THE JOURNAL OF THE ACOUSTICAL SOCIETY OF KOREA Vol.21, No.2E 2002. 6. pp. 63~70

Shape Optimization Technique for Thin Walled Beam of Automotive Structures Considering Vibration

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(Received 24 January 2002; accepted 26 June 2002)

Abstract

In this paper, an optimization technique for thin walled beams of vehicle body structure is proposed. Stiffness of thin walled beam structure is characterized by the thickness and typical section shape of the beam structure. Approximate functions for the section properties such as area, area moment of inertia, and torsional constant are derived by using the response surface method. The approximate functions can be used for the optimal design of the vehicle body that consists of complicated thin walled beams. A passenger car body structure is optimized to demonstrate the proposed technique.

Keywords: *Design optimization, Response surface method, Section Property, Shape optimization, Thin walled beam, Vibration analysis*

I. Introduction

A vehicle body structure is made up of the thin panels with complex shape and assembled by spot welding, bolting, etc. Pillar structures, which mainly influence the static and dynamic characteristics of the vehicle system, consisted of the thin-walled beam structures with closed or open loop sections[1-3]. Therefore, the section properties of the pillars are determined by the typical section shape and the thickness of the thin walled steel plates.

In the automotive industries, the design of pillar sections had been performed by the trial and error approach based on experiences. This approach is neither reliable nor effective. Design issues occur too late in the development process to efficiently affect the design. These problems can be solved by using the optimization methods. The optimization is a useful and challenging activity in vehicle structural design. It provides engineers with tools for producing better design while saving time in the vehicle design process. Recently, researches of the optimal design technique for the vehicle body structure have been made.

In this paper, a new optimal design technique for the thin walled beam sections of the vehicle body structure is proposed. Numerical approximate functions for the section properties such as area, moment of inertia, and torsional constant are derived by using the response surface method [4]. The approximate functions are utilized for the optimization of vehicle body structures, which consist of complicated thin walled beam structures. Optimization for a passenger car body structure is shown to demonstrate the proposed optimization technique.

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II. Design Optimization Technique for Thin Walled Beam Structures

In this study, the optimal design technique that can be applied to the typical pillar sections is presented. Generally, in the vehicle structural optimization, the mass of the vehicle body structure, which influences the weight and cost of the vehicle, is set up for an objective function. Natural frequencies for the specific vibration modes that must be controlled for stiffness and vibration of the car are defined as design constraints. The thickness of the panels of the thin-walled beam structure and nodal coordinates of the shape of the cross section must be defined as design variables so that the drawing of the typical section may be made. However, in the finite element model of the car body structure, which is generally used in the initial design stage, many thin walled beam structures such as pillars, roof rails, rockers, are modeled with beam elements. The input data of the beam elements is not the nodal coordinate values representing the section shape but the section properties such as area, moment of inertia, and torsional constant. Therefore, to perform the pillar section optimization considering the vibration of the vehicle body structure, the proper technique is required between the optimal design program and the finite element analysis program.

2.1. Design Variables

In general, the pillars structures consist of inner panel, outer panel, and reinforcement panel. For optimization, thickness of the thin panels and a scale vector are defined as design variables. A scale vector is introduced to consider the shape design of the cross section of the pillars, instead of using the nodal coordinates. With these design variables, section properties such as the area, area moment of inertia, and torsional constant of the pillars are computed and used for FEM analysis. As shown in Fig. 1, the thickness and the scale vector are used as the design parameters to formulate the approximate function for the section properties. By using the scale vector, it is possible to perform the shape optimal design with the reduced the number of design variables.

In Fig. 1, the coordinate value of the node 12 for the y'z' coordinate system can be expressed as Eq. (1).

$$\begin{pmatrix} y'_{12} \\ z'_{12} \end{pmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} y_{12} \\ z_{12} \end{pmatrix}_{init} = \begin{bmatrix} \cos\theta(y_{12})_{init} + \sin\theta(z_{12})_{init} \\ -\sin\theta(y_{12})_{init} + \cos\theta(z_{12})_{init} \end{bmatrix} \quad (1)$$

In this case, when the scale vector SV_1 is set up in the design variable, the new coordinate value of the node 12 for the yz coordinate system can be written as Eq. (2).

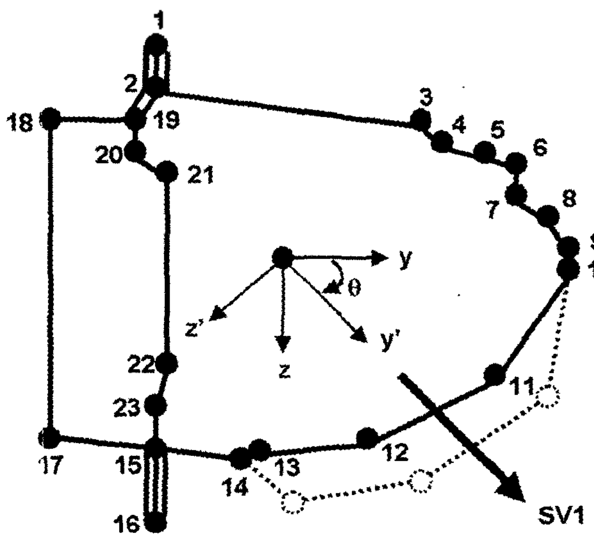

Figure 1. Scale vector for shape design variables.

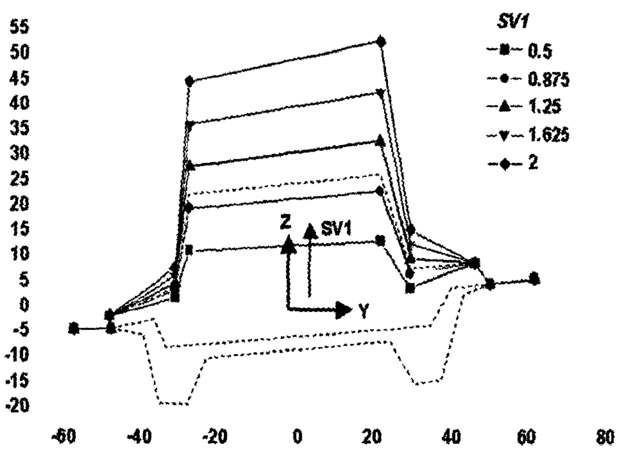

Figure 2. Section shapes with variable scale vectors.

$$\begin{pmatrix} y_{12} \\ z_{12} \end{pmatrix}_{new} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} y'_{12} SV_1 \\ z'_{12} \end{pmatrix} = \begin{bmatrix} \cos\theta\, y'_{12} SV_1 - \sin\theta\, z'_{12} \\ \sin\theta\, y'_{12} SV_1 + \cos\theta\, z'_{12} \end{bmatrix} \quad (2)$$

As shown in Eqs. (1)-(2), the new section shapes can be obtained when the rotation angle θ of the y'z' coordinate system and the scale vector SV_1 are known. Figure 2 illustrates the section shapes changing according to the scale vector SV_1 when the rotation angle θ is 90°.

2.2. Application of Response Surface Method

The most common global approximation is the response surface method. Using the RSM, approximate functions for the section properties of the pillars are derived in terms of the design variables defined in the previous section. The function values are sampled at a number of design points, and then an analytical expression called response surface is fitted to the data. The approximation contains a number of unknown parameters (such as polynomial coefficients) that must be adjusted to match the function to be approximated.

The experiment data or the exact values, which are obtained from the exact equations, are needed to use the response surface method described above. In this study, the exact equations are utilized to use the response surface method. To formulate the exact equations for the pillar section properties, a pillar section is divided into a number of the finite elements as several rectangular with length L and thickness t as shown in Fig. 3. Equations (3)-(9) show the exact equations of the area, the moment of inertia, and the torsional constant for the pillar structures.

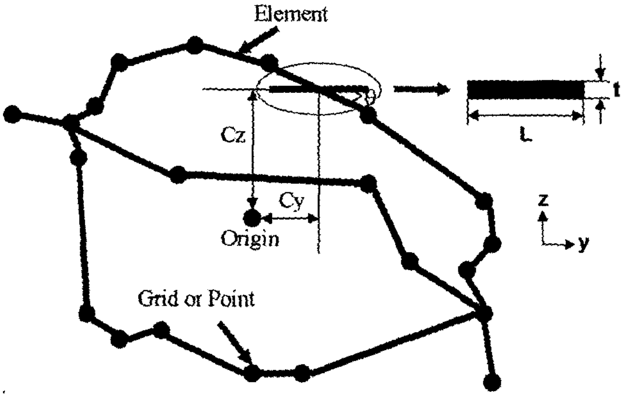

Figure 3. Finite elements for section analysis.

Equation (3) represents the sectional area for the open section and the closed section; Eqs. (4)-(6) represent the moment of inertia for the open section and the closed section; Eq. (7) represents the torsional constant for the open section; Eq. (8) represents the torsional constant for the closed section with single cell; Eq. (9) represents the torsional constant for the closed section with multi cell.

$$A = \sum(Lt) \quad (3)$$

$$I_{yy} = \sum\left\{\frac{Lt^3}{12}\cos^2\theta + \frac{L^3t}{12}\sin^2\theta + tLc_y^2\right\} \quad (4)$$

$$I_{zz} = \sum\left\{\frac{Lt^3}{12}\sin^2\theta + \frac{L^3t}{12}\cos^2\theta + tLc_z^2\right\} \quad (5)$$

$$I_{yz} = \sum\left\{\frac{L^3t - Lt^3}{24}\sin^2 2\theta + tLc_yc_z\right\} \quad (6)$$

$$J_o = \sum\frac{Lt^3}{3} \quad (7)$$

$$J_{cs} = \frac{4\bar{A}^2}{\oint\frac{dL}{t}} \quad (8)$$

$$J_{cm} = \sum 2\bar{A}_j q_j \quad (9)$$

In Eqs. (3)-(9), L, t, and θ respectively denote the length, the thickness and the rotation angle of the each element; C_y and C_z respectively denote the y-direction distance and the z-direction distance from the centroid of the pillar section to the center point of the each element; $\bar{A}$ denotes the area of the closed section; q denotes the shear flow; and j denotes the number of closed section.

The approximation function of a typical section is assumed as polynomials of the design variables. In the approximate functions, polynomials are based on the

Table 1. Parameters for approximate function of section properties.

Section property		Design variable	
		Thickness	Scale vector
Area (A)	Open & single & multi cell section	t_i	SV
Moment of inertia (I_{yy}, I_{zz}, I_{yz})	Open & single & multi cell section	t_i, t_i^3	SV, SV^2
Torsional constant (J)	Open & single cell section	t_i^3	SV, SV^2
	Multi cell section	t_i, t_i^2, t_i^3	SV, SV^2

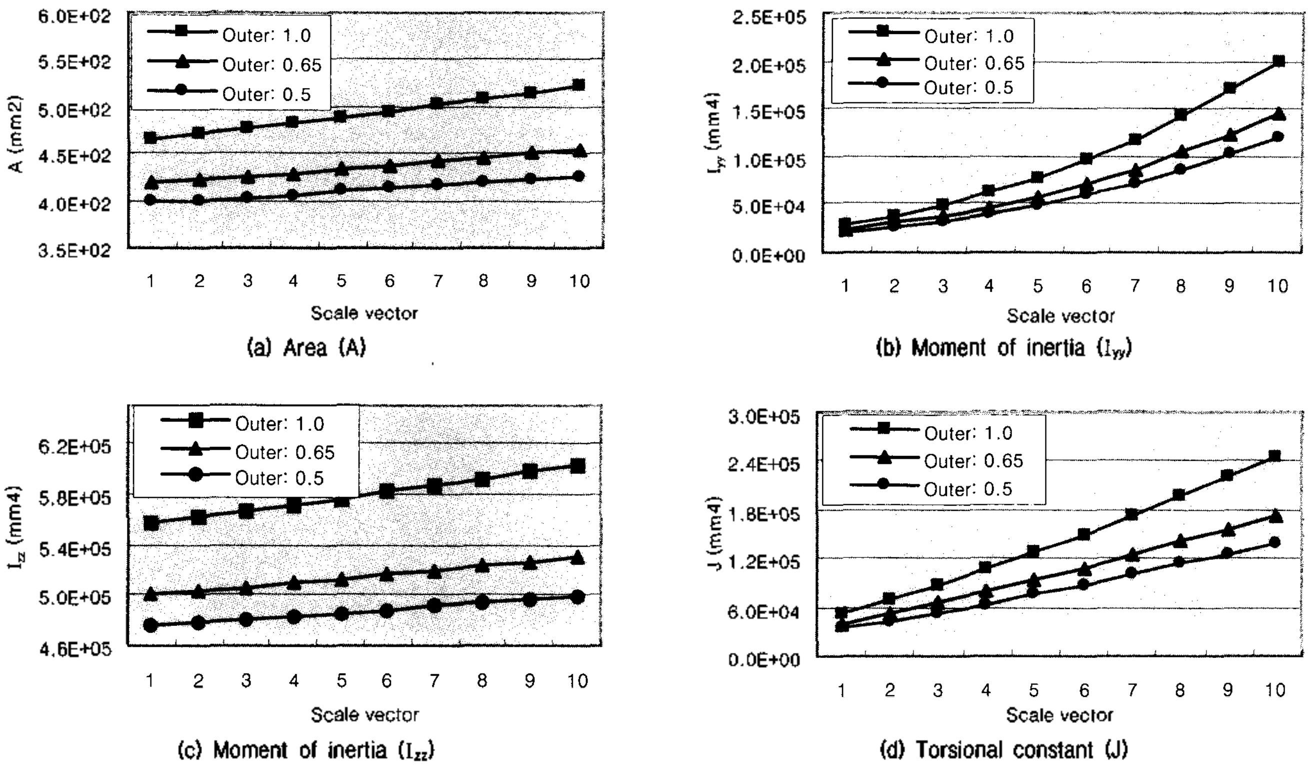

Figure 4. Section properties for various scale vector.

polynomial order of the thickness t and the length L expressed in Eqs. (3)-(9). Table 1 shows the parameters for the approximate function of the section properties. Figure 4 illustrates the variation of the section properties according to the change of the scale vector. Coefficients of the polynomials can be determined using an error minimization method. The errors between the exact values and the values using the approximate function are set up in the objective function. The coefficients of the approximate functions are set up in the design variables, and then the optimal design for error minimization is performed. The coefficients, which are design variables, are determined through the optimization process and the error, which is the objective function, is minimized.

Equation (10) shows the approximate function of the moment of inertia with respect to the y-direction for the

Table 2. Parameter study for the moment of inertia I_{yy}.

No.	Design variable			Exact value (mm^4)	Approx. value (mm^4)	Error (%)
	Inner panel thick. (t_1)	Reinf. panel thick. (t_3)	Scale vector (SV)			
1	0.933	0.733	0.667	2.748E4	2.752E4	-0.15
2	1.367	1.367	1.000	5.125E4	5.085E4	0.77
3	1.367	0.733	1.333	7.449E4	7.315E4	1.80
4	0.933	1.367	0.667	2.903E4	2.879E4	0.81
5	0.933	0.733	1.000	4.385E4	4.482E4	-2.20
6	1.367	1.367	1.333	7.655E4	7.441E4	2.79
7	1.367	0.733	0.667	3.215E4	3.229E4	-0.42
8	0.933	1.367	1.000	4.581E4	4.609E4	-0.60
9	0.933	0.733	1.333	6.716E4	6.838E4	-1.82
10	1.367	1.367	0.667	3.359E4	3.356E4	0.11
11	1.367	0.733	1.000	4.964E4	4.958E4	0.11
12	0.933	1.367	1.333	6.993E4	6.965E4	0.40

section represented in Fig. 2.

$$I_{yy} = (1.099E4)t_1 + (0.000E0)t_1^3 + (2.002E3)t_3 + (0.000E0)t_3^3 + (4.893E3)SV + (2.820E4)SV^2 + (0.000E0) \quad (10)$$

In this equation, t_1 and t_3 respectively denote the thickness of the inner panel and the reinforcement panel. SV denotes the scale vector, and the maximum rotation angle of SV is set up to 90°.

Table 2 shows the results of the parameter study that are calculated with Eq. (4) and Eq. (10), respectively.

III. Development of Optimization System

Figure 5 shows the procedure of the optimization system developed in this study. As shown in Fig. 5, this system consists of pre-processor, solver, and post-processor. MSC/NASTRAN[5] is used to perform the vibration analysis of B.I.W. ADS and DOT is used to perform the optimal design[6-8]. SECOPT[9] is used to calculate the section properties of the pillars.

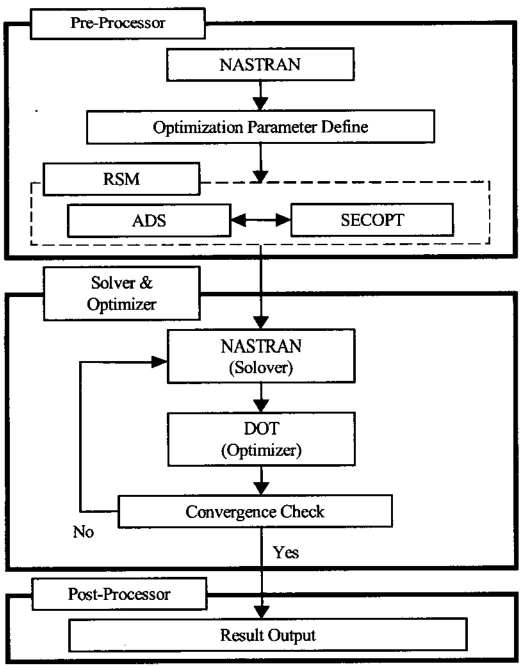

Figure 5. Design optimization procedure.

IV. Application and Estimation of Developed Optimization System

4.1. Application Model

Figure 6 illustrates the B.I.W. finite element model used in this study. As shown in this figure, this vehicle model consists of beam, shell, rigid, and spring elements. Figures 7-8 show the 1st torsional vibration mode and the 1st bending vibration mode, respectively. Table 3 shows the mass and the natural frequencies of this vehicle model before the optimal design.

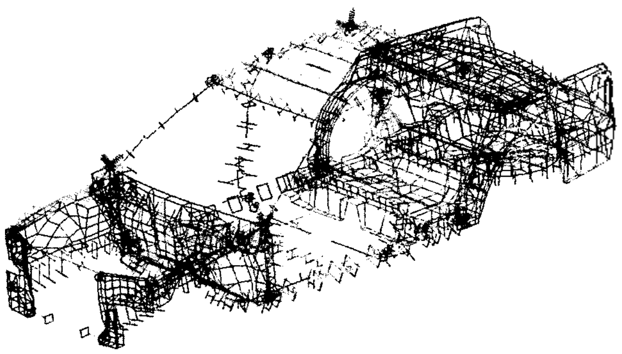

Figure 6. B.I.W. finite element model.

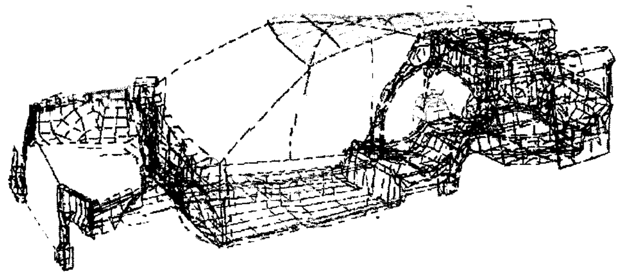

Figure 7. 1st torsional vibration mode.

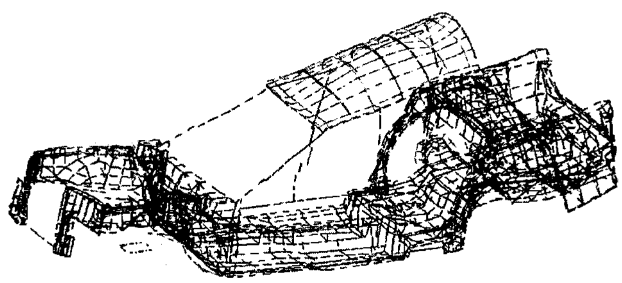

Figure 8. 1st bending vibration mode.

Table 3. Mass and natural frequencies of B.I.W.

Total mass	291.2 kg
1st torsional mode frequency	31.7 Hz
1st bending mode frequency	47.4 Hz

4.2. Determination of Design Variables and Design Constraints

The pillar sections that mainly affect the 1st torsional mode and the 1st bending mode of the vehicle structure are optimized. Table 4 illustrates the design variables and their limits for the pillar sections. The natural frequencies of the 1st torsional mode and the 1st bending mode are set up in the design constraints.

4.3. Results of Optimal Design

Figure 9 and 10 respectively show the objective function history and the design constraint histories. Figures 11-13 illustrate the optimized pillar section shapes. The optimized results of the pillar sections are listed in Table 5.

Table 4. Design variables and their limits.

Section name	Design variable	Variable name	Lower bound	Initial value	Upper bound
A-pillar upper & middle	Inner panel thick.	T_1	0.50	0.75	1.50
	Reinf. panel thick.	T_3	0.10	1.60	2.00
	Scale vector	SV_1	1.00	1.00	1.50
A-pillar lower	Inner panel thick.	T_1	0.50	0.75	1.50
	Reinf. panel thick.	T_3	0.10	1.60	2.00
	Scale vector	SV_1	0.80	1.00	1.50
B-pillar upper	Inner panel thick.	T_1	0.50	1.00	1.80
	Reinf. panel thick.	T_3	0.10	1.40	2.00
	Scale vector	SV_1	0.50	1.00	1.50
B-pillar middle	Inner panel thick.	T_1	0.50	1.00	1.50
	Reinf. panel thick.	T_3	0.10	1.40	2.00
	Scale vector	SV_1	0.80	1.00	1.50
B-pillar lower	Inner panel thick.	T_1	0.50	1.00	1.50
	Reinf. panel thick.	T_3	0.10	1.60	2.00
Body side outer	Inner panel thick.	T_1	0.10	1.20	2.00
	Reinf. panel thick.	T_3	0.10	1.60	2.00
	Scale vector	SV_1	-2.00	1.00	2.00
Rear roof rail	Inner panel thick.	T_1	0.50	0.75	1.50
	Reinf. panel thick.	T_3	0.50	0.75	1.50
	Scale vector	SV_1	0.50	1.00	2.00

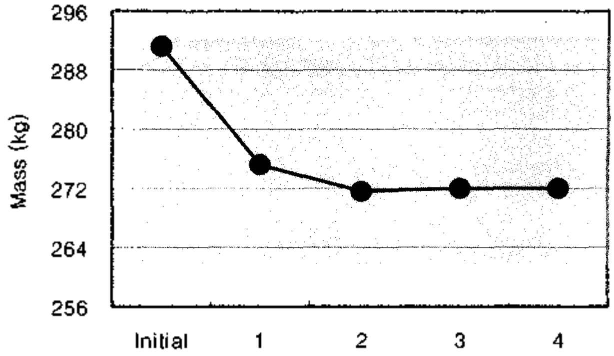

Figure 9. Objective function history.

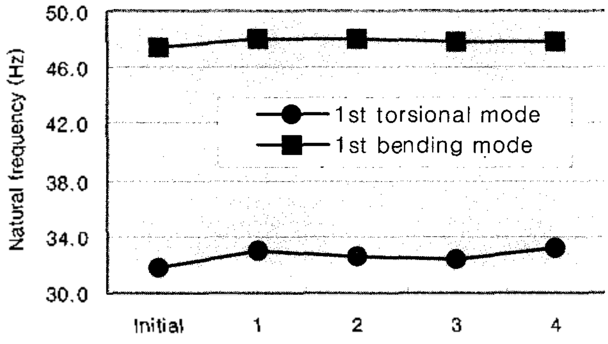

Figure 10. Design constraint histories.

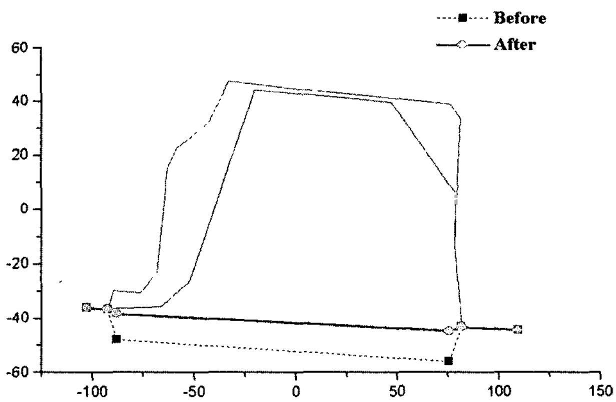

Figure 11. Optimized B-pillar lower section.

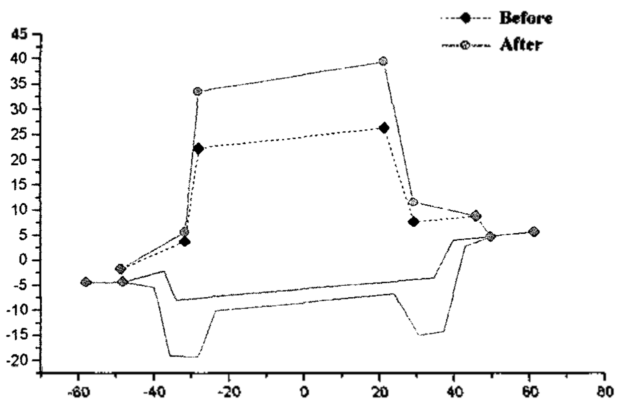

Figure 12. Optimized B-pillar upper section.

Table 5. Optimal design result.

Objective function				
			Initial	Optimum
Total mass (kg)			291.2	271.9
Design constraints				
			Initial	Optimum
1st torsional mode frequency (Hz)			31.7	33.2
1st bending mode frequency (Hz)			47.4	47.9
Design variable				
Section name	Design variable		Initial	Optimum
A-pillar upper & middle	Inner panel thick.	T_1	0.75	0.50
	Reinf. panel thick.	T_3	1.60	0.10
	Scale vector	SV_1	1.00	1.00
A-pillar lower	Inner panel thick.	T_1	0.75	0.50
	Reinf. panel thick.	T_3	1.60	0.10
	Scale vector	SV_1	1.00	0.80
B-pillar upper	Inner panel thick.	T_1	1.00	1.07
	Reinf. panel thick.	T_3	1.40	1.53
	Scale vector	SV_1	1.00	1.50
B-pillar middle	Inner panel thick.	T_1	1.00	1.50
	Reinf. panel thick.	T_3	1.40	0.21
	Scale vector	SV_1	1.00	1.50
B-pillar lower	Inner panel thick.	T_1	1.00	1.50
	Reinf. panel thick.	T_3	1.60	2.00
Body side outer	Inner panel thick.	T_1	1.20	1.71
	Reinf. panel thick.	T_3	1.60	0.10
	Scale vector	SV_1	1.00	1.55
Rear roof rail	Inner panel thick.	T_1	0.75	0.50
	Reinf. panel thick.	T_3	0.75	0.50
	Scale vector	SV_1	1.00	1.27

V. Conclusions

In this paper, a new optimal design technique of thin walled beam structures is present to perform effectively the section design for the pillars of vehicle body structures. Approximate functions for the section properties such as area, area moment of inertia, and torsional constant are derived by the response surface method. The approximate functions are used for the optimization of the vehicle body structure, which consists of the complicated thin walled beam structures. A passenger car body structure is optimized to demonstrate the optimal design technique proposed in this paper. The method can effectively apply to designing the vehicle structure.

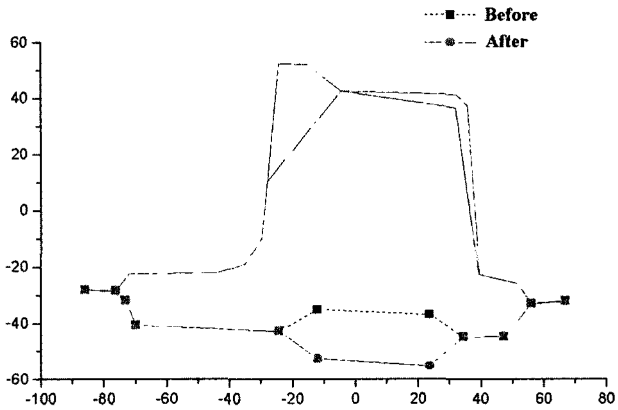

Figure 13. Optimized B-pillar middle section.

Acknowledgements

This work was supported by the Brain Korea 21 Project in 2002.

References

1. M. M. Kamal and J. A. Wolf, *Modern Automotive Structural Analysis*, Van Nostrand Co., 1982.
2. S. B. Lee, "Optimal Design Methods for the Vehicle Structures Using Simple Finite Element Modeling Technique," Ph.D Thesis, Kookmin University, 1996.
3. H. J. Yim and S. B. Lee, "Technology for initial design and analysis of vehicle pillar structures for vibration," *Journal of Korean Society for Noise and Vibration Engineering*, **11** (3), 395-402, 1995.
4. R. T. Haftka and Z. Gürdal, *Elements of Structural Optimization*, 3rd Revised and Expanded Edition, Kluwer Academic Publishers, 209-254, 1992.
5. *MSC/NASTRAN Basic Dynamic Analysis User's Guide*, The Macneal Schwendler Co., 1993.
6. G. J. Moore, *MSC/NASTRAN Design Sensitivity and Optimization User's Guide*, The Macneal Schwendler Co., 1993.
7. G. N. Vanderplaats, *ADS - A Fortran Program for Automated Design Synthesis*, Engineering Design Optimization Inc., 1985.
8. G. N. Vanderplaats, *Numerical Optimization Techniques for Engineering Design*, McGraw-Hill Publishing Co., 1984.
9. H. Y. Lee, "Stiffness Analysis and Optimal Design of Thin Walled Beam Structures," Master Thesis, Kookmin University, 1996.

[Profile]

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