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Evaluation of Ultra-high and High Degree Geopotential Models for Improving the KGEOID98

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Abstract

Recent development of ultra-high and high degree Earth geopotential model opens new avenues to determine the Earth gravity field through spectral techniques to a very high accuracy and resolution. However, due to data availability, quality, and type, the performance of these new EGMs needs to be validated in regional or local scale geoid modeling. For establishing the best reference surface of geoid determination, recent geopotential models are evaluated using GPS/Leveling-derived geometric geoid and the Korean gravimetrical GEOID (KGEOID98) developed by National Geography Institute in 1998. Graphical and statistical comparisons are made for EGM96, GFZ97, PGM2000A and GPM98A models. The mean and standard deviation of difference between geometric height and geoid undulation calculated from GFZ97 are 1.9±46.7 cm. It is shown that the GFZ97 and the GPM98A models are better than the others in the Korean peninsula because the GFZ97 has a smaller bias. It means that the KGEOID98 needs some improvement using the GFZ97 instead of EGM96.

Keywords : *Ultra-high Geopotential, GPS/Leveling, Geoid, GPS/derived Height*

1. Introduction

The separation between the geoid and the ellipsoid, or the geoid undulation (N), is required for many geodetic and land surveying applications, the most notable of which is the transformation between GPS-derived ellipsoidal heights and orthometric heights. The geoid can be broadly defined as the equipotential surface of the Earth's gravity field that corresponds most closely with mean sea level (MSL) in the open oceans, ignoring oceanographic effects. The geoid forms the reference surface for orthometric heights and can, in practice, be realized as the local vertical datum through geodetic leveling from tide-gauge measurements of MSL.

The determination of precise gravimetric geoid is usually performed using an Earth geopotential model (EGM), together with a set of detailed terrestrial gravity data and digital terrain data. Therefore, the development of a high-quality EGM is of vital importance for physical geodesy, determination of accurate gravimetric geoid in particular. An accurate EGM also aids in the transformation of GPS-derived heights in a more convenient, cost saving and time-effective way for surveyors. Also, the spectral decomposition of the gravity field and its relations to other geophysical objects are of interest (Christou et al., 1989;Vanicek and Christou, 1994, Svensson, 1994; Zhang and Feathersone, 2000).

Advances in terrestrial gravimetry, satellite tracking, satellite altimetry (SA) and gravity solution have led to the development of many improved EGMs (Lemonie et al., 1998; Lerch et al., 1999; Schwintzer et al., 1997; Tapley, et al., 1996). The evolution of these models has given higher degree and more accurate solutions with improved long wavelength components. Recent studies show that a high degree EGM can be obtained via a tailoring process (Basic et al., 1990; Kearsly and Forsberg, 1990) without the aid of large computer systems and its accuracy could be significantly improved in the areas that high resolution/ precision gravity information is used (Wenzel, 1998, 1999).

The derived EGMs are provided as coefficients of a truncated expansion in terms of surface spherical harmonic basis functions, which can be used to compute the gravitational potential (V), acceleration (g) and gradient tensor (Γ). Current models of the Earth's

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gravitational field can be divided into three classes: (1) satellite-only EGMs derived from ground-based tracking of artificial satellites; (2) combined EGMs derived from a combination of satellite-only models and terrestrial gravimetry and satellite altimeter-derived gravity data in marine areas; (3) tailored EGMs derived from a refinement of existing EGMs (satellite or combined) using regional and/or higher resolution gravity data (Feathestone, 2001).

The estimation of geopotential coefficient from measurements of satellite orbital perturbations is described by, for example, Reigber (1989) and Lemoine *et al.* (1998). Though some recent satellite-only EGMs are available to degree and order 120 (see Table 1), the higher degree coefficients, say greater than 30, heavily contaminated by noise. The gradiometry-derived EGMs are classified as satellite-only EGMs.

The EGMs of satellite derived coefficients can be extended to higher order if combined with terrestrial gravity data and altimeter-derived gravity data. When computing the combined EGMs, the satellite-only coefficients are also adjusted as part of the combination process. Rapp (1997) describes the general philosophies behind the computation of combined EGMs, though specific details for each model can be found in the references cited in Table 2, since the exact techniques differ among groups. The primary limitation to the accuracy of the combined EGMs is the spatial coverage and quality of the terrestrial gravity, satellite altimetry and terrain data used to refine the satellite-only EGMs.

A tailoring process may refine satellite-only and combined EGMs, where the existing spherical harmonic coefficients are adjusted, and often extended to higher degrees, using gravity and terrain data that may not necessarily have been used in the EGM. These tailored coefficients can be used with the computer software described earlier after some minor modifications to account for higher degree and order terms. Tailored EGMs can be developed either globally or over a particular region.

Table 1. Satellite-only global geopotential models published since 1990

Model	Max. degree	Citation
GEM-T2S	36*	Marsh et al. (1990)
GEM-T3S	50	Lerch et al. (1994)
JGM-1S	60	Nerem et al. (1994a and b)
JGM-2S	60	Nerem et al. (1994a and b)
PGTF-4	50	Shum et al. (1990)
GRIM4-S1	50	Schwintzer et al. (1991)
GRIM4-S2	50	Schwintzer et al. (1992)
GRIM4-S3	50	Schwintzer et al. (1993)
GRIM4-S4	60*	Schwintzer et al. (1997)
GRIM5-S1	120	Biancale et al. (2000)
EGM96S	70	Lemoine et al. (1998)

Wenzel (1998) has computed the GPM98A, B and C global EGMs to spherical harmonic degree 1800. It is debatable whether these should be classified as combined EGMs or globally tailored EGMs because they are based on the degree-20 expansion of EGM96 and global $5' \times 5'$ grids of terrestrial gravity anomalies. Specific details for each model can be found in the reference cited in Table 1.

In this paper the latest high degree geopotential

Table 2. Combined global geopotential models published since 1990

Model	Max. degree	Citation
GEM-T2	36*	March et al. (1990)
GEM-T3	50	Lerch et al. (1994)
JGM-1	70	Nerem et al. (1994a and b)
JGM-2	70	Nerem et al. (1994a and b)
JGM-3	70	Tapley et al. (1996)
PGTF-4A	50	Shum et al. (1990)
TEG-2	54	Tapley et al. (1991)
TEG-2B	54	Tapley et al. (1991)
TEG-3	70	Tapley et al. (1997)
GRIM4-C1	50	Reigber et al. (1993) Schwintzer et al. (1991)
GRIM4-C2	50	Reigber et al. (1993) Schwintzer et al. (1993)
GRIM4-C3	60	Schwintzer et al. (1993)
GRIM4-C4	72	Schwintzer et al. (1997)
GRIM5-C1	120	Gruber et al. (2000)
OSU89A	360	Rapp and Pavlis (1990)
OSU89B	360	Rapp and Pavlis (1990)
OSU91A	360	Rapp et al. (1991)
OGE12	360	Gruber et al. (1992)
GFZ93A	360	Gruber et al. (1993)
GFZ93B	360	Gruber et al. (1993)
GFZ95A	360	Gruber et al. (1996)
GFZ96	359	Gruber et al. (1997)
GFZ97	359	Gruber et al. (1993)
EGM96	360	Lemoine et al. (1998)
GAO-98	360	Demianov et al. (2000)
PGM2000A	360	Pavils et al. (2001)

solutions which are EGM96, GPM98, GFZ97 and PGM2000A models were evaluated in terms of the undulation and anomaly differences to propose the optimal reference surface for the refinement of existing KGEOID98 model. The potential coefficients solutions can be evaluated in many ways, but one of which will be applied here. Thus, the geopotential solution is evaluated through the comparison of quasigeoid undulation (N_{GPS}) with undulations from the selected geopotential models (N_{MOD}) by

$$\delta N(i) = N_{GPS}(i) - N_{MOD}(i) \tag{1}$$

2. Comparison of Geopotential Models

The geopotential model is available with coefficients C_{nm} and S_{nm} complete up to degree and order n, m. Then the geoid undulations above the reference (normal) ellipsoid are computed from the fully normalized spherical harmonic coefficients to degree n using the truncated formula:

$$N = \frac{GM}{r\gamma} \sum_{n=2}^{n=\max} \left[\frac{a}{r}\right]^n \sum_{m=0}^{n} \left[\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda\right] \bar{P}_{nm}(\sin\phi) \tag{2}$$

where

$\bar{C}_{nm}$, $\bar{S}_{nm}$ are the fully normalized geopotential coefficients of degree and order n, m

$\bar{P}_{nm}$ is the fully normalized associated Legendre function of degree and order n, m

$n_{\max}$ is the maximum degree of the geopotential model

γ is the normal gravity on the surface of the reference ellipsoid

r is the radial distance from Earth's mass center

ϕ, λ are the geocentric latitude and longitude

Normally a reference ellipsoid is chosen so that its centre coincides with geocenter and its mass equals to the mass of the Earth. According to its definition, GRS80 satisfies these two conditions so that the 1st (n=0) and 2nd (n=1) order terms are zero. The geoid undulations referring to GRS80 were computed from geopotentila models as mentioned above within in the area [123°<φ<132°; 32°<λ<45°].

Fig. 1, 2, 3 and 4 shows a map of geoid undulation computed from the newly developed geopotential models, GPM98A, EGM96, GFZ97 and PGM2000A up to degree and order 360. Fig. 5 and 6 also shows a map of geoid undulation computed from GPM98A up to degree and order 720 and 1800 respectively. Their

Table 3. Statistics of geoid undulations calculated from different geopotential models (unit:meter)

Models	Degree	Min.	Max.	Mean	RMS
EGM96	360	6.157	33.553	22.898	5.755
GPM98A	360	5.739	32.818	22.855	5.737
	720	5.751	32.840	22.855	5.737
	1800	5.757	32.829	22.855	5.738
GFZ97	360	6.202	33.963	22.655	5.765
PGM2000A	360	6.074	33.492	22.884	5.756

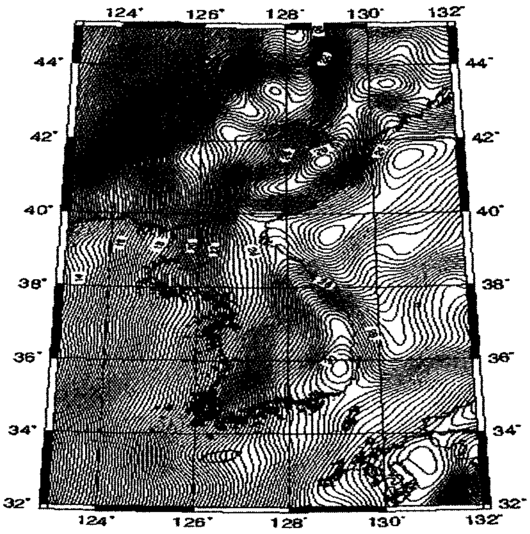

Fig. 1. Geoid undulation map generated from EGM96 to degree 360 (C. I.:0.2m).

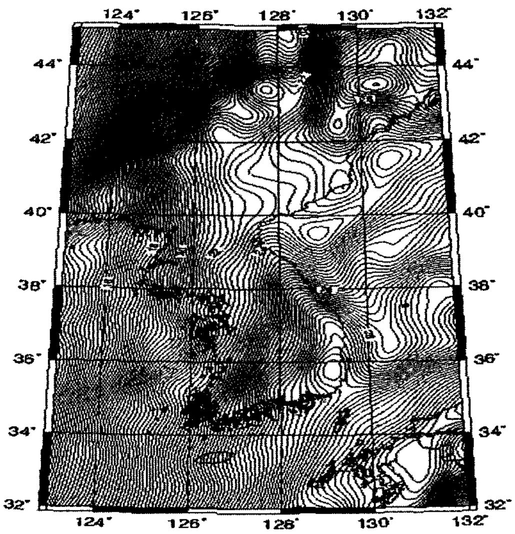

Fig. 2. Geoid undulation map generated from GFZ97 to degree 359 (C.I.:0.2m).

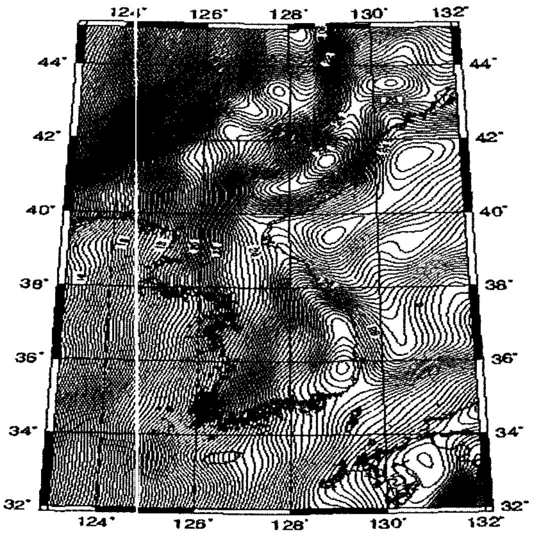

Fig. 3. Geoid undulation map generated from PGM2000A to degree 360 (C.I.:0.2m).

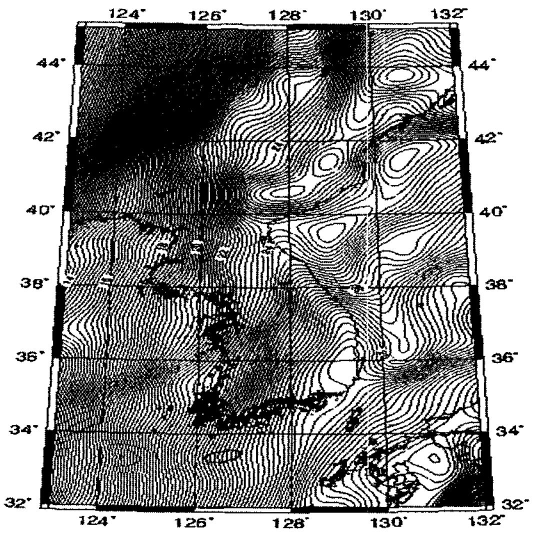

Fig. 4. Geoid undulation map generated from GPM98A to degree 360 (C.I.:0.2m).

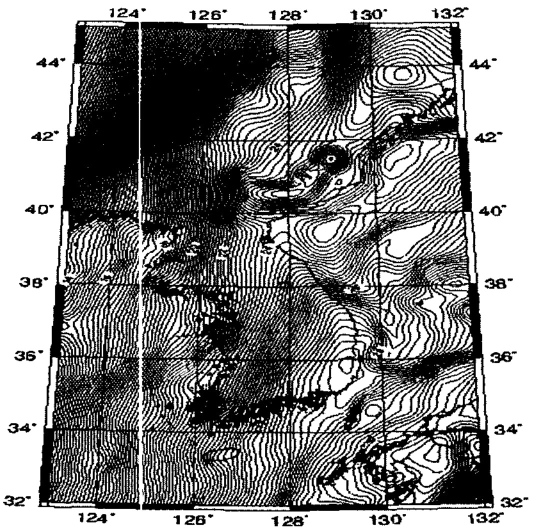

Fig. 5. Geoid undulation map generated from GPM98A to degree 720 (C.I.:0.2m).

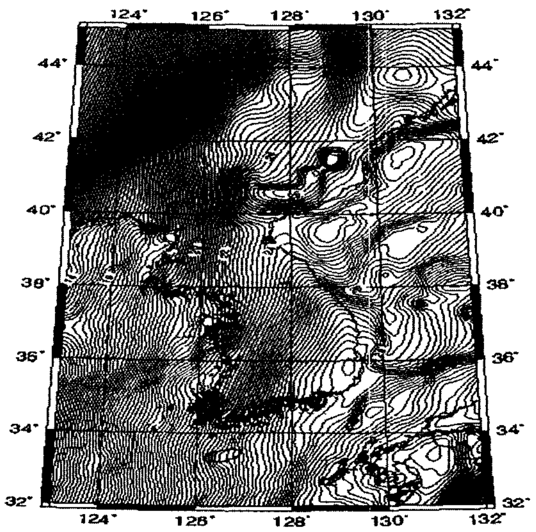

Fig. 6. Geoid undulation map generated from GPM98A to degree 1800 (C.I.:0.2m).

statistics listed in Table 3. The geoid undulation derived from geopotential models as shown in Fig. 1 ~ Fig. 6 range from about 6.0 m to 33.5 m for the test area. The geoid undulation maps show that the difference from the north-west to the south-east is about 14m across the Korean peninsula.

3. Computation and Evaluation

The performance of the GPM98A, GFZ97 and PGM2000A models in Korea is evaluated against that of EGM96. Statistical and graphical comparisons are made and verified by using the following computation:

- Comparing the differences in the geoid undulation between different geopotential models statistically.
- Comparing geoid undulations computed from various geopotential models with geometrically derived geoid undulations at GPS/Leveling stations.

Fig. 7, 8, 9, 10 and 11 shows a surface of geoid undulation differences from GFZ97, PGM2000A and GPM98A with different degree and order compared with the EGM96 in the test area. Their statistical differences are listed in Table 4.

In Table 4 and Figures, we can see that large discrepancies

Table 4. Statistics comparison of geoid undulation differences between geopotential models against EGM96 according to the degree and order (unit:meter)

Models	Degree	Min.	Max.	Mean	RMS
EGM96-GFZ97	360	-0.908	5.101	0.243	0.736
EGM96-GPM98A	360	-1.874	2.454	0.052	0.598
	720	-2.190	2.780	0.043	0.609
	1800	-2.103	2.791	0.043	0.610
EGM96-PGM2000A	360	-0.096	0.098	0.014	0.041

in the northern part of the peninsula exist in most of geopotential models used except only PGM2000A model. Figure 7 shows that the surfaces generated from GFZ97 model show range of -0.9 ~ 15.1m and a mean of 0.243 m with a standard deviation of 0.74 m. Especially, large discrepancy show in the northern part of peninsula because of no data in the northern part. It means that GFZ97 model is less improved in the northern part of the Korean peninsula. The comparison between EGM96 and PGM2000A show that PGM2000A model is not improved in the test area. But, GPM98A

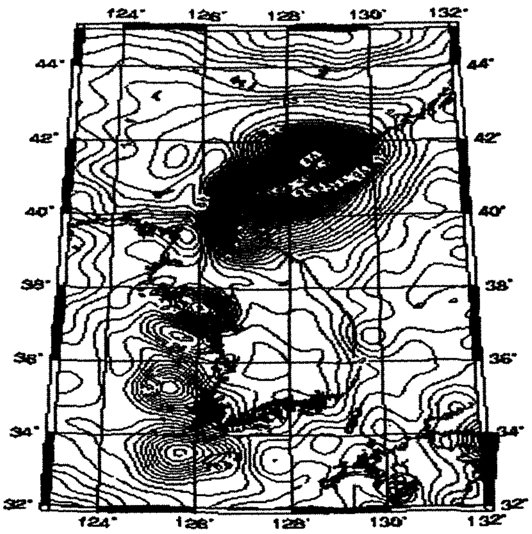

Fig. 7. The difference of geoid undulations between EGM96 and GFZ97 up to degree and order 360 (C.I.:10cm).

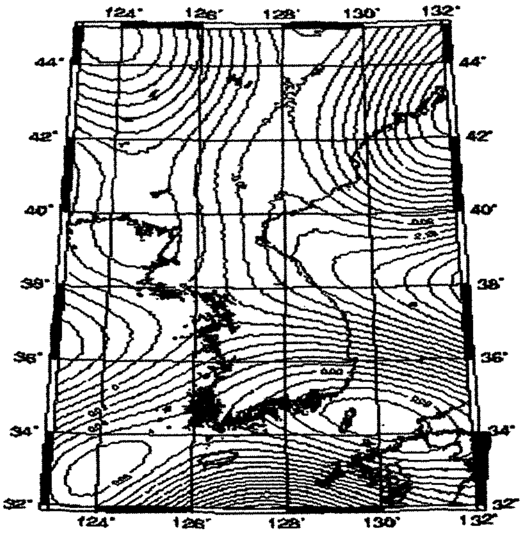

Fig. 8. The difference of geoid undulations between EGM96 and PGM2000A up to degree and order 360 (C.I.:10cm).

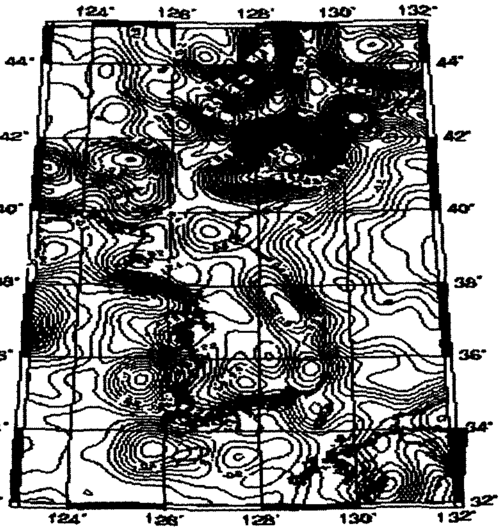

Fig. 9. The difference of geoid undulations between EGM96 and GPM98A up to degree and order 360 (C.I.:10cm).

Fig. 10. The difference of geoid undulations between EGM96 and GPM98A up to degree and order 720 (C.I.:10cm).

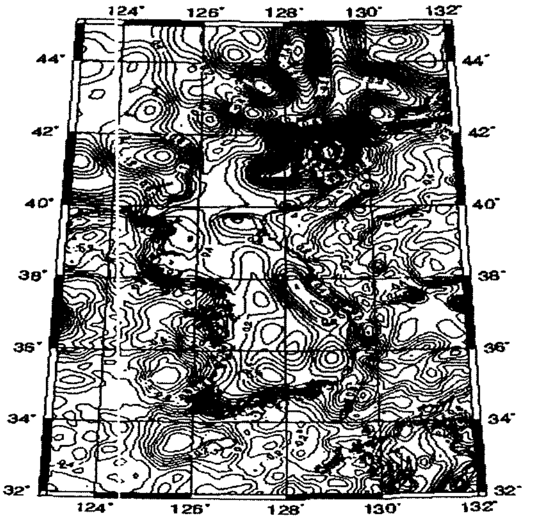

Fig. 11. The difference of geoid undulations between EGM96 and GPM98A up to degree and order 1800 (C.I.:2cm).

shows that much improvement has performed by altimetry data and terrain corrections according to different degree and order. Large discrepancies show in the ocean and a hilly mountain area. The surfaces of Fig 9, 10 and 11 show a range of about -2.0~2.8 m and a standard deviation of 0.61 m.

In the second investigation, the performance of EGM96, GFZ97, PGM2000A and GPM98A models is evaluated using relative carrier phase GPS and spirit leveling data. This investigation is made by comparing GPS-derived Korean Height Datum (KHD) height differences (ΔH_{GPS}) computed using each geoid model with spirit leveled KHD height differences (ΔH_{KHD}).

This is equivalent to comparing geoid gradients for each model to those derived geometrically from the difference of GPS and leveling data.

Fourteen permanent GPS sites are also used for these comparisons. The first set of ITRF values were computed using 11 days (9th November, 2000~20th November, 2000) at 14 permanent GPS sites. This work was used a processing strategy compliant with IGS standards. The nearest seven IGS sites (Tsukuba, Beijing, Taejun, Wuhan, Usuda, IRKT, YSSK) are constrained in the solution that use the IGS products (precise ephemeredes, orientation parameters and coordinates) to produce results with an accuracy of a few centimeters. For each site eleven-day mean was computed from continuous daily GPS solutions. Daily output solutions included a vector of all estimated parameters and a full covariance matrix of errors. The daily position estimate vectors and covariance matrices were combined to yield one final position estimate vector and covariance matrix. The coordinates computed in terms of ITRF97 at epoch 2000.343 for the Korean GPS fiducial network as shown in Table 5. The accuracy of GPS-drived ellipsoidal heights at 14 permanent GPS stations is estimated as 3 mm (NGI, 2001). The orthometric height of each station was determined by spirit leveling with 5 mm accuracy.

The orthometric height of a point can be determined by

$$H \approx h - N \qquad (3)$$

The approximation is due to the non-linearity of the plumb line along which the orthometric height, H, is measured. The ellipsoidal height, h, and the geoid undulation, N, are both measured normal to the ellipsoid. The orthometric height is, however, sufficiently equivalent to h-N to justify a full equality.

Table 5 listed the coordinates and geometrical geoid (h-H) referred to ITRF97. Table 6 shows the comparison of geoid undulations at 14 permanent GPS sites, which was calculated from KGEOID98 and each model described earlier. As shown in Table 7, the geoid differences between the geometrical geoid and each model are similar, and the GFZ97 solution gives the best fit solution. Some solutions seem to indicate a little improvement. But the EGM96, as a reference surface for detailed geoid determination, is worse than the others. Apparent differences between EGM96 and GFZ97 appear in the northern part and the western ocean area of the Korean peninsula. The EGM96 and the PGM2000A solutions give apparently the same results. The GPM98A solutions with various degree and order gives similar accuracy in terms of standard deviation and give rather improvement compared to the EGM96. It is shown that the GFZ97 and the GPM98A models are superior to the others in the test area. It means that the KGEOID98 needs some improvement using the GFZ97 or GPM98A instead of EGM96.

4. Conclusions

The recently developed geopotential models have been evaluated using GPS/Leveling-derived geoids and the Korean gravimetric geoid (KGEOID98). It is shown that no improvement is found in EGM96 and PGM 2000A. In reality, EGM96 and PGM2000A models present slightly detrimental effects over Korea. This is mainly due to the fact that the actual Korea gravity field data is not much used in the generation of the

Table 5. Coordinates and height differences referred to ITRF97

Station Name	ITRF97 Coordinates		Ellipsoidal Hight (*h*)	Orthometric Height (*H*)	Geometrical Geoid (*h*-*H*)
	Latitude	Longitude			
JINJ	35-10-23.1139	128-02-58.8240	122.032 m	93.796 m	28.236 m
JUNJ	35-50-36.4277	127-08-06.4470	77.199	52.184	25.015
KANR	37-46-15.3395	128-52-05.6180	57.084	29.983	27.101
KWNJ	35-10-42.1504	126-54-36.8490	71.717	45.324	26.393
SEOS	36-46-35.0736	126-29-39.1260	52.292	29.892	22.400
SUWN	37-16-31.8536	127-03-15.2620	83.867	60.112	23.755
TEGN	35-54-22.7037	128-48-07.0790	106.429	77.422	29.007
WNJU	37-20-13.9459	127-56-49.5150	180.244	154.283	25.961
WULJ	36-59-31.1154	129-24-46.7800	80.756	52.235	28.521
CHJU	33-30-50.1367	126-31-47.3490	50.335	25.280	25.055
CNJU	36-37-36.8211	127-27-40.4150	93.535	68.104	25.431
SNJU	36-22-44.9927	128-08-40.1150	111.656	84.212	27.444
SOUL	37-37-46.8977	127-04-47.0060	59.162	35.505	23.657
TABK	37-09-39.1269	128-58-32.1670	763.330	734.404	28.926

Table 6. Geoid Undulations computed from KGEOID98 and different models (Unite:meter)

Station Name	h	H	N_{GPS} (=h-H)	KGEOID	EGM96	GFZ97	PGM2000A	GPM98 -360	GPM98 -720	GPM98 -1800
JINJ	122.032	93.796	28.236	28.803	28.285	28.557	28.766	28.242	28.178	28.189
JUNJ	77.199	52.184	25.015	25.307	24.810	25.015	25.392	25.176	25.138	25.238
KANR	57.084	29.983	27.101	27.140	26.656	27.306	27.352	26.852	27.063	27.063
KWNJ	71.717	45.324	26.393	25.173	24/635	25.120	25.284	25.131	25.176	25.191
SEOS	52.292	29.892	22.400	22.595	22.128	22.503	22.769	22.372	22.263	22.271
SUWN	83.867	60.112	23.755	23.667	23.174	23.548	23.746	23.545	23.530	23.521
TEGN	106.429	77.422	29.007	29.758	29.309	29.578	29.798	29.125	29.045	29.028
WNJU	180.244	154.283	25.961	25.976	25.425	25.734	25.946	25.577	25.580	25.576
WULJ	80.756	52.235	28.521	28.948	28.482	29.063	28.918	28.527	28.440	28.420
CHJU	50.335	25.280	25.055	26.029	25.550	25.416	26.049	25.879	25.880	25.894
CNJU	93.535	68.104	25.431	25.556	25.042	25.189	25.553	25.233	25.180	25.181
SNJU	111.656	84.212	27.444	27.460	26.886	27.160	27.430	27.184	27.208	27.202
SOUL	59.162	35.505	23.657	23.402	23.001	23.611	23.611	23.442	23.365	23.375
TABK	763.330	734.404	28.926	29.379	28.621	29.042	29.144	28.268	28.170	28.157

Table 7. Statistics of geoid differences between geometric geoid and geopotential solutions (unit:meter)

	$N_{GPS/LEV}$ KGEOID98	$N_{GPS/LEV}$ - N_{EGM96}	$N_{GPS/LEV}$-N_{GFZ97}	$N_{GPS/LEV}$-$N_{PGM2000A}$	$N_{GPS/LEV}$-N_{GPM98} (360)	$N_{GPS/LEV}$-N_{GPM98} (720)	$N_{GPS/LEV}$-N_{GPM98} (1800)
MAX.	0.974	1.054	1.273	1.109	1.262	1.217	1.202
MIN.	-1.220	-1.032	-0.571	-0.994	-0.824	-0.825	-0.839
MEAN	0.164	-0.218	0.019	-0.204	0.168	0.195	0.193
STDEV.	0.422	0.492	0.467	0.489	0.460	0.451	0.453

model and the complexity of the Korea gravity field, particularly in coastal and the northern part of test area.

The GPM98A model is apparently improved in the costal and the northern part of the test area. But, the comparison by GPS/Leveling-derived geoid shows about 20 cm bias and the standard deviation of 45 cm. This is due to the use of EGM96 as a reference surface for tailoring the ultra-high degree GPM98A model.

It is shown that the GFZ97 is better than the others for use as a reference surface of detailed geoid modeling in the Korean peninsula because the GFZ97 has a small bias. It means that the KGEOID98 needs some improvement using the GFZ97 instead of EGM96.

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