Decentralized Output Feedback Robust Passive Control for Linear Interconnected Uncertain Time-Delay Systems

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Abstract: We consider a class of large-scale interconnected time delay systems and investigate a decentralized robust passive control problem. Sufficient conditions for unforced interconnected uncertain systems with time delay to be robustly stable with extended strictly passivity is given in terms of algebraic Riccati inequality and linear matrix inequality. The decentralized robust passive control problem for norm-bounded and positive real uncertainty is shown to be converted to extended strictly positive real control problem for a modified system which contains neither time delay nor uncertainty.

Keywords: interconnected system, time delay, uncertainty, extended strictly passive, extended strictly positive real

I. Introduction

Decentralized control of large-scale interconnected systems has been an important topic more than two decades(see [1,3,7,9,10] and references therein). Much concern for the decentralized control comes from the fact that many control problems of modern industrial society are associated with the control of complex interconnected systems such as electric power systems, transportation systems, chemical process systems etc. The concept of decentralized control is that control inputs for local systems use local measurement only such that the stability and performance of the whole interconnected system should be guaranteed. During the last decade, considerable attention has been paid to stability analysis and control problem for time delay systems(see references of [12]), and recently more attention has been devoted to uncertain systems with time delay. This trend also has been applied to the decentralized control problem. Many research has been done on decentralized control problem for interconnected systems which contain time delay[3], uncertainty[1,10], and both time delay and uncertainty[9]. The objective of decentralized robust control problem is to obtain closed-loop stability in spite of uncertainty and time delay, and additionally H_∞ norm-boundedness between disturbance and controlled output[9,10]. For systems with time delay and uncertainty, there are some research for positive real (or passivity) control. Xie et. al.[11] focuses on positive real control of linear time-invariant systems with norm-bounded uncertainty and shows that the solution can be obtained by solving a scaled strict positive real control problem. Mahmoud[4] investigate the robust passivity synthesis problem for a class of uncertain time-delay systems and provides a sufficient condition for the uncertain time delay system to be robustly stable and strictly passive for all uncertainty in terms of linear matrix inequality.

In this paper we consider linear interconnected uncertain systems with time delay. The uncertainty may be norm-bounded or of linear fractional form of positive real uncertainty. The objective of the problem is to design output feedback decentralized controller such that the whole interconnected system is robustly stable and the system from the disturbance to controlled output is extended strictly passive. A sufficient condition for unforced interconnected system to be robustly stable with extended strictly passivity is given in terms of algebraic Riccati inequality and linear matrix inequality. The decentralized robust passive control problem is shown to be converted to extended strictly positive real control problem for a modified system which contains neither time delay nor uncertainty.

The concept of passivity or positive realness has played important role in system stability and control theory. The main motivation of studying the passivity or positive realness comes from robust and nonlinear control problem. It is well-known that closed loop stability is guaranteed by means of negative feedback with strictly passive compensation for passive plant even though the plant has uncertainty or nonlinearity.

In Section II, decentralized robust passive control problem is formulated. Analysis result is given in Section III, and decentralized robust stabilization for norm-bounded uncertainty and linear fractional form of positive real uncertainty is given in Section IV and V respectively. Conclusions are given in Section VI.

II. Problem formulation and definitions

Consider large-scale linear systems consisting of N subsystems containing time-varying uncertainty and unknown time delay as follows:

$$\begin{aligned} \dot{x}_i(t) &= (A_i + \Delta A_i(t))x_i(t) + B_{1i}w_i(t) + (B_{2i} + \Delta B_{2i}(t))u_i(t) \\ &\quad + \sum_{j=1, j\neq i}^{N} (A_{ij} + \Delta A_{ij}(t))x_j(t - \tau_{ij}) \qquad (1) \\ z_i(t) &= C_{1i}x_i(t) + D_{11i}w_i(t) + D_{12i}u_i(t) \\ y_i(t) &= (C_{2i} + \Delta C_{2i}(t))x_i(t) \\ &\quad + D_{21i}w_i(t) + (D_{22i} + \Delta D_{22i}(t))u_i(t) x_i(t) = \phi_i(t), \quad t \le 0 \end{aligned}$$

where for i=1,2,...N, $x_i(t) \in R^{n_i}$ is the state, $w_i(t) \in R^{p_i}$ is the disturbance, $u_i(t) \in R^{m_i}$ is the control input, and $A_i, B_{1i}, B_{2i}, C_{1i}, C_{2i}, D_{11i}, D_{12i}, D_{21i}, D_{22i}$ and A_{ij} are known constant matrices of appropriate dimensions, τ_{ij} is time delay from j-th subsystem to i-th subsystem which is unknown but constant. Assume that the parameter uncertainties $\Delta A_i(t), \Delta B_{2i}(t), \Delta C_{2i}(t), \Delta D_{22i}(t)$ and $\Delta A_{ij}(t)$ have the following forms:

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$$\begin{bmatrix} \Delta A_i(t) & \Delta B_{2i}(t) \\ \Delta C_{2i}(t) & \Delta D_{22i}(t) \end{bmatrix} = \begin{bmatrix} L_{1i} \\ L_{2i} \end{bmatrix} F_i(t) \begin{bmatrix} E_{1i} & E_{2i} \end{bmatrix} \quad (2a)$$

$$\Delta A_{ij}(t) = H_{ij} F_{ij}(t) J_{ij} \quad (2b)$$

where $L_{1i}, L_{2i}, E_{1i}, E_{2i}, H_{ij}$ and J_{ij} are known constant matrices of appropriate dimensions. For the time-varying uncertainty matrices $F_i(t)$ and $F_{ij}(t)$, which are Lebesgue measurable, we consider two cases of uncertainties as follows:

$$\text{i)} \quad F_i^T(t)F_i(t) \le I, \; F_{ij}^T(t)F_{ij}(t) \le I, \; \forall t, \forall i, j \in \{1,2,\ldots,N\} \quad (3)$$

$$\text{ii)} \quad F_i(t) = \overline{F_i}(t)(I + D_i\overline{F_i}(t))^{-1}, \; F_{ij}(t) = \overline{F_{ij}}(t)(I + D_{ij}\overline{F_{ij}}(t))^{-1} \quad (4)$$

where $\overline{F_i}(t) + \overline{F_i}^T(t) \ge 0$, $\overline{F_{ij}}(t) + \overline{F_{ij}}^T(t) \ge 0$, $\forall t, \forall i, j \in \{1,2,\ldots,N\}$ and D_i and D_{ij} are constant known matrices with $D_i + D_i^T > 0$, $D_{ij} + D_{ij}^T > 0$.

We will consider uncertainty (3) in Section IV and uncertainty (4) in Section V, respectively.

The decentralized robust passive control problem we consider in this paper can be described as follows:

Design a decentralized linear output feedback controller $G_C(s) = diag\{G_{C_1}, G_{C_2}, \ldots, G_{C_N}\}$ for system (1) with uncertainty (2) such that with $u_i = G_{C_i}(s)y_i$, i=1,2,...,N, the resulting closed-loop system is robustly stable and the system $T_{z_i w_i}$ from w_i to z_i is extended strictly passive for any nonzero $w_i \in L_2[0,\infty)$ and for all admissible uncertainties.

The notion of extended strictly passive and extended strictly positive real is defined as follows.

Definition 1: A dynamic system is said to be passive if

$$\int_0^\infty z^T(t)w(t)dt > \beta, \qquad \forall w \in L_2[0,\infty)$$

where z(t) and w(t) are the output and input of the system respectively and β is some constant. A system is said to be **extended strictly passive(ESP)** if it is passive and $D + D^T > 0$ where D is the feedthrough matrix from w to z.

Definition 2[8]: A system is said to be **extended strictly positive real(ESPR)** if its transfer function G(s) is analytic in $\mathrm{Re}(s) \ge 0$ and satisfies $G(j\omega) + G^*(j\omega) > 0$ for $\omega \in [0,\infty]$.

We introduce some matrix inequalities which will be used to obtain main results.

Lemma 1[10] : Let L, F, E and G be real matrices of appropriate dimensions with $F^T F \le I$. Then

(i) For any scalar $\varepsilon > 0$,

$LFE + E^T F^T L^T \le \varepsilon LL^T + \varepsilon^{-1} E^T E$

(ii) For any scalar $\varepsilon > 0$ such that $\varepsilon E^T E < I$,

$(G + HFJ)(G + HFJ)^T \le G(I - \varepsilon J^T J)^{-1} G^T + \varepsilon^{-1} HH^T$.

The following lemma gives an equivalent condition for **ESPR**ness of an LTI system in terms of **algebraic Riccati inequality(ARI)** and **linear matrix ineqality(LMI) conditions.**

Lemma 2[8] : Consider the linear time-invariant system $\Sigma: \; \dot{x} = Ax + Bw, z = Cx + Dw$.

The following statements are equivalent.

i) The system Σ is **ESPR** and A is stable.

ii) $D + D^T > 0$ and the **ARI** $A^T P + PA + (C - B^T P)^T (D + D^T)^{-1} (C - B^T P) < 0$ has a positive definite solution P.

iii) $D + D^T > 0$ and the **LMI** $\begin{bmatrix} A^T P + PA & (C - B^T P)^T \\ (C - B^T P) & -(D + D^T) \end{bmatrix} < 0$ has a positive definite solution P.

III. Robust stabilization with extended strictly passivity

In this section we consider the analysis result of the decentralized robust passive control problem for system (1) with uncertainty (2) and (3).

Consider partially interconnected system in which only some parts of states in a subsystem affects other subsystem. For example, when we use decentralized output feedback control, the original states of a subsystem affect other subsystem, but the states of controllers do not. We call the states which affect other subsystem active states, the states which do not affect other subsystem passive states. For convenience, we decompose the states $x_i(t) \in R^{n_i}$ of i-th subsystem into active states $x_{i_1}(t) \in R^{n_{i_1}}$ and passive states $x_{i_2}(t) \in R^{n_{i_2}}$ with $x_i(t) = \begin{bmatrix} x_{i_1}^T & x_{i_2}^T \end{bmatrix}^T$ and $n_{i_1} + n_{i_2} = n_i$.

Consider the following partially interconnected unforced system described as (5):

$$\dot{x}_i(t) = A_i x_i(t) + B_{1i} w_i(t) + \sum_{j=1, j\ne i}^{N} A_{ij} x_{j_1}(t - \tau_{ij}) \quad (5)$$
$$z_i(t) = C_{1i} x_i(t) + D_{11i} w_i(t)$$

The following theorem provides sufficient **ARI** conditions for the partially interconnected system (5) to be asymptotically stable with **ESP**.

Theorem 1 : Consider the interconnected system (5). Suppose that $D_{11i} + D_{11i}^T > 0$ and if the matrices A_i, for i=1,2,...N, are stable and there exist real symmetric matrices $P_i > 0$ such that for i=1,2,...N,

$$A_i^T P_i + P_i A_i + P_i\Big(\sum_{j=1, j\ne i}^{N} A_{ij} A_{ij}^T\Big) P_i + (C_{1i} - B_{1i}^T P_i)^T (D_{11i} + D_{11i}^T)^{-1}$$
$$\bullet (C_{1i} - B_{1i}^T P_i) + (N-1) \begin{bmatrix} I_{i_1} & 0 \\ 0 & 0_{i_2} \end{bmatrix} < 0 \quad (6)$$

where $I_{i_1} \in R^{n_{i_1} \times n_{i_1}}$ is an identity matrix, $0_{i_2} \in R^{n_{i_2} \times n_{i_2}}$ is a zero matrix, and N is the number of subsystems, then the interconnected system (5) is asymptotically stable with ESP.

Proof : To prove asymptotic stability of the interconnected system (5), we assume $w_i(t) \equiv 0$. Let the Lyapunov function of interconnected system be

$$V(x,t) = \sum_{i=1}^{N} x_i^T(t) P_i x_i(t) + \sum_{i=1}^{N} \sum_{j=1, j\ne i}^{N} \int_{t-\tau_{ij}}^{t} x_{j_1}^T(\sigma) x_{j_1}(\sigma) d\sigma$$

We obtain the derivative of V(x,t) with respect to time as follows.

$$\dot{V}(x,t)=\sum_{i=1}^{N} x_i^T(t)(A_i^T P_i+P_iA_i)x_i$$
$$+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\{x_{j_1}^T(t-\tau_{ij})A_{ij}^T P_i x_i(t)+x_i^T(t)P_i A_{ij}x_{j_1}(t-\tau_{ij})\}$$
$$+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\{x_{j_1}^T(t)x_{j_1}(t)-x_{j_1}^T(t-\tau_{ij})x_{j_1}(t-\tau_{ij})\}$$
$$<-\sum_{i=1}^{N} x_i^T(t)(C_{1i}-B_{1i}^T P_i)^T(D_{11i}+D_{11i}{}^T)^{-1}(C_{1i}-B_{1i}^T P_i)x_i(t)$$
$$-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(x_{j_1}(t-\tau_{ij})-A_{ij}^T P_i x_i(t))^T(x_{j_1}(t-\tau_{ij})-A_{ij}^T P_i x_i(t))\le 0$$

The inequality above comes from ARI (6) and $\dot{V}(x,t)<0$ holds always for all $x_i(t)\neq 0$. Thus the partially interconnected system (5) is asymptotically stable by the well- known Lyapunov stability theorem.

To prove the **ESP**ness of interconnected system (5), let $J=\int_0^\infty 2z^T(t)w(t)dt$. Under zero initial condition, we obtain after some manipulation using ARI (6)

$$J=\sum_{i=1}^{N}\int_0^\infty 2z_i^T(t)w_i(t)dt$$
$$=\sum_{i=1}^{N}\int_0^\infty\{2z_i^T(t)w_i(t)-\frac{d}{dt}(x_i^T(t)P_ix_i(t)\}dt$$
$$>\sum_{i=1}^{N}\int_0^\infty\{\eta_i^T(t)(D_{11i}+D_{11i}^T)\eta_i(t)+\sum_{i=1}^{N}\xi^T(t)\xi(t)\}dt>0$$

where $\eta_i(t)=w_i(t)+(D_{11i}+D_{11i}^T)^{-1}(C_{1i}-B_{1i}^T P_i)x_i(t)$, $\xi_i(t)=x_{j_1}(t-\tau_{ij})-A_{ij}^T P_i x_i(t)$.

We have $D_{11i}+D_{11i}^T>0$ from the assumption, thus the interconnected system (5) is asymptotically stable with ESP by the Definition 2. ■

Remark 1 : The condition that there exists a real symmetric matrix $P_i>0$ satisfying ARI (6) is equivalent to the **ESPR**ness of the following system:

$$\dot{x}=F_ix+G_iv$$
$$z=H_ix+J_iv$$

where $F_i=A_i, G_i=\begin{bmatrix}B_{1i} & B_i & 0\end{bmatrix}, B_iB_i^T=\sum_{j=1,j\neq i}^{N}A_{ij}A_{ij}^T$,

$$J_i=\begin{bmatrix}D_{11i} & 0 & 0\\ 0 & \frac{1}{2}I & 0\\ 0 & 0 & \frac{1}{2}I\end{bmatrix}\text{ and } H_i=\begin{bmatrix}C_{1i}\\ 0\\ \sqrt{N-1}\begin{bmatrix}I_{i_1} & 0\end{bmatrix}\end{bmatrix}.$$

1. ARI condition

Consider the following partially interconnected uncertain system as follows:

$$\dot{x}_i(t)=(A_i+L_{1i}F_iE_{1i})x_i(t)+B_{1i}w_i(t)+\sum_{j=1,j\neq i}^{N}(A_{ij}+H_{ij}F_{ij}J_{ij})x_{j_1}(t-\tau_{ij})$$
$$z_i(t)=C_{1i}x_i(t)+D_{11i}w_i(t) \tag{7}$$

where uncertain matrices F_i and F_{ij} are norm-bounded, i.e., $F_i^TF_i\le I$, $F_{ij}^TF_{ij}\le I$. According to the results of Theorem 1 and Lemma 1, we obtain the sufficient condition for the interconnected uncertain system (7) to be robustly stable with **ESP**.

Theorem 2 : Consider the interconnected uncertain system (7). If the matrices A_i, for i=1,2,…,N, are stable and conditions (i) through (iii) below are satisfied, then system (7) is robustly stable with **ESP**.

i) $D_{11i}+D_{11i}^T>0$

ii) $\lambda_{ij}^2J_{ij}^TJ_{ij}<I$

iii) there exist real symmetric matrices $P_i>0$ and parameters $\varepsilon_i>0$ and $\lambda_{ij}>0$ such that for i=1,2,…,N,

$$A_i^TP_i+P_iA_i+P_i\{\varepsilon_i^2L_{1i}L_{1i}^T+\sum_{j=1,j\neq i}^{N}(A_{ij}(I-\lambda_{ij}^2J_{ij}^TJ_{ij})^{-1}A_{ij}^T$$
$$+\frac{1}{\lambda_{ij}^2}H_{ij}H_{ij}^T)\}P_i+(C_{1i}-B_{1i}^TP_i)^T(D_{11i}+D_{11i}^T)^{-1}(C_{1i}-B_{1i}^TP_i)+\frac{1}{\varepsilon_i^2}E_{1i}^TE$$
$$+(N-1)\begin{bmatrix}I_{i_1} & 0\\ 0 & 0_{i_2}\end{bmatrix}<0 \tag{8}$$

Proof : According to Theorem 1, a sufficient condition for system (7) to be robustly stable with ESP is that there exist real symmetric matrices $P_i>0$ such that for i=1,2,…,N,

$$(A_i+L_{1i}F_iE_{1i})^TP_i+P_i(A_i+L_{1i}F_iE_{1i})+P_i$$
$$\{\sum_{j=1,j\neq i}^{N}(A_{ij}+H_{ij}F_{ij}J_{ij})(A_{ij}+H_{ij}F_{ij}J_{ij})^T\}$$
$$+(C_{1i}-B_{1i}^TP_i)^T(D_{11i}+D_{11i}^T)^{-1}(C_{1i}-B_{1i}^TP_i)+(N-1)\begin{bmatrix}I_{i_1} & 0\\ 0 & 0_{i_2}\end{bmatrix}<0 \tag{9}$$

From Lemma 1 we obtain the following inequalities

$$E_{1i}^TF_i^TL_{1i}^TP_i+P_iL_{1i}F_iE_{1i}\le\varepsilon_i^2P_iL_{1i}L_{1i}^TP_i+\frac{1}{\varepsilon_i^2}E_{1i}^TE_{1i} \tag{10}$$

$$(A_{ij}+H_{ij}F_{ij}J_{ij})(A_{ij}+H_{ij}F_{ij}J_{ij})^T\le A_{ij}(I-\lambda_{ij}^2J_{ij}^TJ_{ij})^{-1}$$
$$A_{ij}^T+\frac{1}{\lambda_{ij}^2}H_{ij}H_{ij}^T \tag{11}$$

Applying (10) and (11) to (9), we obtain that

Left side of ARI (9) ≤ Left side of ARI (8) < 0. This completes the proof. ■

Remark 2 : The condition that there exists a real symmetric matrix $P_i>0$ such that satisfies ARI (8) is equivalent to the ESPRness of the following system

$$\dot{\xi}_i=F_i\xi_i+\overline{G_i}v_i$$
$$z_i=\overline{H_i}\xi_i+\overline{J_i}v_i$$

where,

$$F_i = A_i, \overline{G_i} = \begin{bmatrix} B_{1i} & \varepsilon_i L_{1i} & \overline{B}_i & 0 & 0 \end{bmatrix}, \overline{H_i} = \begin{bmatrix} C_{1i} \\ 0 \\ 0 \\ \frac{1}{\varepsilon_i} E_{1i} \\ \sqrt{N-1}\begin{bmatrix} I_{i_1} & 0 \end{bmatrix} \end{bmatrix},$$

$$\overline{J_i} = diag\{D_{11i}, \frac{1}{2}I, \frac{1}{2}I, \frac{1}{2}I, \frac{1}{2}I\}$$ and

$$\overline{B_i B_i}^T = \sum_{j=1, j\neq i}^{N} (A_{ij}(I - \lambda_{ij}^2 J_{ij}^T J_{ij})^{-1} A_{ij}^T + \frac{1}{\lambda_{ij}^2} H_{ij} H_{ij}^T)$$

2. LMI condition

Consider the partially interconnected uncertain system (7) with norm-bounded uncertainty $F_i^T F_i \le I$, $F_{ij}^T F_{ij} \le I$. In this sub-section we obtain the sufficient condition for the interconnected uncertain system (7) to be robustly stable with **ESP** using **LMI** technique.

Theorem 3 : Consider the interconnected uncertain system (7). If the matrices A_i, for i=1,2,...,N, are stable there exist real symmetric matrices $P_i > 0$ and real parameters $\varepsilon_i > 0$ satisfying the LMI below, then system (7) is robustly stable with **ESP**.

$$\begin{bmatrix} \Omega_1 & \Omega_2 \\ \Omega_2^T & \Omega_3 \end{bmatrix} < 0 \qquad (12)$$

where $\Omega_1 = A_i^T P_i + P_i A_i$,

$$\Omega_2 = \begin{bmatrix} (C_i - B_{1i}^T P_i)^T & \varepsilon_i P_i \begin{bmatrix} L_{1i} & \overline{H_i} \end{bmatrix} & P_i \overline{A_i} & \frac{1}{\varepsilon_i} E_{1i} \sqrt{N-1}\begin{bmatrix} I \\ 0 \end{bmatrix} \end{bmatrix}$$

$$\Omega_3 = diag\{-(D_{11i} + D_{11i}^T), -I, -I + \frac{1}{\varepsilon_i^2} \overline{J_i J_i}^T, -I, -I\}$$

$$\overline{A_i} = [A_{i1} A_{i2} \cdots A_{i(i-1)} A_{i(i+1)} \cdots A_{iN}]$$

$$\overline{H_i} = [H_{i1} H_{i2} \cdots H_{i(i-1)} H_{i(i+1)} \cdots H_{iN}]$$

$$\overline{J_i} = diag\{J_{i1}, J_{i2}, \cdots, J_{i(i-1)}, J_{i(i+1)}, \cdots, J_{iN}\}$$

Proof : According to Theorem 1, a sufficient condition for system (7) to be robustly stable with ESP is the existence of real symmetric matrices $P_i > 0$, i=1,2,...,N, satisfying the ARI (9), which is equivalent to the following LMI,

$$\Lambda = \begin{bmatrix} A_i^T P_i + P_i A_i + (N-1)\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \Gamma_1 & P_i \overline{A}_i + \Gamma_2 & (C_i - B_i^T P_i)^T \\ \overline{A}_i^T P_i + \Gamma_2^T & -I & 0 \\ (C_i - B_i^T P_i) & 0 & -(D_{11i} + D_{11i}^T) \end{bmatrix} < 0$$

where $\Gamma_1 = E_{1i}^T F_i^T L_{1i}^T P_i + P_i L_{1i} F_i E_{1i}$ and $\Gamma_2 = P_i \begin{bmatrix} L_{i1} F_{i1} E_{i1} & \cdots & L_{iN} F_{iN} E_{iN} \end{bmatrix}$

Matrix Λ can be decomposed into two parts, one having no uncertainty and the other having uncertainty, $\Lambda = \Lambda_1 + \Lambda_2$ where

$$\Lambda_1 = \begin{bmatrix} A_i^T P_i + P_i A_i + (N-1)\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} & P_i \overline{A_i} & (C_i - B_i^T P_i)^T \\ \overline{A_i}^T P_i & -I & 0 \\ (C_i - B_i^T P_i) & 0 & -(D_{11i} + D_{11i}^T) \end{bmatrix}$$ and

$$\Lambda_2 = \begin{bmatrix} \Gamma_1 & \Gamma_2 & 0 \\ \Gamma_2^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Substituting Γ_1 and Γ_2 in Λ_2, we obtain from Lemma 1,

$$\Lambda_2 = \begin{bmatrix} P_i \begin{bmatrix} L_{1i} & \overline{L_i} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_i & 0 \\ 0 & \overline{F_i} \end{bmatrix} \begin{bmatrix} E_{1i} & 0 \\ 0 & \overline{E_i} \end{bmatrix} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} P_i \begin{bmatrix} L_{1i} & \overline{L_i} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_i & 0 \\ 0 & \overline{F_i} \end{bmatrix} \begin{bmatrix} E_{1i} & 0 \\ 0 & \overline{E_i} \end{bmatrix} & 0 \\ 0 & 0 \end{bmatrix}^T$$

$$\le \begin{bmatrix} \varepsilon_i P_i \begin{bmatrix} L_{1i} & \overline{L_i} \end{bmatrix} & \frac{1}{\varepsilon_i} E_{1i}^T & 0 \\ 0 & 0 & \frac{1}{\varepsilon_i} \overline{E_i} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \varepsilon_i P_i \begin{bmatrix} L_{1i} & \overline{L_i} \end{bmatrix} & \frac{1}{\varepsilon_i} E_{1i}^T & 0 \\ 0 & 0 & \frac{1}{\varepsilon_i} \overline{E_i} \\ 0 & 0 & 0 \end{bmatrix}^T$$

for some $\varepsilon_i > 0$.

Let $\Lambda_3 = \begin{bmatrix} \varepsilon_i P_i \begin{bmatrix} L_{1i} & \overline{L_i} \end{bmatrix} & \frac{1}{\varepsilon_i} E_{1i}^T & 0 \\ 0 & 0 & \frac{1}{\varepsilon_i} \overline{E_i} \\ 0 & 0 & 0 \end{bmatrix}$ and $\Lambda_4 = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$

If there exist matrices $P_i > 0$ and parameters $\varepsilon_i > 0$ such that $\Lambda_1 + \Lambda_3 \Lambda_4 \Lambda_3^T < 0$, then system (7) is robustly stable with ESP. We can obtain LMI (12) by arranging $\Lambda_1 + \Lambda_3 \Lambda_4 \Lambda_3^T < 0$ and using well-known Schur inequality. ■

IV. Decentralized robust stabilization for norm-bounded uncertainty

In this section we present decentralized robust output feedback controllers for the interconnected system (1) and (2) containing norm-bounded uncertainty (3), $F_i^T(t)F_i(t) \le I$, $F_{ij}^T(t)F_{ij}(t) \le I$, $\forall t, \forall i, j \in \{1,2,...,N\}$.

1. Output feedback control from ARI

Consider the following modified system from system (1) and (2) with uncertainty (3).

$$\dot{\eta}_i = A_i \eta_i + \begin{bmatrix} B_{1i} & \varepsilon_i L_{1i} & \overline{B_i} & 0 & 0 \end{bmatrix} \overline{w_i} + B_{2i} u_i$$

$$z_i = \begin{bmatrix} C_{1i} \\ 0 \\ 0 \\ \frac{1}{\varepsilon_i} E_{1i} \\ \sqrt{N-1} I \end{bmatrix} \eta_i + \begin{bmatrix} D_{11i} & 0 & 0 & 0 & 0 \\ 0 & 0.5I & 0 & 0 & 0 \\ 0 & 0 & 0.5I & 0 & 0 \\ 0 & 0 & 0 & 0.5I & 0 \\ 0 & 0 & 0 & 0 & 0.5I \end{bmatrix} \overline{w_i} + \begin{bmatrix} D_{12i} \\ 0 \\ 0 \\ \frac{1}{\varepsilon_i} E_{2i} \\ 0 \end{bmatrix} u_i \qquad (13)$$

$$y_i = C_{2i} \eta_i + \begin{bmatrix} D_{21i} & \varepsilon_i L_{2i} & 0 & 0 & 0 \end{bmatrix} \overline{w_i} + D_{22i} u_i$$

where $\overline{B_i B_i}^T = \sum_{j=1, j\neq i}^{N} (A_{ij}(I - \lambda_{ij}^2 J_{ij}^T J_{ij})^{-1} A_{ij}^T + \frac{1}{\lambda_{ij}^2} H_{ij} H_{ij}^T)$

The following theorem is one of main results in this paper, which shows the decentralized robust stabilization with **ESP** for interconnected system which contain norm-bounded uncertainty and time delay using **ARI**.

Theorem 4 Consider the interconnected system (1) with uncertainty (2) and (3). This system is robustly stabilizable with **ESP** via decentralized strictly proper output feedback controller $G_C(s) = diag\{G_{C_1}(s), G_{C_2}(s), ..., G_{C_N}(s)\}$ if for i=1,2,...,N, there exist parameters $\varepsilon_i > 0$ and $\lambda_{ij} > 0$ such that

i) $D_{11i} + D_{11i}^T > 0$

ii) $\lambda_{ij}^2 J_{ij}^T J_{ij} < I$

iii) each closed-loop system of system (13) with control law $u_i = G_{C_i} y_i$ is **ESPR**.

Proof : Suppose that for i=1,2,...,N, the controller from i-th subsystem has the following realization

$$\begin{aligned} \dot{x}_{C_i}(t) &= A_{C_i} x_{C_i}(t) + B_{C_i} y_i(t) \\ u_i(t) &= C_{C_i} x_{C_i}(t) \end{aligned} \tag{14}$$

where $x_{C_i} \in R^{n_{C_i}}$ is the state of the controller of the i-th subsystem.

The closed-loop system of the i-th subsystem (1) and (2) with the controller (14) is given by the following equation

$$\begin{aligned} \dot{\xi}_i(t) &= (\hat{A}_i + \overline{L_{1i}} F_i \overline{E_{1i}}) \xi_i(t) + \overline{B_{1i}} w_i(t) \\ &+ \sum_{j=1, j\neq i}^{N} (\overline{A_{ij}} + \overline{H_{ij}} F_{ij} \overline{J_{ij}}) \xi_{j1}(t - \tau_{ij}) \end{aligned} \tag{15}$$

$$z_i(t) = \overline{C_{1i}} \xi_i(t) + D_{11i} w_i(t)$$

where

$$\xi_i = \begin{bmatrix} x_i(t) \\ x_{C_i}(t) \end{bmatrix} \in R^{n_i + n_{C_i}}, \ \xi_{j1} = x_j(t),$$

$$\hat{A}_i = \begin{bmatrix} A_i & B_{2i} C_{C_i} \\ B_{C_i} C_{2i} & A_{C_i} + B_{C_i} D_{22i} C_{C_i} \end{bmatrix}, \ \overline{L_{1i}} = \begin{bmatrix} L_{1i} \\ B_{C_i} L_{2i} \end{bmatrix},$$

$$\overline{E_{1i}} = \begin{bmatrix} E_{1i} & E_{2i} C_{C_i} \end{bmatrix}, \ \overline{B_{1i}} = \begin{bmatrix} B_{1i} \\ B_{C_i} D_{21i} \end{bmatrix}, \ \overline{A_{ij}} = \begin{bmatrix} A_{ij} \\ 0 \end{bmatrix},$$

$$\overline{H_{ij}} = \begin{bmatrix} H_{ij} \\ 0 \end{bmatrix}, \ \overline{J_{ij}} = J_{ij}, \ \overline{C_{1i}} = \begin{bmatrix} C_{1i} & D_{12i} C_{C_i} \end{bmatrix}$$

Consider the closed-loop system of modified system (13) for i-th subsystem with the controller (14), which can be described by the following equation.

$$\begin{aligned} \dot{x}_i &= \hat{A}_i x_i + \hat{B}_i w_i \\ z_i &= \hat{C}_i x_i + \hat{D}_i w_i \end{aligned} \tag{16}$$

where $\hat{B}_i = \begin{bmatrix} B_{1i} & \varepsilon_i L_{1i} & \overline{B_i} & 0 & 0 \\ B_{C_i} D_{21i} & \varepsilon_i B_{C_i} L_{2i} & 0 & 0 & 0 \end{bmatrix}$,

$$\hat{C}_i = \begin{bmatrix} C_{1i} & D_{12i} C_{C_i} \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\varepsilon_i} E_{1i} & \frac{1}{\varepsilon_i} E_{2i} C_{C_i} \\ \sqrt{N-1} I & 0 \end{bmatrix},$$

$\hat{D}_i = diag\{D_{11i}, \frac{1}{2} I, \frac{1}{2} I, \frac{1}{2} I, \frac{1}{2} I\}$ and

$$\overline{B_i}\,\overline{B_i}^T = \sum_{j=1, j\neq i}^{N} (A_{ij}(I - \lambda_{ij}^2 J_{ij}^T J_{ij})^{-1} A_{ij}^T + \frac{1}{\lambda_{ij}^2} H_{ij} H_{ij}^T)$$

Since system (16) is **ESPR**, we obtain from Lemma 2 that $\hat{D}_i + \hat{D}_i^T > 0$ and there exists a symmetric matrix $\overline{P_i} > 0$ such that

$$\hat{A}_i^T \overline{P_i} + \overline{P_i} \hat{A}_i + (\hat{C}_i - \hat{B}_i^T \overline{P_i})^T (\hat{D}_i + \hat{D}_i^T)^{-1} (\hat{C}_i - \hat{B}_i^T \overline{P_i}) < 0 \tag{17}$$

ARI (17) can be rearranged as follows

$$\begin{aligned} &\hat{A}_i^T \overline{P_i} + \overline{P_i} \hat{A}_i + \overline{P_i} \widetilde{G}_i \widetilde{G}_i^T \overline{P_i} \\ &+ (\overline{C_{1i}} - \overline{B}_{1i}^T \overline{P}_i)^T (D_{11i} + D_{11i}^T)^{-1} (\overline{C_{1i}} - \overline{B}_{1i}^T \overline{P}_i) \\ &+ \frac{1}{\varepsilon_i^2} \overline{E}_{1i}^T \overline{E}_{1i} + (N-1) \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0_{n_{C_i}} \end{bmatrix} < 0 \end{aligned} \tag{18}$$

where

$$\widetilde{G}_i \widetilde{G}_i^T = \varepsilon_i^2 \overline{L}_{1i} \overline{L}_{1i}^T + \sum_{j=1, j\neq i}^{N} (\overline{A}_{ij} (I - \lambda_{ij}^2 \overline{J}_{ij}^T \overline{J}_{ij})^{-1} \overline{A}_{ij}^T + \frac{1}{\lambda_{ij}^2} \overline{H}_{ij} \overline{H}_{ij}^T)$$

Note that $D_{11i} + D_{11i}^T > 0$ and from condition ii) $\lambda_{ij}^2 J_{ij}^T J_{ij} < I$ we obtain that $\lambda_{ij}^2 \overline{J}_{ij}^T \overline{J}_{ij} = \lambda_{ij}^2 \begin{bmatrix} J_{ij}^T J_{ij} & 0 \\ 0 & 0 \end{bmatrix} < I$

According to ARI (18) and Theorem 2, the interconnected system (15) is robustly stable with ESP. Therefore the interconnected system (1) with uncertainty (2) and (3) is robustly stabilizable with ESP via decentralized strictly proper output feedback controller which is used for the augmented system (13) such that the given closed-loop system is ESPR. ■

Remark 3 : Theorem 4 says that the decentralized robust passive control problem for interconnected system with uncertainty (2) and (3) can be solved in terms of ESPR control problem for N modified decoupled linear systems which do not contain uncertainty nor time delay. Refer to [8] for the solution to the ESPR control problem and [5] for removing the " D_{22} " term.

2. Output feedback control from LMI

Consider the following modified system from system (1) and (2) with uncertainty (3).

$$\begin{aligned} \dot{\eta}_i &= A_i \eta_i + \begin{bmatrix} B_{1i} & \varepsilon_i \begin{bmatrix} L_{1i} & \overline{H}_i \end{bmatrix} & \overline{A_i} & 0 & 0 \end{bmatrix} \overline{v_i} + B_{2i} u_i \\ z_i &= \begin{bmatrix} C_{1i} \\ 0 \\ 0 \\ \frac{1}{\varepsilon_i} E_{1i} \\ \sqrt{N-1} I \end{bmatrix} \eta_i + \begin{bmatrix} D_{11i} & 0 & 0 & 0 & 0 \\ 0 & 0.5I & 0 & 0 & 0 \\ 0 & 0 & 0.5(I - \frac{1}{\varepsilon_i^2} \overline{J}_i \overline{J}_i^T) & 0 & 0 \\ 0 & 0 & 0 & 0.5I & 0 \\ 0 & 0 & 0 & 0 & 0.5I \end{bmatrix} \overline{v_i} \\ &+ \begin{bmatrix} D_{12i} \\ 0 \\ 0 \\ \frac{1}{\varepsilon_i} E_{2i} \\ 0 \end{bmatrix} u_i \\ y_i &= C_{2i} \eta_i + \begin{bmatrix} D_{21i} & \varepsilon_i \begin{bmatrix} L_{2i} & 0 \end{bmatrix} & 0 & 0 & 0 \end{bmatrix} \overline{v_i} + D_{22i} u_i \end{aligned} \tag{19}$$

where

$$\overline{A_i} = [A_{i1} A_{i2} \cdots A_{i(i-1)} A_{i(i+1)} \cdots A_{iN}],$$

$$\overline{H_i} = [H_{i1} H_{i2} \cdots H_{i(i-1)} H_{i(i+1)} \cdots H_{iN}],$$

$$\overline{J_i} = diag\{J_{i1}, J_{i2}, \cdots, J_{i(i-1)}, J_{i(i+1)}, \cdots, J_{iN}\}$$

The following theorem is one main result of this paper, which shows the decentralized robust stabilization with **ESP** for interconnected system which contain norm-bounded uncertainty and time delay.

Theorem 5 : Consider the interconnected system (1) with uncertainty (2) and (3). This system is robustly stabilizable with ESP via decentralized strictly proper output feedback controller $G_C(s) = diag\{G_{C_1}(s), G_{C_2}(s), \ldots, G_{C_N}(s)\}$ if for i=1,2,...,N, there exist parameters $\varepsilon_i > 0$ such that

i) $D_{11i} + D_{11i}^T > 0$

ii) $\frac{1}{\varepsilon_i^2}\overline{J_i}\overline{J_i}^T < I$

iii) each closed-loop system of system (19) with control law $u_i = G_{C_i} y_i$ is **ESPR**.

Proof : The procedure of the proof is similar to that of Theorem 4 and thus the proof is omitted. ■

Remark 4 : System (13) and system (19) are scaled systems which come from by using ARI and LMI, respectively. They have similar forms and system (19) has larger dimension than system (19).

V. Decentralized robust stabilization for positive real uncertainty

In this section we present the decentralized robust output feedback Controller for the interconnected system (1) and (2) containing positive real uncertainty (4),

$$F_i(t) = \overline{F_i}(t)(I + D_i\overline{F_i}(t))^{-1}, \quad F_{ij}(t) = \overline{F_{ij}}(t)(I + D_{ij}\overline{F_{ij}}(t))^{-1}$$

Lemma 3[6] : Consider the uncertainty set $\widetilde{F}$

$$\widetilde{F} := \{\hat{F} = F(I + DF)^{-1} : D + D^T > 0, F + F^T \geq 0, F \in R^{m\times m}\} \tag{20}$$

Then the $\widetilde{F}$ is equivalent to the set $\widetilde{Q}$

$$\widetilde{Q} := \{\hat{Q} = (D + D^T)^{-\frac{1}{2}}(I + Q)(D + D^T)^{-\frac{1}{2}} : Q^T Q \leq I, Q \in R^{m\times m},$$
$$\det(I - (D + D^T)^{-\frac{1}{2}}(I + Q)(D + D^T)^{-\frac{1}{2}}D) \neq 0\} \tag{21}$$

In (20), $\hat{F}$ is also positive real, i.e., $\hat{F} + \hat{F}^T \geq 0$ [2]. Lemma 3 shows that positive real uncertainty in the linear fractional form of positive real uncertainty as in (20) can be transformed to the uncertainty in the form of (21) given in terms of norm-bounded uncertainty.

Consider the interconnected system (1) and uncertainty (2) and (4). Applying Lemma 3, we obtain the following interconnected system which contains norm-bounded uncertainty.

$$\dot{x}_i(t) = (\widetilde{A}_i + \widetilde{L}_{1i}Q_i\widetilde{E}_{1i})x_i(t) + B_{1i}w_i(t) + (\widetilde{B}_{2i} + \widetilde{L}_{1i}Q_i\widetilde{E}_{2i})u_i(t) + \sum_{j=1, j\neq i}^{N}(\widetilde{A}_{ij} + \widetilde{H}_{ij}Q_{ij}\widetilde{J}_{ij})x_{j_1}(t - \tau_{ij}) \tag{22}$$

$$z_i(t) = C_{1i}x_i(t) + D_{11i}w_i(t) + D_{12i}u_i(t)$$

$$y_i(t) = (\widetilde{C}_{2i} + \widetilde{L}_{2i}Q_i\widetilde{E}_{1i})x_i(t) + D_{21i}w_i(t) + (\widetilde{D}_{22i} + \widetilde{L}_{2i}Q_i\widetilde{E}_{2i})u_i(t)$$

where

$$\begin{bmatrix}\widetilde{A}_i & \widetilde{B}_{2i}\\ \widetilde{C}_{2i} & \widetilde{D}_{22i}\end{bmatrix} = \begin{bmatrix}A_i & B_{2i}\\ C_{2i} & D_{22i}\end{bmatrix} + \begin{bmatrix}L_{1i}\\ L_{2i}\end{bmatrix}(D_i + D_i^T)^{-1}\begin{bmatrix}E_{1i} & E_{2i}\end{bmatrix},$$

$$\begin{bmatrix}\widetilde{L}_{1i}\\ \widetilde{L}_{2i}\end{bmatrix} = \begin{bmatrix}L_{1i}\\ L_{2i}\end{bmatrix}(D_i + D_i^T)^{-\frac{1}{2}}, \widetilde{A}_{ij} = A_{ij} + H_{ij}(D_{ij} + D_{ij}^T)^{-1}J_{ij},$$

$$\begin{bmatrix}\widetilde{E}_{1i} & \widetilde{E}_{2i}\end{bmatrix} = (D_i + D_i^T)^{-\frac{1}{2}}\begin{bmatrix}E_{1i} & E_{2i}\end{bmatrix},$$

$$\widetilde{H}_{ij} = H_{ij}(D_{ij} + D_{ij}^T)^{-\frac{1}{2}}, \ \widetilde{J}_{ij} = (D_{ij} + D_{ij}^T)^{-\frac{1}{2}}J_{ij}.$$

Consider the following modified system for system (22).

$$\dot{\varsigma}_i = \widetilde{A}_i\varsigma_i + \begin{bmatrix}B_{1i} & \varepsilon_i\widetilde{L}_{1i} & \widetilde{B}_i & 0 & 0\end{bmatrix}\widetilde{w}_i + \widetilde{B}_{2i}u_i$$

$$z_i = \begin{bmatrix}C_{1i}\\ 0\\ 0\\ \frac{1}{\varepsilon_i}\widetilde{E}_{1i}\\ \sqrt{N-1}I\end{bmatrix}\varsigma_i + \begin{bmatrix}D_{11i} & 0 & 0 & 0 & 0\\ 0 & 0.5I & 0 & 0 & 0\\ 0 & 0 & 0.5I & 0 & 0\\ 0 & 0 & 0 & 0.5I & 0\\ 0 & 0 & 0 & 0 & 0.5I\end{bmatrix}\widetilde{w}_i + \begin{bmatrix}D_{12i}\\ 0\\ 0\\ \frac{1}{\varepsilon_i}\widetilde{E}_{2i}\\ 0\end{bmatrix}u_i \tag{23}$$

$$y_i = \widetilde{C}_{2i}\varsigma_i + \begin{bmatrix}D_{21i} & \varepsilon_i\widetilde{L}_{2i} & 0 & 0 & 0\end{bmatrix}\widetilde{w}_i + \widetilde{D}_{22i}u_i$$

where $\widetilde{B}_i\widetilde{B}_i^T = \sum_{j=1, j\neq i}^{N}(\widetilde{A}_{ij}(I - \lambda_{ij}^2\widetilde{J}_{ij}^T\widetilde{J}_{ij})^{-1}\widetilde{A}_{ij}^T + \frac{1}{\lambda_{ij}^2}\widetilde{H}_{ij}\widetilde{H}_{ij}^T)$

Theorem 6 : Consider the interconnected system (1) with positive real uncertainty (2) and (4). This system is robustly stabilizable with **ESP** via decentralized strictly proper output feedback controller $G_C(s) = diag\{G_{C_1}(s), G_{C_2}(s), \ldots, G_{C_N}(s)\}$ if for i=1,2,...,N, there exist parameters $\varepsilon_i > 0$ and $\lambda_{ij} > 0$ *such that*

i) $D_{11i} + D_{11i}^T > 0, D_i + D_i^T > 0, D_{ij} + D_{ij}^T > 0$

ii) $\lambda_{ij}^2\widetilde{J}_{ij}^T\widetilde{J}_{ij} < I$

iii) each closed-loop system of system (23) with control law $u_i = G_{C_i}y_i$ is **ESPR**.

Proof : Theorem 6 comes from theorem 4 directly and thus the proof is omitted. ■

VI. Conclusions

This paper considers decentralized robust passive control problem for large-scale interconnected uncertain systems with time delay. The uncertainty may be norm-bounded or of linear fractional form of positive real uncertainty. We address the problem of designing a linear output feedback controller such that the whole interconnected system is robustly and the system from the disturbance to controlled output is extended strictly passive. It is shown that the decentralized robust passive control problem can be converted to extended strictly positive real control problem for a modified system which contains neither time delay nor uncertainty.

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