$

where the harmonic dependency, $\exp(-i\omega t)$, will be omitted for convenience. Using the expansion coefficients z_n, the transfer matrix analysis of swaged panel can be formulated by determining the relationship between the state vectors at an arbitrary location and another.

The formulation of transfer matrix for a swaged part can be briefly outlined as follows[2,3]: The swaged panel model in Fig. 1 can be subdivided into three parts as two flat panels, designated by 1 and 2, and the swage, which is regarded as an incomplete cylindrical shell. According to the Love-Timoshenko plate and shell theory[8], the state vector $\mathbf{z}$ of each part can be defined as

$$\mathbf{z}_f(y)=[u,\ v,\ w,\ \phi\ ,\ M,\ V,\ -N,\ S]_n, \tag{2a}$$

$$\mathbf{z}_s(\phi)=[u,\ v,\ w,\ \phi\ ,\ M,\ V,\ -N,\ S]_n, \tag{2b}$$

where the subscripts 'f' and 's' represent 'flat panel' and 'swage', respectively. Here, u, v, and w indicate the displacement along x, y and z, respectively, and ϕ is the rotation angle about x-axis. M is the bending moment along y-axis, $V(=Q+\partial T/\partial x)$ is the first kind Kirchhoff shear force, N is the in-plane tension along y-axis, and S is the in-plane shear in x-axis. Figure 1 shows the sign convention assigned to these displacements and forces. Note that T and Q in Fig. 2 mean the bending moment along x-axis and shear force along z-axis.

Substituting the preceding states into the governing equations of flat panel and cylindrical shell[9], eight first-order differential equations can be obtained in terms of the state variables, which can be reduced to a matrix differential equation. The solution to this equation is the desired field transfer matrix, $\mathbf{T}$, which represents the state transfer across each sub-part[2]. Consequently, the states at the boundaries of each sub-part can be joined together through transfer matrices as follows:

$$\mathbf{z}_{f,1}^{R}=\mathbf{T}_{f,1}\,\mathbf{z}_{f,1}^{L},\ \ \mathbf{z}_{s}^{R}=\mathbf{T}_{s}\,\mathbf{z}_{s}^{L},\ \ \text{and}$$

$$\mathbf{z}_{f,2}^{R}=\mathbf{T}_{f,2}\,\mathbf{z}_{f,2}^{L} \tag{3a-c}$$

Here, the superscripts 'R' and 'L' indicate 'right-' and 'left-' boundary of each sub-part, respectively.

In addition, the states should satisfy the boundary conditions at the joints of flat panels and swage. For the present model, joints are considered as rigid connections, thus, only coordinate transformations occur. The point transfer matrix $\mathbf{G}$, which represents the state transformation across the joints, can be imposed to Eqs. (3) and, consequently, the total transfer matrix can be constructed as

$$\begin{aligned}\mathbf{z}_{f,2}^{R}&=[\ \mathbf{T}_{f,2}\cdot\ \mathbf{G}\cdot\ \mathbf{T}_{s}\cdot\ \mathbf{G}\cdot\ \mathbf{T}_{f,1}]\ \cdot\ \mathbf{z}_{f,1}^{L}\\ &=\mathbf{T}_{total}\ \cdot\ \mathbf{z}_{f,1}^{L};\\ \mathbf{z}_{s}^{L}&=\mathbf{G}\ \cdot\ \mathbf{z}_{f,1}^{R},\ \ \mathbf{z}_{f,2}^{L}=\mathbf{G}\ \cdot\ \mathbf{z}_{s,1}^{R}\end{aligned} \tag{4}$$

where $\mathbf{z}_{f,1}^{L}$ and $\mathbf{z}_{f,2}^{R}$ denote the states at boundaries of the whole swaged panel end.

The fact that u, w, M, and N are vanished for the simply supported edges leads to the determinant equation deduced from Eq. (4), from which the natural frequencies and the corresponding mode shapes are obtained as solutions [3]. Figure 2 shows several lower order natural modes obtained from the aforementioned manner. Here, h/R means the ratio between the thickness of plate and the radius of cylindrical shell. The mode index (m,n) indicates that the number of half wavelengths in x-direction is m, and the order of corresponding natural frequency for the swaged panel is n. One can find that the modal order is changed from that of the flat panel, by swaging: for example, the swaged mode (1,1) is corresponds approximately to the (1,2) mode of flat panel, and (2,1) to (2,2). In this work, the mode numbering for flat panel is based on the number of half wavelengths in each axis, and, for swaged panel, the modal order is based on the order of natural frequencies.

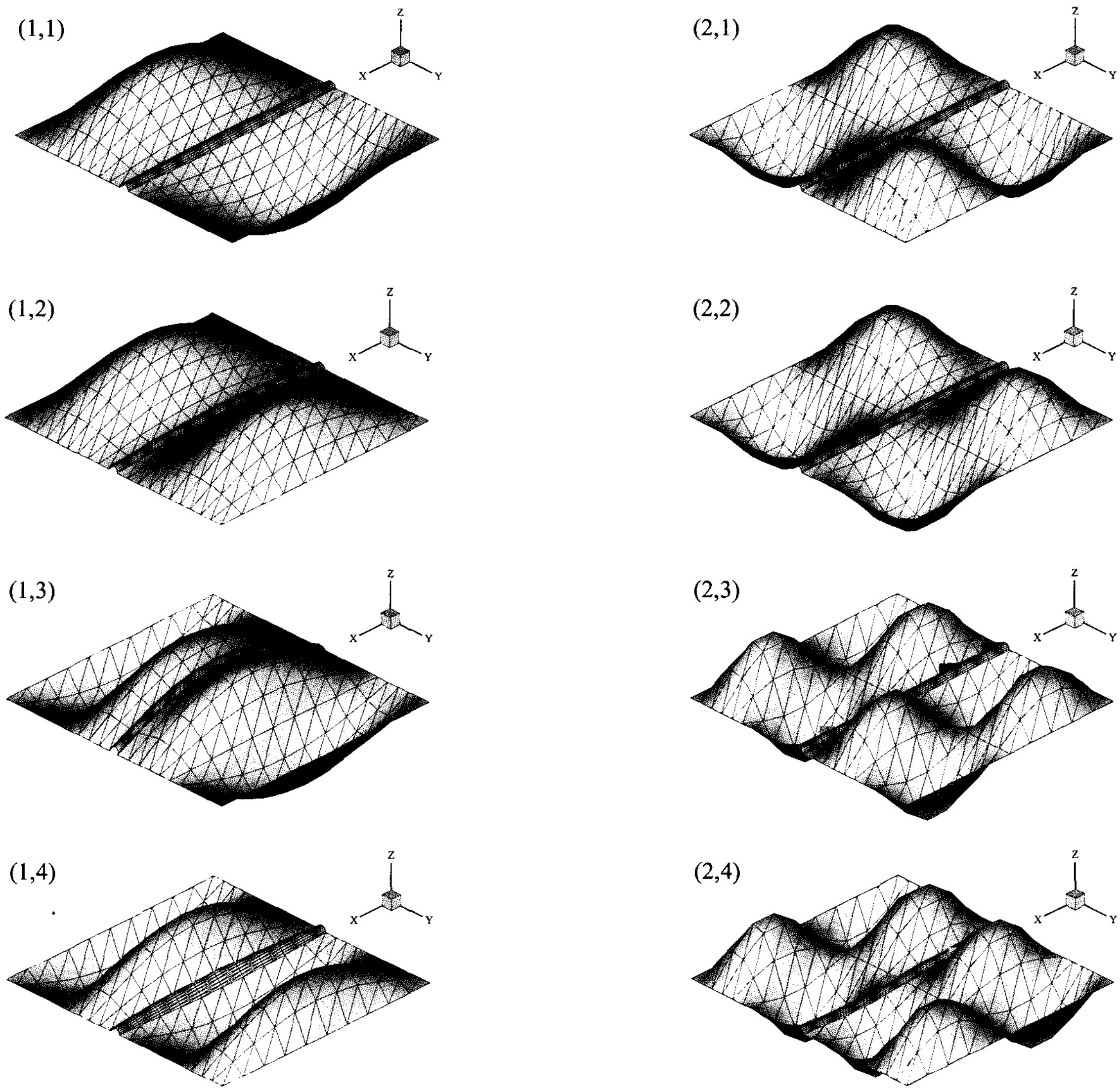


Figure 2. Low order natural modes of a swaged panel (l_x/l_y = 2.0, R/l_x = 0.02, h/R = 0.05).

2.2 Moda Radiation Efficiency

The structure-borne sound field should satisfy the linearized wave equation and boundary condition that the particle velocity of acoustic medium and normal velocity of the vibrating surface should coincide. The velocity potential of field, $\Psi(\mathbf{r})$, must satisfy the following conditions for a source vibrating harmonically in time:

$$(\nabla^2+k^2)\Psi(\mathbf{r})=0,\quad \frac{\partial \Psi(\mathbf{r})}{\partial \mathbf{n}}\bigg|_{\mathbf{r}=\mathbf{r}_s}=v_n(\mathbf{r}_s). \tag{5}$$

Here, v_n is the surface normal velocity, k the acoustic wave number, and $\mathbf{n}$ the outward unit normal vector from the surface. It has been known that, if the vibrating surface can be completely defined in terms of a single coordinate such as in the cases of an infinite cylinder or a sphere, the velocity potential satisfying Eq. (5) can be obtained explicitly[9]. However, unfortunately, most of practical structures do not satisfy this condition, including the present case also. In order to overcome this difficulty in the analysis, several methods have been attempted. Williams *et. al.*[10] investigated the acoustic radiation produced by a finite cylinder using the least-squares approximation scheme. Suzuki[11] calculated the radiation field generated by a dome in an infinite baffle by utilizing the similar method.

In this paper, to obtain the radiated field from swaged panel, several assumptions are made and the superposition concept is employed. The superposition concept in the case of multiple sources is that the total sound field can be obtained as linear summation of the field born by each source and the scattered field due to each other. Consequently, it can be said that the total field is the sum of the radiation fields from two flat panels and a swage.

First, if the radius of swage is much smaller than the acoustic wavelength corresponding to frequencies of interest, the scattering from swage can be neglected. Thus, the sound radiation from the flat panel can be formulated by using the Rayleigh's integral equation[12] as

$$p(r,\theta,\phi)=2i\omega\rho\int_S v_n(\mathbf{r}_s)\cdot g(\mathbf{r}\mid\mathbf{r}_s)\mathrm{d}S$$
$$=-2i\omega\rho\int_S v_n(\mathbf{r}_s)\cdot\frac{e^{ikW}}{4\pi W}\mathrm{d}S \qquad (6)$$

where W denotes the distance between a field point and a surface point which can be expressed in the far field as

$$W=\sqrt{(r\cos\phi-x_s)^2+(r\sin\phi\cos\theta-y_s)^2+(r\cos\phi\cos\theta)^2}$$
$$\cong r-x_s\cos\phi-y_s\sin\phi\cos\theta. \qquad (7)$$

Hence, Eq. (6) can be reduced to

$$p(r,\theta,\phi)\cong-i\omega\rho\frac{e^{ikr}}{2\pi r}\int_S\int v_n(\mathbf{r}_s)$$
$$\exp(-ikx_s\cos\phi-iky_s\sin\phi\cos\theta)\,\mathrm{d}x_s\,\mathrm{d}y_s. \qquad (8)$$

Considering only the radiation from one vibratory mode of the panel, the surface normal velocity $v_n(\mathbf{r}_s)$ can be represented by using separation of variables in the form of

$$v_{n,mn}(x,y)=\tilde{v}_{mn}\cdot\varphi_n(y)\sin\left(\frac{m\pi}{l_x}x\right), \qquad (9)$$

where sine function is for simple supports along y-axis and $\varphi_n(y)$ is the y-directional shape of (m,n) mode which has been obtained in section 2.1.

The radiation from swage can be approximated by that from the equivalent strip of width of $2R\sin\theta_o$ as shown in Fig. 3, provided that the radius is small and the normal velocity is approximately uniform over the swage or its amplitude is much smaller than that of flat panel as

$$v'_{n,mn}\sim v_{n,mn}/\cos\theta'\ ;\ \theta'=\sin^{-1}(y_s/R). \qquad (10)$$

Because the swaged part is normally small in circumferential direction and it has a high stiffness in axial direction, the aforementioned assumptions can be easily met in practice. Figure 2 can be viewed as an evidence of this statement and thus one can use Eq. (10) as the approximated surface velocity.

Substituting Eqs. (9) and (10) into Eq. (8) and integrating over $-l_x/2\leq x\leq l_x/2$ yields

$$p_{mn}(\mathbf{r})=i\omega\rho\left(\frac{l_x}{m\pi}\right)\frac{\tilde{v}_{mn}e^{ikr}}{2\pi r}I_{mn}(\theta,\phi). \qquad (11)$$

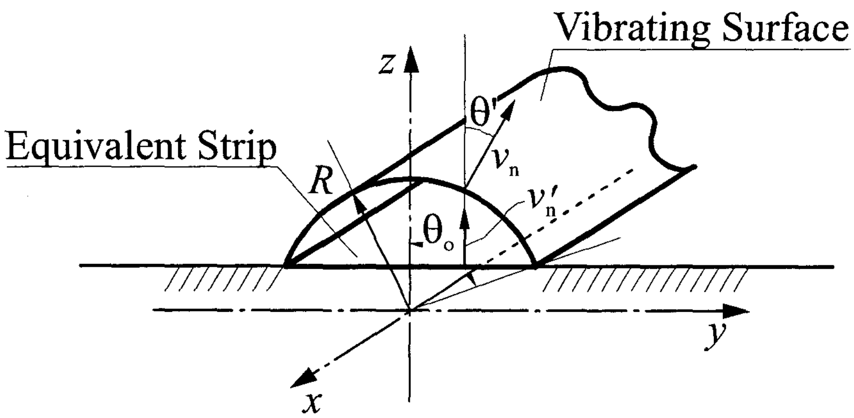


Figure 3. Cross section of a swaged part for the acoustic radiation analysis.

Here, $I_{mn}(\theta,\phi)$ represents the directivity pattern for the radiated field

$$I_{mn}(\theta,\phi)=\left[\frac{(-1)^m e^{-ikl_x\cos\phi}-1}{(kl_x\cos\phi/m\pi)^2-1}\right] \cdot e^{ikl_x\sin\phi/2}\int_{-l_y-R\sin\theta_o}^{l_y+R\sin\theta_o}\varphi'_n(y_s)e^{-iky_s\sin\phi\cos\theta}\,dy_s, \quad (12)$$

where $\varphi'_n(y_s)$ is given by

$$\varphi'_n(y_s)=\begin{cases}\varphi_n(y_s), & \text{for } |y_s|>R\sin\theta_0,\\ \varphi_n/\cos\theta', & \text{for } |y_s|<R\sin\theta_0.\end{cases} \quad (13)$$

From these equations, the radiated sound power, Π_{mn}, and the modal radiation efficiency, σ_{mn}, can be obtained as

$$\Pi_{mn}=\frac{1}{2}\,\mathrm{Re}\left\{\int_\Omega p_{mn}\cdot v^*_{mn}\,d\Omega\right\}=\frac{1}{2\rho c}\int_\Omega|p_{mn}|^2 d\Omega. \quad (14)$$

$$\sigma_{MN}=\frac{\Pi_{mn}}{\rho cS\langle\overline{v}\rangle^2} =\frac{1}{2}\frac{(kl_x)^2}{m^2\pi^4}\frac{1}{\Lambda_n l_x}\int_0^{2\pi}\int_0^{\pi/2}|I_{mn}(\theta,\phi)|^2\sin\phi\,d\phi\,d\theta, \quad (15)$$

where Ω is the arbitrary hemispherical surface, $d\Omega=r^2\sin\phi\,d\phi\,d\theta$, and Λ_n is given by

$$\Lambda_n=\int_{-l_y-R\sin\theta_o}^{l_y+R\sin\theta_o}|\varphi'_n(y_s)|^2\,dy_s. \quad (16)$$

III. Numerical Results

Numerical calculation after substituting the mode shape function, as was obtained in section 2.1, into Eq. (15) yields the modal radiation efficiencies as shown in Figs. 4 and 5. In these figures, the radiation index, $10\log10\,\sigma$, is used instead of the radiation efficiency, σ. The results are compared with those of the flat panel of same size[13], in which $k_{b,mn}$ denotes the wave number corresponding to the natural frequency of the flat panel, *i.e.*, it means the coincidence frequency.

According to Fig. 2, the (1,1) mode, known as the most effective radiating mode of flat panels, does not appear due to the swage. This fact leads to a significant decrease of the radiation efficiency corresponding to the 'lowest' mode of swaged panel, as shown in Fig. 4 (a). On the other

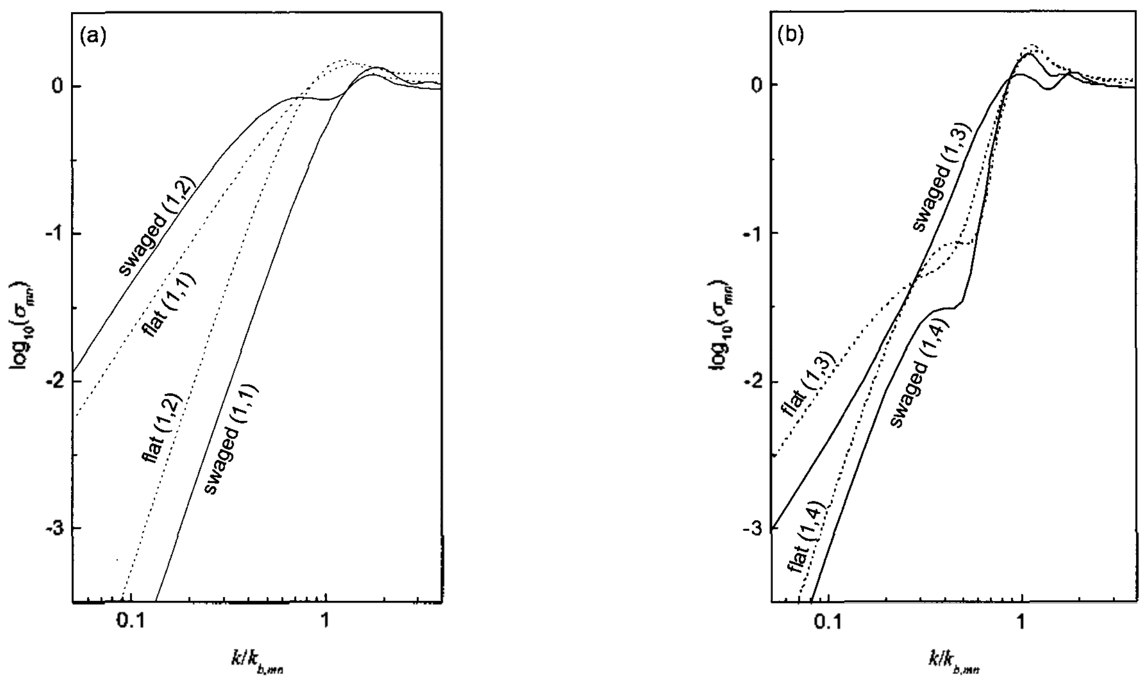


Figure 4. Radiation indices of several low order modes of a single swaged panel as a function of normalized wave number (m = 1).

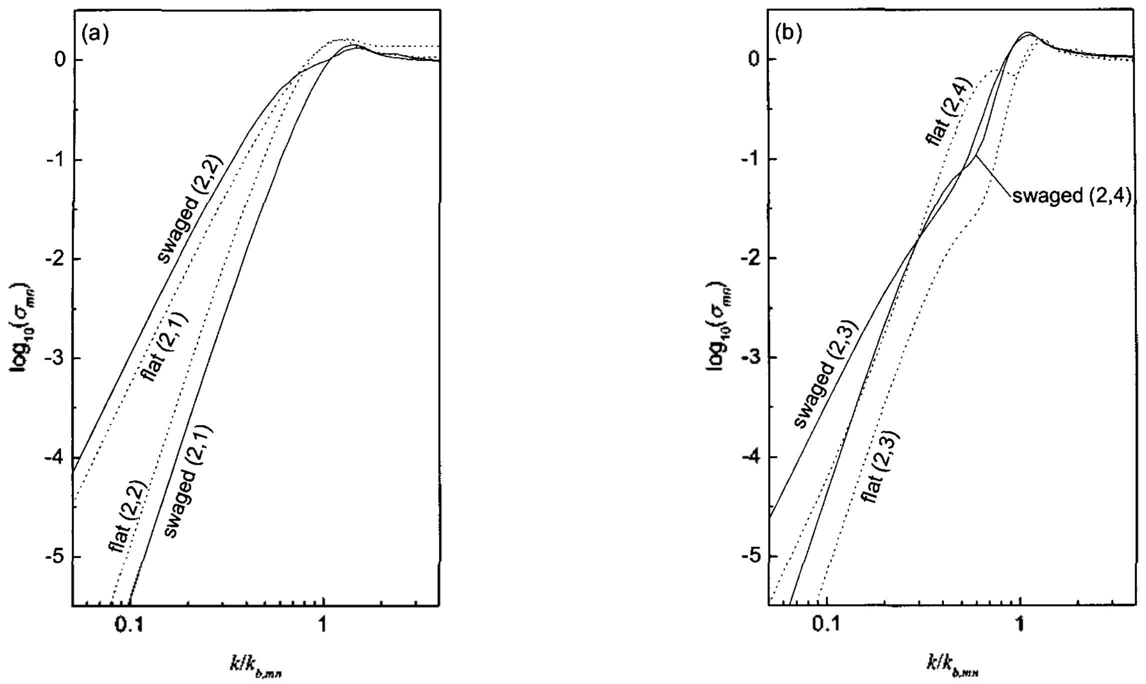


Figure 5. Radiation indices of several low order modes of a single swaged panel as a function of normalized wave number (m = 2).

hand, the 'lowest' mode of swaged panel seems approximately like the (1,2) mode of flat panel, and Fig. 4 (a) shows the slight increase of radiation efficiency compared with the (1,2) mode of flat panel. This increase of radiation efficiency comes from the incomplete intercellular cancellation of oscillating airflow between adjacent cells divided by vibrational phase, which governs the sound radiation of the low order modes of flat panels[12].

Another noticeable result is the slight shift of the coincidence frequency. In general, this frequency represents the demarcation between the two frequency regions; one over which the acoustic power is radiated and the other no real power is radiated or the radiated power is negligibly small. Thus, the change of the coincidence frequency indicates enlarging (or narrowing) of the effective frequency region in the viewpoint of the radiated sound power, or spreading the transition region. Consequently, it is possible that the total radiated sound power can be increased in spite of the decrease of the efficiency, or *vice versa*. These phenomena can be observed apparently for (1,1) mode as shown in Fig. 4 (a) and the spreading can be seen in other modes.

IV. Concluding Remarks

There are two concerning points due to swaging. One is the suppression of vibration level and another is the change of radiation characteristics, that is, the variation of radiation efficiencies and coincidence frequencies. In general, the swage is known to increase the effective stiffness of plate, and thus, suppress the vibrational level. However, according to the present analysis, the swage changes the radiation characteristics of plate. In the present analysis, the most effective radiating mode, *i.e.*, (1,1) mode, of flat panels does not appear by imposing the artificial nodal lines. This fact implies that, if undesirable resonance effects occur, they can be avoided effectively by swaging technique without increasing the total weight of panels. However, the presence of the swage can cause the change of coincidence frequency and the increase of modal radiation efficiency of the involved modes, thus results the unexpected sound radiation. Therefore, in order to modify the structures desirably, the vibrational and acoustic characteristics of the involved structures should be known a priori, from which the troublesome resonance effects can be properly suppressed by the swaging technique.

Acknowledgements

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