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Performance Evaluation of Warm Standby Redundant Systems

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Abstract. In this paper, we consider the warm standby redundant system(WSRS) which is consisted of an active unit, a standby unit and a switchover device. In addition, the switchover processing is controlled by a control module. The effect of failure of the control module is taken into account to develop our reliability model for the redundant structure. For the performance evaluation of a redundant system with the function of switchover processing which is assumed to cause the increase of the failure rate of the system, some reliability indices, such as availability, average availability, reliability and steady state availability, are considered.

Key Words : *redundancy, availability, steady state availability, standby redundant system, transition probability*

1. INTRODUCTION

To improve the system reliability, the standby redundant structures such as electric power generator and airplane jet engines have been widely adopted by the system designers. In a two-unit repairable standby redundant system, the standby unit starts operating immediately upon the failure of the active unit and once the failed active unit is repaired, it assumes the position of standby unit. Thus, these two units alternate their positions either active or standby whenever the failure

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or repair occurs. Depending on readiness(or consequently, the failure rate) of the standby unit, it is classified as hot, cold or warm standby unit. The cold standby unit does not fail when it remains standby and thus, the failure rate of cold standby unit equals zero. The failure rate of hot standby unit is the same as that of active unit, while the warm standby unit has a lower failure rate than the active unit, but its failure rate is greater than zero.

The performance of the system is analyzed with respect to its reliability characteristics, such as reliability function, availability, MTBF, mean residual life function and so on. One of the most widely used performance criteria of repairable systems is an availability which is defined as the probability that a system is operating satisfactorily when it is required to perform the given mission. Because of the fact that the availability is an important measure to evaluate the performance of the system, many researchers have worked on these subjects quite extensively.

Lim (1996) and Lim and Koh (1997) study a redundant system with the function of switchover processing which consists of three units ; an active unit, a standby unit, a switchover device. Shin et al.(2001) consider a hot standby redundant system and evaluate the performance of the system by time dependent availability and its related measures. Figure 1 shows a reference model for such a redundant system. These articles assume that the failure rate of the system increases by installing the switchover processing, since the failure of the control module can cause the failure of the system. In order to develop a reliability model, they distribute such increment of the failure rate to each unit of the system in such a way that the failure rate of each unit increases by $\lambda_\alpha = \alpha\lambda$, where $0 \leq \alpha \leq 1$ and λ is the failure rate of the unit without the switchover processing. Note that $\alpha = 0$ implies no failure of the control module.

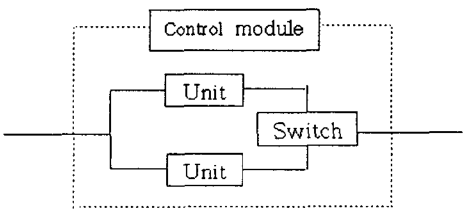


Figure 1. A model of redundant structure with a function of switchover processing.

In this paper, we consider a redundant system with the switchover processing discussed in Lim (1996) and Lim and Koh (1997). In Section 2, we derive several reliability measures, such as availability, average availability, reliability and steady state availability for the warm standby redundant system. Section 3 presents numerical examples to illustrate our results.

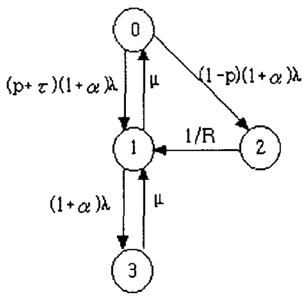
2. WSRS MODELLING AND ITS RELATED PERFORMANCE MEASURES

2.1 Assumptions

1) All units are independent and have exponential life distributions, each unit having a mean life of $1/\lambda$, and the repair times of unit and switch are exponentially distributed with a mean of $1/\mu$ and $1/\gamma$, respectively.
2) The probability of successful switchover operation is assumed to be equal to p.
3) The type of standby unit in the redundant system is a warm standby unit whose failure rate $\lambda_W = \tau\lambda$, where $0 \le \tau \le 1$.

2.2 WSRS Model

We define four states of the system and the state transition diagram(STD) is shown in Figure 2. The states 2 and 3 represent the failure of the system. The state 2, which represents uncoverage outage, is caused by the failure of active unit while the control module is in the failure state and the state 3 is due to the failures of both units.



State	Description
0	Duplex
1	Simplex
2	Uncoverage Outage
3	Failure

Figure 2. State transition diagram of WSRS.

2.3 Time dependent Transition Probabilities and Related Measures

Let i, $i = 0, 1, 2, 3$, represent the state of warm standby redundant system(WSRS) and let $p_i(t)$ be the probability that the system is in state i at time t. The flow rate equations can be established by consideration of the fact that the flow rate out of the system must be equal to the flow rate into the state. Thus, we have

$$\begin{aligned} p_0'(t) &= -(1+\tau)(1+\alpha)\lambda p_0(t) + \mu p_1(t) \\ p_1'(t) &= -(\mu + (1+\alpha)\lambda)p_1(t) + (p+\tau)(1+\alpha)\lambda p_0(t) + \gamma p_2(t) + \mu p_3(t) \quad (2.1) \\ p_2'(t) &= -\gamma p_2(t) + (1-p)(1+\alpha)\lambda p_0(t) \\ p_3'(t) &= -\mu p_3(t) + (1+\alpha)\lambda p_1(t), \end{aligned}$$

where $p_i'(t) = dp_i(t)/dt$, $i = 0, 1, 2, 3$.

To solve these equations simultaneously for $p_0(t)$ and $p_1(t)$, we may apply the Laplace and the inverse Laplace transformations. The Laplace transform of a function $f(t)$ is defined as

$$\tilde{f}(s) = \int_0^\infty \exp(-st)f(t)dt.$$

Note that for $i = 0, 1, 2, 3$, the Laplace transform of $p_i^{'}(t)$ is obtained as $\tilde{p}_i^{'}(s) = \tilde{p}_i(0) + s\tilde{p}_i(s)$. Using the initial conditions, $p_0(0) = 1$ and $p_1(0) = p_2(0) = p_3(0) = 0$, it is straightforward to obtain

$$p_0(t) = N_1 \exp(-\beta_1 t) + N_2 \exp(-\beta_2 t) + N_3 \exp(-\beta_3 t) + N_4 \tag{2.2}$$

and

$$p_1(t) = M_1 \exp(-\beta_1 t) + M_2 \exp(-\beta_2 t) + M_3 \exp(-\beta_3 t) + M_4, \tag{2.3}$$

where $-\beta$'s are the roots of the following third order equation

$$\begin{aligned} s^3 &+ (2\mu + (2+\tau)(1+\alpha)\lambda + \gamma)s^2 + \Big(\mu^2 + \mu\lambda(1+\alpha)(2+\tau-p) \\ &+ (2\mu + (2+\tau)(1+\alpha)\lambda)\gamma + (1+\tau)(1+\alpha^2)\lambda^2\Big)s \\ &+ \mu^2(\lambda(1+\alpha)(1-p)+\gamma) + (1+\tau)(1+\alpha)\lambda(\mu + (1+\alpha)\lambda)\gamma \\ &= 0 \end{aligned}$$

and N_j and M_j are

$$N_j = (\gamma - \beta_j)(r_1 - \beta_j)(r_2 - \beta_j) \Big/ \prod_{\substack{k=1 \\ k\neq j}}^{4} (\beta_k - \beta_j)$$

and

$$M_j = \lambda(1+\alpha)(p+\tau)((1+\tau)\gamma/(p+\tau) - \beta_j)(\mu - \beta_j) \Big/ \prod_{\substack{k=1 \\ k\neq j}}^{4} (\beta_k - \beta_j)$$

for $j = 1, 2, 3, 4$ and r_1 and r_2 are $(-((1+\alpha)\lambda + 2\mu) + \sqrt{\lambda(1+\alpha)(\lambda(1+\alpha)+4\mu)})/2$ and $(-((1+\alpha)\lambda + 2\mu) - \sqrt{\lambda(1+\alpha)(\lambda(1+\alpha)+4\mu)})/2$, respectively.

Given the expressions for $p_0(t)$ and $p_1(t)$, we evaluate the following reliability measures for WSRS.

Availability and Average Availability of WSRS

The availability of WSRS at time t, $A(t)$, and average availability of the system in $(0, t]$, $\bar{A}(t)$, are obtained as

$$\begin{aligned} A(t) &= p_0(t) + p_1(t) \\ &= \sum_{i=1}^{3} (N_i + M_i)\exp(-\beta_i t) + (N_4 + M_4) \end{aligned}$$

and

$$\begin{aligned}\bar{A}(t) &= \frac{1}{t}\int_0^t A(u)du \\ &= \sum_{i=1}^{3}(N_i + M_i)(1 - \exp(-\beta_i t))/(\beta_i t) + (N_4 + M_4),\end{aligned}$$

respectively. The expected operating time of WSRS during the time interval $(0, t]$ can be evaluated by $E_w(t) = \int_0^t [p_0(u) + p_1(u)]du$. It follows that the expected down time of WSRS during $(0, t]$, $E_d(t)$, is $t - E_w(t)$.

Reliability and Mean Time to First Failure of WSRS

Letting μ and γ approach to zero, $A(t)$ can be interpreted as the reliability of WSRS. In this case, the system is considered as a nonrepairable system and its reliability is obtained as

$$R(t) = [(p+\tau)\exp(-(1+\alpha)\lambda t) - p\exp(-(1+\tau)(1+\alpha)\lambda t)]/\tau$$

for $t \geq 0$. Thus, the mean time to first failure(MTTFF) of the system is

$$\begin{aligned}\text{MTTFF} &= \int_0^\infty R(u)du \\ &= \frac{p+\tau+1}{(1+\tau)(1+\alpha)\lambda}.\end{aligned}$$

2.4 Steady-State Transition Probabilities and Related Measures

To obtain the steady-state transition probabilities, we replace $p_i(t)$, $i = 0, 1, 2, 3$, by constants p_i(and thus, $p_i'(t)$ by 0) in equations of (2.1) and solve these equations simultaneously for p_0 and p_1, where p_i can be interpreted as a steady-state transition probability of state i. These equations are given as

$$\begin{aligned}\mu p_1 &= (1+\tau)(1+\alpha)\lambda p_0 \\ (\mu + (1+\alpha)\lambda)p_1 &= (p+\tau)(1+\alpha)\lambda p_0 + \gamma p_2 + \mu p_3 \\ (1-p)(1+\alpha)\lambda p_0 &= \gamma p_2 \\ (1+\alpha)\lambda p_1 &= \mu p_3.\end{aligned} \tag{2.4}$$

Straightforward calculation yields

$$p_0 = \frac{\mu^2}{\mu^2 + 2\mu(1+\alpha)\lambda + (\mu^2/\gamma)(1-p)(1+\alpha)\lambda + 2(1+\alpha)^2\lambda^2} \tag{2.5}$$

and

$$p_1 = \frac{(1+\tau)\mu(1+\alpha)\lambda}{\mu^2 + 2\mu(1+\alpha)\lambda + (\mu^2/\gamma)(1-p)(1+\alpha)\lambda + 2(1+\alpha)^2\lambda^2}, \tag{2.6}$$

respectively. Therefore, the steady-state availability of the system, denoted by A_W, is equal to $p_0 + p_1$. Although it may not be feasible to prove analytically, $A(t)$ can be shown to converge to A_W as t becomes sufficiently large numerically.

The following theorem compares a single component system and a WSRS with regard to its availability. It is well known that the availability of a single component, denoted by A_s, is equal to $\mu/(\lambda+\mu)$.

Theorem 1. Let $\rho = 4(1+\tau)\lambda\mu$ and $\delta = (1+\tau)\lambda + \mu^2/\gamma$. Given that $(-\delta + \sqrt{\delta^2 - \rho((\mu/\gamma)-1)})/2(1+\tau)\lambda < \alpha < (-(1+\tau)\lambda + \sqrt{(1+\tau)^2\lambda^2+\rho})/2(1+\tau)\lambda$, there exist a $p^* \in [0,1]$ such that $A_S \geq A_W$ for $0 \leq p \leq p^*$ and $A_S \leq A_W$ for $p^* \leq p \leq 1$, where $p^* = 1 - (\mu - (1+\tau)\alpha(1+\alpha)\lambda)\gamma/\mu^2(1+\alpha)$.

Proof. We note that A_W is non-decreasing in p because each of p_0 and p_1 is non-decreasing in p and A_S is a constant. Hence, it is sufficient to show that when $p=0$, $A_S \geq A_W$ and when $p=1$, $A_S \leq A_W$. It is somewhat tedious but straightforward to show that when $p=0$, $A_S \geq A_W$ if $\alpha > (-\delta + \sqrt{\delta^2-\rho((\mu/\gamma)-1)})/2(1+\tau)\lambda$. Similarly, we can show that when $p=1$, $A_S \leq A_W$ if $(-(1+\tau)\lambda + \sqrt{(1+\tau)^2\lambda^2+\rho})/2$ $(1+\tau)\lambda$. Thus, the existence and uniqueness of p^* is established.

3. NUMERICAL EXAMPLES

In this section, we evaluate the values of $p_0(t)$, $p_1(t)$, $A(t)$, $\bar{A}(t)$, $E_w(t)$ and $R(t)$. To estimate the parameters λ, μ and θ, we use the following data for times to failure(Data 1) and the data for times to repair(Data 2), which are obtained under the assumption that both times follow exponential distributions with means of $1/\lambda$ and $1/\mu$, respectively.

Data 1	95.7	98.5	93.4	97.2	100.5
Data 2	6.2	3.8	7.9	7.3	5.4

Based on these data, the MLE's of λ and μ are obtained as 0.0103 and 0.163399, respectively.

For Tables 1-4, we consider the case when the values of p, α and τ are 0.0, 0.3, 0.6 and 1.0. Table 1 presents the behaviors of $A(t)$ for various p when $\alpha = 0.3$ and $\tau = 0.5$, and it shows the availability increases as the value of p becomes higher.

Table 2 shows that the availability decreases as the value of α increases for $p = 0.9$ and $\tau = 0.5$. Table 3 shows that the availability increases as the value of τ decreases for $p = 0.9$ and $\alpha = 0.3$. Table 4 gives the value of $p_0(t)$, $p_1(t)$, $A(t)$, $\bar{A}(t)$, $E_w(t)$ and $R(t)$ for $\alpha = 0.3$, $p = 0.9$ and $\tau = 0.5$. It is easy to obtain for $\alpha = 0.3$, $p = 0.9$ and $\tau = 0.5$ that the MTTF is 119.458 and the steady-state availability is 0.98399. Setting $\tau = 1$, the values coincides with the one discussed by Shin et al.(2001).

Table 1. Availability of WSRS, $A(t)$, for various p's with $\alpha = 0.3$ and $\tau = 0.5$.

		Time (t)					
		0	0.3	0.7	1	3	7
p	1.0	1.0000	1.0000	1.0000	0.9999	0.9992	0.9970
	0.6	1.0000	0.9984	0.9964	0.9950	0.9870	0.9765
	0.3	1.0000	0.9973	0.9938	0.9914	0.9779	0.9613
	0.0	1.0000	0.9961	0.9912	0.9877	0.9688	0.9462
		10	15	20	25	30	∞
p	1.0	0.9955	0.9936	0.9923	0.9918	0.9915	0.9911
	0.6	0.9718	0.9674	0.9653	0.9642	0.9638	0.9632
	0.3	0.9544	0.9483	0.9456	0.9443	0.9438	0.9433
	0.0	0.9372	0.9296	0.9265	0.9252	0.9246	0.9242

Table 2. Availability of WSRS, $A(t)$, for various α's with $p = 0.9$ and $\tau = 0.5$.

		Time (t)					
		0	0.3	0.7	1	3	7
α	1.0	1.0000	0.9994	0.9985	0.9979	0.9934	0.9856
	0.6	1.0000	0.9995	0.9988	0.9983	0.9950	0.9893
	0.3	1.0000	0.9996	0.9991	0.9987	0.9961	0.9919
	0.0	1.0000	0.9997	0.9993	0.9990	0.9971	0.9942
		10	15	20	25	30	∞
α	1.0	0.9811	0.9761	0.9734	0.9719	0.9711	0.9702
	0.6	0.9862	0.9826	0.9807	0.9796	0.9790	0.9785
	0.3	0.9895	0.9870	0.9856	0.9848	0.9844	0.9840
	0.0	0.9926	0.9909	0.9900	0.9895	0.9892	0.9889

Table 3. Availability of WSRS, $A(t)$, for various τ's with $p = 0.9$ and $\alpha = 0.3$.

		Time (t)					
		0	0.3	0.7	1	3	7
τ	1.0	1.0000	0.9996	0.9990	0.9986	0.9959	0.9910
	0.6	1.0000	0.9996	0.9991	0.9987	0.9961	0.9917
	0.3	1.0000	0.9996	0.9991	0.9987	0.9962	0.9922
	0.0	1.0000	0.9996	0.9991	0.9987	0.9964	0.9927
		10	15	20	25	30	∞
τ	1.0	0.9883	0.9853	0.9836	0.9827	0.9822	0.9818
	0.6	0.9893	0.9866	0.9852	0.9844	0.9840	0.9836
	0.3	0.9900	0.9877	0.9864	0.9857	0.9853	0.9849
	0.0	0.9908	0.9888	0.9877	0.9871	0.9868	0.9840

Table 4. Values of $p_0(t)$, $p_1(t)$, $A(t)$, $\bar{A}(t)$, $E_w(t)$ and $R(t)$ with $\alpha = 0.3$, $p = 0.9$ and $\tau = 0.5$.

Time(t)	$p_0(t)$	$p_1(t)$	$A(t)$	$\bar{A}(t)$	$E_w(t)$	$R(t)$
3	0.9528	0.0433	0.9961	0.9980	2.994	0.9950
5	0.9323	0.0615	0.9938	0.9968	4.984	0.9906
7	0.9175	0.0744	0.9919	0.9957	6.970	0.9856
10	0.9025	0.0870	0.9895	0.9941	9.942	0.9766
20	0.8824	0.1032	0.9856	0.9907	19.81	0.9376
30	0.8778	0.1066	0.9844	0.9887	29.66	0.8883
40	0.8766	0.1075	0.9841	0.9876	39.50	0.8328

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