

Sliding Mode Control for a Robot Manipulator with Passive Joints

Won Kim, Jin-Ho Shin, and Ju-Jang Lee

Abstract: In this paper, we propose a sliding mode controller for a robot manipulator with passive joints. A robot manipulator with passive joints which are not equipped with any actuators is a kind of underactuated system. Underactuated systems have some advantages compared to fully-actuated ones. For example, they weigh less and consume less energy because they have smaller number of components than fully-actuated ones. However the control of an underactuated manipulator is much more difficult than that of fully-actuated robot manipulator. In this paper a complex dynamic model of a manipulator with passive joints is manipulated for sliding mode control. Sliding mode controllers are designed for this complex system and the stability of the controllers is proved mathematically. Finally a simulation for this control system is executed for evaluating the effectiveness of the designed sliding mode controller.

Keywords: sliding mode control, robot manipulator, variable structure control, passive joint, sliding surface

I. Introduction

Underactuated systems are systems in which the dimension of the configuration space exceeds that of the control input space. For example, when one or more of robot manipulators' actuators fail to work properly, it is said to be in the underactuated states.

Control of this type of an underactuated manipulator is important from a fault-tolerance point of view, for sometimes it is necessary that the robot completes its task before it can be attended to and repairs can be performed.

Moreover, underactuated systems have some advantages compared to fully-actuated ones. First, they weigh less and consume less energy because they have smaller number of components than fully-actuated ones. This characteristics is suitable for a special machine such as a manipulator attached to space shuttle. Second, reliable or fault-tolerant design of the manipulator is possible for hazardous areas such as space, nuclear power plants, etc. In case one of a manipulator's joints has failed during operating time, the failed joint - passive joint - still can be controlled via the dynamic coupling with active joints. The dynamics and control schemes of underactuated systems have been studied from the 1990's. Because the control is much more difficult than the control of a fully-actuated robot manipulator, not much active researches about this area have emerged. H. Arai and S. Tachi, G.Oriolo and Y. Nakamura, E. Papadopoulos and S. Dubowsky[1][2][3] concentrated on designing a controller based on accurate dynamic modeling. However gathering accurate parameters from a large scale robot manipulator is not easy, and also load parameters vary according to the payload. Bergeman applied VSS control scheme to underactuated manipulators for overcoming modeling errors and disturbances[4]. J. Shin and J. Lee has been doing research about robust adaptive control scheme[5].

II. System dynamics

Using the Lagrangian formulation, the dynamics equation of an n-link underactuated robot manipulator with r -actuated joints and p -unactuated joints can be written in joint space as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u + d(t) = \begin{pmatrix} \tau_a + d_a \\ O_{p+d_p} \end{pmatrix} \quad (1)$$

where $q = (q_a^T q_p^T)^T (\in R^{n=r+p})$ is the joint variables in which $q_a \in R^r$ is the position vector of the active joints and $q_p \in R^p$ is the position vector of the passive joints. $M(q) \in R^{n \times n}$ is the inertial matrix, $C(q, \dot{q}) \in R^n$ is the centrifugal and Coriolis torques, $G(q) \in R^n$ is vector gravitational torques, $u = (\tau_a^T O_p^T)^T (\in R^n)$ is the control torque input vector, $\tau_a (\in R^r)$ is the actual control input, $O_p (\in R^p)$ is the zero input vector to passive joints, $n (= r + p)$ is the number of total joints, r is the number of active joints, p is the number of passive joints, and $d(t) = (d_a^T d_p^T)^T (\in R^n)$ is the norm-bounded external disturbance vector for which $d_a \in R^r$, $d_p \in R^p$, $\|d_a\| \leq d_{am}$, and $\|d_p\| \leq d_{pm}$.

Equation (1) can be partitioned as

$$\begin{pmatrix} M_{aa} & M_{ap} \\ M_{pa} & M_{pp} \end{pmatrix} \begin{pmatrix} \ddot{q}_a \\ \ddot{q}_p \end{pmatrix} + \begin{pmatrix} F_a \\ F_p \end{pmatrix} = \begin{pmatrix} \tau_a + d_a \\ O_{p+d_p} \end{pmatrix} \quad (2)$$

where $M_{aa} \in R^{r \times r}$, $M_{pp} \in R^{p \times p}$, and $F(q, \dot{q}) = (F_a^T F_p^T)^T = C(q, \dot{q})\dot{q} + G(q)$.

Some useful properties are given below.

Property 1 [6] [7] $M(q)$ is a symmetric, bounded, invertible and positive definite matrix.

Property 2 [6] [7] $\dot{M}(q) - 2C(q, \dot{q})$ is a skew-symmetric matrix.

Property 3 [7] There exist positive constants m_{min} , m_{max} , c_{max} , g_{max} , f_g and f_c such that

$$\begin{aligned} m_{min} &\leq \|M(q)\| \leq m_{max}, \|C(q, \dot{q})\| \leq c_{max} \|\dot{q}\|, \\ \|G(q)\| &\leq g_{max}, \text{ and } \|F(q, \dot{q})\| \leq f_g + f_c \|\dot{q}\|^2 \end{aligned} \quad (3)$$

where $\|M(q)\|$ and $\|C(q, \dot{q})\|$ are induced matrix norm, and are vector norm.

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Property 4 By Property 1, both $M_{aa} \in R^{r \times r}$ and $M_{pp} \in R^{p \times p}$ are also symmetric, bounded, invertible and positive definite matrices.

Property 5 Effective inertial matrices are symmetric invertible positive definite matrices.

$$\tilde{M}_{aa} = M_{aa} - M_{ap}M_{pp}^{-1}M_{pa} \quad (4)$$

$$\tilde{M}_{pp} = M_{pp} - M_{pa}M_{aa}^{-1}M_{ap} \quad (5)$$

Definition [8] Pseudo inverse matrix (for $p \times r$ matrix A , where $r > p$) is as follows :

$$AA^\# = I_p, A^\#A \neq I_r, A^\# = A^T(AA^T)^{-1}. \quad (6)$$

III. Design of sliding mode controller

1. Control of passive joints

The dynamic equation of the underactuated robot manipulator given in partitioned form in equation (2) can be rewritten for each joint as

$$\ddot{q}_a = -M_{aa}^{-1}(M_{ap}\ddot{q}_p + F_a - \tau_a - d_a) \quad (7)$$

and

$$\ddot{q}_p = -M_{pp}^{-1}(M_{pa}\ddot{q}_a + F_p - d_p). \quad (8)$$

Substituting (7) in (8), we can get equation (9) as follows :

$$M_p\ddot{q}_p = M_{\tau_a}\tau_a + M_{F_a}F_a + M_{F_p}F_p + M_{d_a}d_a + M_{d_p}d_p \quad (9)$$

where

$$\begin{aligned} M_p &= M_{pp}^{-1}\tilde{M}_{pp} = M_{pp}^{-1}(M_{pp} - M_{pa}M_{aa}^{-1}M_{ap}) \in R^{p \times p}, \\ M_{\tau_a} &= -M_{pp}^{-1}M_{pa}M_{aa}^{-1} \in R^{p \times r}, \\ M_{F_a} &= M_{pp}^{-1}M_{pa}M_{aa}^{-1} \in R^{p \times r}, \\ M_{d_a} &= -M_{pp}^{-1}M_{pa}M_{aa}^{-1} \in R^{p \times r}, \\ M_{F_p} &= -M_{pp}^{-1} \in R^{p \times p} \text{ and} \\ M_{d_p} &= M_{pp}^{-1} \in R^{p \times p}. \end{aligned}$$

Therefore, the dynamic equation for the passive joints can be rewritten as follows :

$$\ddot{q}_p = M_{p\tau_a}\tau_a + H_p \quad (10)$$

where

$$\begin{aligned} M_{p\tau_a} &= M_p^{-1}M_{\tau_a} = -\tilde{M}_{pp}^{-1}M_{pa}M_{aa}^{-1} \in R^{p \times r} \text{ and} \\ H_p &= M_p^{-1}(M_{F_a}F_a + M_{F_p}F_p + M_{d_a}d_a + M_{d_p}d_p) \in R^p. \end{aligned}$$

We define a sliding mode controller as

$$\tau_a = \hat{M}_{p\tau_a}^\#(V_{pr} - \hat{H}_p) \quad (11)$$

where $\hat{M}_{p\tau_a}^\# \in R^{r \times p}$ is a pseudoinverse matrix of $\hat{M}_{p\tau_a}$ ($= -\tilde{M}_{pp}^{-1}\hat{M}_{pa}\hat{M}_{aa}^{-1}$) with guessed nominal values for the dynamic parameters of the underactuated manipulator.

The nominal values are assigned as

$$\hat{H}_p = \hat{M}_p^{-1}(\hat{M}_{F_a}\hat{F}_a + \hat{M}_{F_p}\hat{F}_p)$$

where $\hat{M}_p, \hat{M}_{F_a}, \hat{F}_a, \hat{M}_{F_p}$ and \hat{F}_p are the guessed nominal values for $M_p, M_{F_a}, F_a, M_{F_p}$ and F_p , respectively.

Applying (10) to (11) gives

$$\ddot{q}_p = V_{pr} + \eta_p \quad (12)$$

where the lumped uncertainty term is as follows:

$$\eta_p = (M_{p\tau_a}\hat{M}_{p\tau_a}^\# - I_p)V_{pr} + (H_p - M_{p\tau_a}\hat{M}_{p\tau_a}^\#\hat{H}_p). \quad (13)$$

The controller for the passive joints, V_{pr} is denoted as

$$V_{pr} = V_p + \Delta V_p$$

where ΔV_p is a robust control input term.

Tracking error can be given by

$$e_p = q_p - q_{pd} \in R^p$$

where q_{pd} is the desired set points vector.

Sliding surface is given by

$$s_p = \dot{e}_p + \Lambda_p e_p \in R^p$$

where Λ_p is a positive definite diagonal constant gain matrix.

Then outer loop input is given by

$$V_p = \ddot{q}_{pd} - (K_p + \Lambda_p)\dot{e}_p - K_p\Lambda_p e_p$$

where K_p is a positive definite diagonal constant matrix.

At this moment the error dynamics becomes

$$\dot{s}_p = -K_p s_p + \Delta V_p + \eta_p.$$

Here the norm-bound of η_p which includes parameter uncertainties, disturbances and the control input can be derived as

$$\|\eta_p\| \leq \|(M_{p\tau_a}\hat{M}_{p\tau_a}^\# - I_p)V_{pr}\| + \|(H_p - M_{p\tau_a}\hat{M}_{p\tau_a}^\#\hat{H}_p)\|$$

Assumption 1 By property 3 and the norm-bound property of $d(t)$, it is assumed that there exist unknown positive constants such that

$$\begin{aligned} \|M_{p\tau_a}\hat{M}_{p\tau_a}^\# - I_p\| &\leq c_0 < 1, \\ \|H_p - M_{p\tau_a}\hat{M}_{p\tau_a}^\#\hat{H}_p\| &\leq c_1 + c_2\|\dot{q}\|^2. \end{aligned}$$

Robust control input is defined as follows :

$$\Delta V_p = -\hat{\rho}_p \frac{a_p}{\|a_p\|}, a_p = R_p s_p \text{ and } \hat{\rho}_p = \hat{\theta}_p^T \Psi_p \quad (14)$$

where $R_p \in R^{p \times p}$ is a positive definite diagonal constant gain matrix, $\hat{\theta}_p \in R^{\beta_p}$ is some constant vector specified later and $\Psi_p \in R^{\beta_p}$ is a known continuous function which is given by

$$\Psi_p = (1 \|\dot{p}\|^2 \|\ddot{q}_{pd}\| \|\dot{e}_p\| \|e_p\|)^T.$$

There may exist positive constants such that

$$\begin{aligned} \|V_{pr}\| &= \|V_p + \Delta V_p\| \\ &\leq \|V_p\| + \|\Delta V_p\| \\ &\leq \|\ddot{q}_{pd}\| + c_3\|\dot{e}_p\| + c_4\|e_p\| + \hat{\rho}_p. \end{aligned}$$

Therefore using assumption 1, $\|\eta_p\|$ can be described as

$$\|\eta_p\| \leq \bar{\theta}_{p1} + \bar{\theta}_{p2}\|\dot{q}\|^2 + \bar{\theta}_{p3}(\|\ddot{q}_{pd}\| + \hat{\rho}_p) + \bar{\theta}_{p4}\|\dot{e}_p\| + \bar{\theta}_{p5}\|e_p\| \quad (15)$$

where

$$\bar{\theta}_{p1} = c_1, \bar{\theta}_{p2} = c_2, \bar{\theta}_{p3} = c_0, \bar{\theta}_{p4} = c_0c_3 \text{ and } \bar{\theta}_{p5} = c_0c_4.$$

The proposed sliding mode controller for the passive joints of underactuated robot manipulator can be summarized as follows :

$$\tau_a = \hat{M}_{p\tau_a}^\# (V_{pr} - \hat{H}_p) \in R^r, \quad (16)$$

$$V_{pr} = V_p + \Delta V_p \in R^p, \quad (17)$$

$$V_p = \ddot{q}_{pd} - (K_p + \Lambda_p)\dot{e}_p - K_p\Lambda_p e_p \in R^p, \quad (18)$$

$$\Delta V_p = -\hat{\rho}_p \frac{a_p}{\|a_p\|}, a_p = R_p s_p, \hat{\rho}_p = \hat{\theta}_p^T \Psi_p \text{ and} \quad (19)$$

$$\Psi_p = (1 \parallel \dot{q} \parallel^2 \parallel \ddot{q}_{pd} \parallel \parallel \dot{e}_p \parallel \parallel e_p \parallel)^T. \quad (20)$$

Theorem 1 Under the assumptions 1 & 2, if we apply the control law (16) ~ (20) to the underactuated robot manipulator system, then the overall system is globally exponentially stable.

Proof: A Lyapunov function candidate is chosen as

$$V = \frac{1}{2} s_p^T R_p s_p.$$

The time derivative of V is

$$\begin{aligned} \dot{V} &= s_p^T R_p \dot{s}_p \\ &= s_p^T R_p (-K_p s_p + \Delta V_p + \eta_p) \\ &\leq -s_p^T R_p K_p s_p + s_p^T R_p \Delta V_p + \parallel R_p s_p \parallel \parallel \eta_p \parallel. \end{aligned} \quad (21)$$

Substituting (19) into the above equation, we can get the derivative of V as follows:

$$\begin{aligned} \dot{V} &\leq -s_p^T R_p K_p s_p - \hat{\rho}_p \frac{\parallel a_p \parallel^2}{\parallel a_p \parallel} + \parallel a_p \parallel \parallel \eta_p \parallel \\ &\leq -s_p^T R_p K_p s_p - \hat{\rho}_p \parallel a_p \parallel + \parallel a_p \parallel (\bar{\theta}_{p1} + \bar{\theta}_{p2} \parallel \dot{q} \parallel^2 \\ &\quad + \bar{\theta}_{p3} (\parallel \ddot{q}_{pd} \parallel + \hat{\rho}_p) + \bar{\theta}_{p4} \parallel \dot{e}_p \parallel + \bar{\theta}_{p5} \parallel e_p \parallel) \\ &= -s_p^T R_p K_p s_p - \hat{\rho}_p \parallel a_p \parallel + \bar{\theta}_{p3} \hat{\rho}_p \parallel a_p \parallel \\ &\quad + \parallel a_p \parallel (\bar{\theta}_{p1} + \bar{\theta}_{p2} \parallel \dot{q} \parallel^2 + \bar{\theta}_{p3} (\parallel \ddot{q}_{pd} \parallel + \hat{\rho}_p) \\ &\quad + \bar{\theta}_{p4} \parallel \dot{e}_p \parallel + \bar{\theta}_{p5} \parallel e_p \parallel) \\ &= -s_p^T R_p K_p s_p - \hat{\rho}_p (1 - \bar{\theta}_{p3}) \parallel a_p \parallel \\ &\quad + (\frac{\bar{\theta}_{p1}}{1 - \bar{\theta}_{p3}} + \frac{\bar{\theta}_{p2}}{1 - \bar{\theta}_{p3}} \parallel \dot{q} \parallel^2 + \frac{\bar{\theta}_{p3}}{1 - \bar{\theta}_{p3}} \parallel \ddot{q}_{pd} \parallel \\ &\quad + \frac{\bar{\theta}_{p4}}{1 - \bar{\theta}_{p3}} \parallel \dot{e}_p \parallel + \frac{\bar{\theta}_{p5}}{1 - \bar{\theta}_{p3}} \parallel e_p \parallel) (1 - \bar{\theta}_{p3}) \parallel a_p \parallel \\ &= -s_p^T R_p K_p s_p - \hat{\rho}_p (1 - \bar{\theta}_{p3}) \parallel a_p \parallel + \rho_p (1 - \bar{\theta}_{p3}) \parallel a_p \parallel \end{aligned}$$

where

$$\begin{aligned} \theta_{pi} &= \frac{\bar{\theta}_{pi}}{1 - \bar{\theta}_{p3}}, (i = 1, 2, \dots, 5), \\ \theta_p &= (\theta_{p1}, \theta_{p2}, \dots, \theta_{p5})^T, \\ \hat{\rho}_p &= \hat{\theta}_p^T \Psi_p, \rho_p \theta_p^T \Psi_p \text{ and} \\ \Psi_p &= (1 \parallel \dot{q} \parallel^2 \parallel \ddot{q}_{pd} \parallel \parallel \dot{e}_p \parallel \parallel e_p \parallel)^T. \end{aligned}$$

At this point, it is observed that if we select $\hat{\rho}_p$ to satisfy the condition, $\hat{\rho}_p > \rho_p$, the derivative of V is always negative semi-definite. This guarantees the globally exponential stability of the system. ■

Design algorithm of controller for passive joints

So we can summarize design algorithm as follows :

1. Choose appropriate c_0, c_1, c_2, c_3 and c_4 satisfying assumption 1.

2. Compute $\bar{\theta}_{p1} = c_1, \bar{\theta}_{p2} = c_2, \bar{\theta}_{p3} = c_0, \bar{\theta}_{p4} = c_0 c_3$ and $\bar{\theta}_{p5} = c_0 c_4$.

3. Compute $\theta_{pi} = \frac{\bar{\theta}_{pi}}{1 - \bar{\theta}_{p3}}, i = 1, 2, \dots, 5$.

4. Choose $\hat{\theta}_{pi}, i = 1, 2, \dots, 5$ satisfying the following condition $\hat{\theta}_{pi} > \theta_{pi}, i = 1, 2, \dots, 5$.

If we choose variables to satisfy the above condition, the inequality $\hat{\rho}_p > \rho_p$ can always be satisfied.

5. Control input component which overcomes parameter uncertainties and external disturbances is $\Delta V_p = -\hat{\rho}_p \frac{a_p}{\|a_p\|}, a_p = R_p s_p$ and $\hat{\rho}_p = \hat{\theta}_p^T \Psi_p$.

2. Control of active joints

Equation (7) can be written as

$$\begin{aligned} \ddot{q}_a &= M_{aa}^{-1} \tau_a - M_{aa}^{-1} F_a + M_{aa}^{-1} d_a \\ &= M_{aa}^{-1} \tau_a + H_a \end{aligned} \quad (22)$$

where

$$\begin{aligned} H_a &= -M_{aa}^{-1} (F_a - d_a), \text{ and} \\ \ddot{q}_a &= M_{aa}^{-1} (\tau_a + d_a - M_{ap} \ddot{q}_p - F_a). \end{aligned} \quad (23)$$

Since $\dot{q}_p = \ddot{q}_p = 0$ by the operation of brakes, tracking error is denoted as follows:

$$e_a = q_a - q_{ad} \in R^a$$

where q_{ad} is the desired set points vector.

Sliding surface is defined as follows :

$$s_a = \dot{e}_a + \Lambda_a e_a \in R^a$$

where Λ_a is a positive definite diagonal constant gain matrix.

A sliding mode controller is then defined as :

$$\tau_a = -K_a s_a + \Delta V_a \in R^r$$

where K_a is a positive definite diagonal constant matrix.

Therefore

$$\begin{aligned} \dot{s}_a &= \ddot{e}_a + \Lambda_a \dot{e}_a = \ddot{q}_a - \ddot{q}_{ad} + \Lambda_a \dot{e}_a \\ &= M_{aa}^{-a} \tau_a + H_a - \ddot{q}_{ad} + \Lambda_a \dot{e}_a. \end{aligned} \quad (24)$$

From equation (24), we can get equation (25).

$$M_{aa} \dot{s}_a = \tau_a - F_a + d_a + M_{aa} (-\ddot{q}_{ad} + \Lambda_a \dot{e}_a) \quad (25)$$

We define a Lyapunov function candidate as

$$V = \frac{1}{2} s_a^T M_{aa} s_a, \text{ and} \quad (26)$$

$$\begin{aligned} \dot{V} &= s_a^T M_{aa} \dot{s}_a + \frac{1}{2} s_a^T \dot{M}_{aa} s_a \\ &= s_a^T M_{aa} \dot{s}_a + s_a^T C_{aa} s_a = s_a^T (\tau_a + \eta_a) \\ &= -s_a^T K_a s_a + s_a^T (\Delta V_a + \eta_a) \end{aligned} \quad (27)$$

where $\eta_a \in R^r$ is the lumped uncertainty and is defined by

$$\eta_a = -F_a + d_a + M_{aa} (-\ddot{q}_{ad} + \Lambda_a \dot{e}_a) + C_{aa} s_a.$$

Property 7 There exist positive constants defined as follows :

$$\begin{aligned} &m_{aa_{min}}, m_{aa_{max}}, c_{aa_{max}}, g_{aa_{max}}, f_{g_a} \text{ and } f_{c_a} \text{ such that} \\ m_{aa_{min}} &\leq \parallel M_{aa}(q) \parallel \leq m_{aa_{max}}, \parallel C_{aa}(q, \dot{q}) \parallel \leq c_{aa_{max}} \parallel \dot{q} \\ &\text{and } \parallel F_a(q, \dot{q}) \parallel \leq f_{g_a} + f_{c_a} \parallel \dot{q} \parallel^2. \end{aligned} \quad (28)$$

Using property 7, the norm-bound is

$$\|\eta_a\| \leq e_0 + e_1 \|\dot{q}\|^2 \quad (29)$$

where $\theta_a \in R^5$ is an unknown non-negative constant vector and $\Psi_a = (1 \|\dot{q}\|^2 \|\ddot{q}_{a,d}\| \|\dot{e}_a\| \|s_a\|)^T$.

Robust control input is defined as follows,

$$\Delta V_a = -\hat{\rho}_a \frac{s_a}{\|s_a\|} \quad (30)$$

where $\hat{\rho}_A = \hat{\theta}_a^T \Psi_a$, $\hat{\theta}_a \in R^5$ is an estimate vector of θ_a .

Summary of the proposed sliding mode controller for the active joints is as follows :

$$\tau_a = -K_a s_a + \Delta V_a \in R^r, \quad (31)$$

$$\Delta V_a = -\hat{\rho}_a \frac{s_a}{\|s_a\|}, \quad (32)$$

$$\hat{\rho}_A = \hat{\theta}_a^T \Psi_a, \text{ and} \quad (33)$$

$$\Psi_a = (1 \|\dot{q}\|^2 \|\ddot{q}_{a,d}\| \|\dot{e}_a\| \|s_a\|)^T.$$

Theorem 2 If we apply the control law (31) ~ (33) to the underactuated robot manipulator system with the locked passive joints at their desired set-points, then the overall system is globally exponentially stable.

Proof: We define a Lyapunov function candidate as in equation (26) as follows :

$$V = \frac{1}{2} s_a^T M_{aa} s_a. \quad (34)$$

$$\begin{aligned} \dot{V} &= -s_a^T K_a s_a + s_a^T (\Delta V_a + \eta_a) \\ &= -s_a^T K_a s_a - \hat{\rho}_a \|s_a\| + s_a^T \eta_a \\ &\leq -s_a^T K_a s_a - \hat{\rho}_a \|s_a\| + \|s_a\| \|\eta_a\|. \end{aligned} \quad (35)$$

By substituting (29) in (35), we can get the following result :

$$\begin{aligned} \dot{V} &\leq -s_a^T K_a s_a - \hat{\rho}_a \|s_a\| + \|s_a\| \rho_a \\ &= -s_a^T K_a s_a - (\hat{\rho}_a - \rho_a) \|s_a\|. \end{aligned}$$

This indicates that if we select $\hat{\rho}_a$ to satisfy the condition, $\hat{\rho}_a > \rho_a$, the derivative of V is always negative semi-definite and this ensures the globally exponential stability of the system. ■

Design algorithm of controller for active joints

1. Choose appropriate e_0, e_1, e_2, e_3 and e_4 satisfying equation (29).

2. Choose $\hat{\theta}_{a_i}, i = 1, 2, \dots, 5$ satisfying the following condition $\hat{\theta}_{a_i} > \theta_{a_i}, i = 1, 2, \dots, 5$.

If we choose variables to satisfy the above condition, the inequality $\hat{\rho}_a > \rho_a$ is always satisfied.

3. Control input component which overcomes parameter uncertainties and external disturbance is $\Delta V_a = -\hat{\rho}_a \frac{s_a}{\|s_a\|}$, $\hat{\rho}_a = \hat{\theta}_a^T \Psi_a$, and $\Psi_a = (1 \|\dot{q}\|^2 \|\ddot{q}_{a,d}\| \|\dot{e}_a\| \|s_a\|)^T$.

IV. Simulation

Simulation is conducted on a three-link planar robot arm ($n = 3$). The number of actuated (active) joints is two ($r = 2$) and the number of underactuated (passive) joints is one ($p = 1$). The passive joint is located at the third link (q_3). $r > p$ condition is satisfied with the definition in the section 2. The figure of the underactuated robot manipulator is given in figure 1.

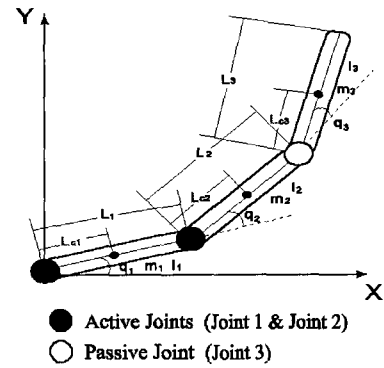


Fig. 1. The configuration of underactuated robot manipulator

To include uncertainty of robot parameters in simulating, the nominal values of parameters are selected to be 70 % of the real values. So the ratio of real value to nominal value is 70 %. Initial conditions are $q_1(0) = q_2(0) = 0[\text{degree}]$, and $q_3(0) = -90[\text{degree}]$.

Desired set points are $q_{1d} = 90[\text{degree}]$, and $q_{2d} = q_{3d} = 0[\text{degree}]$.

The constants of gains (or matrix) are chosen as :

$$K_p = 100, \Lambda_p = 15, R_p = 1, K_a = 3, \text{ and } \Lambda_a = 3.$$

So the slope of the sliding line of passive joint in the phase portrait is -15, and the slope of each of the sliding lines of active joints in the phase portrait is -3.

In figure 2 the trajectories of all the joint angles are displayed. At 0.606 [seconds] joint 3 was moved to desired set point by direct coupling of active joints so brake operated at that time. After passive control was terminated, active control was begun.

The phase portraits of all the joints are displayed in figure 3 ~ 5. There are two stages in the control of active joints. In the first stage active joints contribute to controlling passive joint and in the latter stage they go into self control stage. This fact can be found in figure 3 and 4. The straight lines in these figures are the sliding lines. The states may initially exist in distant positions from the control target points. The states of active joints start to move smoothly in the first stage which is for the control of passive joint by the dynamic link between the two joints. In the second stage active joints are controlled to their objective points. This stage is subjected to chattering phenomena and this is evident in figure 3 and 4. The figures indicate that the states move along the sliding lines to their control points.

The phase portrait of joint 3 is presented in figure 5. The state of joint 3, which initially exists at a distant position from the objective point, moves along the sliding line to the target via the dynamic links of the actuated joints.

V. Conclusions

A robot manipulator with passive joints which are not equipped with any actuators is a kind of underactuated system. However the control of an underactuated manipulator is much more difficult than that of fully-actuated robot manipulator. In this paper a complex dynamic model of a manipulator with passive joints is manipulated for sliding mode control. Sliding mode controllers are designed for this complex system and the stability of the controllers is proved mathematically. The robustness property attributed to sliding mode controller enables

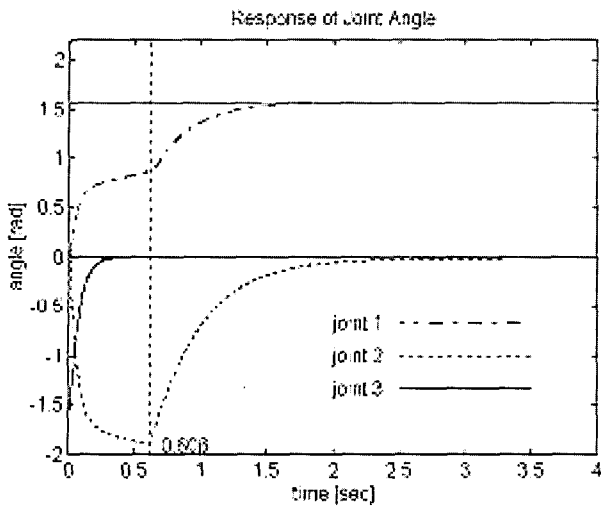


Fig. 2. Joint angles.

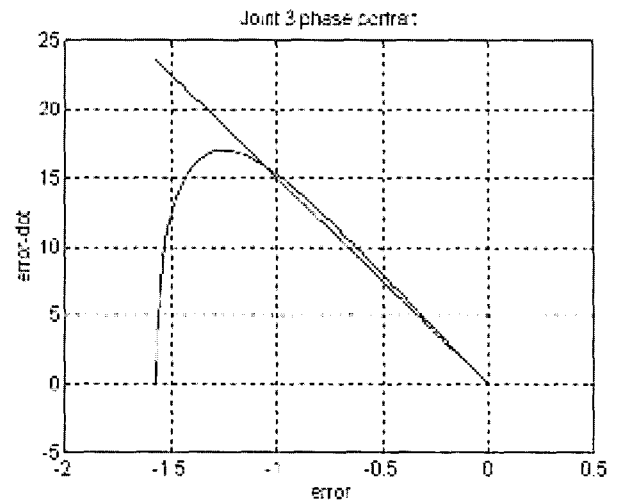


Fig. 5. The phase portrait of joint 3.

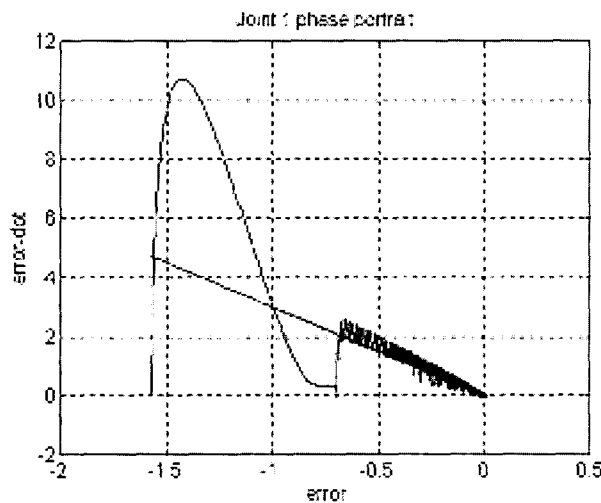


Fig. 3. The phase portrait of joint 1.

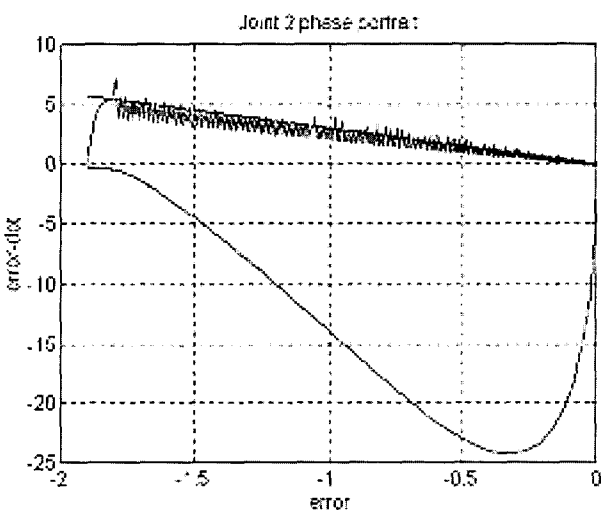


Fig. 4. The phase portrait of joint 2.

the controller to perform well even in the presence of parameter uncertainties. Simulation results are presented to verify the efficiency of the proposed controller. 30% parametric uncertainty was included in the system model to test the robustness prop-

erty. The entire algorithms are simple but the design procedures are rather complex. Stabilities of the two stage controllers have been proved by using Liapunov stability theorem.

In the simulation results the parameters on c_0 , c_1 and c_2 are 0.8, 6.0 and 0.5, respectively. In assumption 1, c_0 should be less than 1 and this fact means careful selection of the nominal value for $\hat{M}_{p\tau\alpha}^\#$ is very important. The condition on c_0 in assumption 1 is not always satisfied and depends on selection of the nominal value for $\hat{M}_{p\tau\alpha}^\#$. Physically the case of $c_0 > 1$ means that the control input can not overcome the lumped uncertain term and the stability can not be ensured in that case. So the value of c_0 in this simulation was calculated in the consideration of physical values of the parameters of robot manipulator.

Because almost physical controllers have limitations on their outputs, a lot of control actuators have bounded outputs. In this bounded case the results of this paper can not be applied directly, because the bounded output may not ensure the stability condition. So for the bounded case the controller should be designed newly in consideration of this bounded condition or the controllers are still effective if the control outputs are in the range of bounded limit values.

One of the disadvantages of this approach is the calculation of norm-bound of lumped-disturbance. Practically norm-bound calculations for choosing c_i, e_i are not easy. The other disadvantage is the chattering phenomena in the control input. To alleviate the chattering phenomena a boundary layer method or fuzzy SMC can be applied. However in solving chattering phenomena with these schemes proving the overall stability is important to ensure the errors go to zeros.

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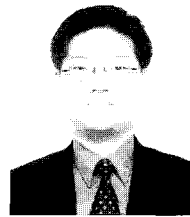
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