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Comparison of Inbred Lines Within Two Groups

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Abstract. Sometimes we have two groups of inbred lines and there are only interest in gca comparisons within two groups of lines of sizes p_1 and p_2, not between two groups. For example, suppose there two Lab, each of the 2 Labs have obtained the best line. For this purpose we now give a method of constructing block designs for diallel cross experiments and we will explain how to calculate efficiency. Then we show the efficiencies in the table.

Key Words : *diallel cross, general combining ability, Latin square.*

1. INTRODUCTION

Genetic properties of inbred lines in plant breeding experiments are investigated by carrying out diallel crosses. Let p denote the number of inbred lines and let a cross between lines i and j be denoted by (i, j) with $i < j = 1, 2, \ldots, p$. Let n_c denote the total number of distinct crosses in the experiment. Our interest lies in comparing the lines with respect to their general combining ability (gca) parameters. The complete diallel cross (CDC) involves all possible crosses among p parental lines with $n_c = p(p-1)/2$. Gupta and Kageyama(1994) gave a method of constructing balanced block designs for CDCs using the nested balanced incomplete block (BIB) designs of Preece(1967). Subsequently, Dey and Midha(1996), among others, gave further methods of constructing balanced diallel cross block designs.

We consider the case where there are two groups of inbred lines, such as those coming from two different regions or laboratories. When the total number of lines in the two groups is large, sometimes it becomes impractical to carry out a complete diallel cross. In such situations, only a subset of all possible crosses is used in the experiment, which is called a partial diallel cross (PDC). Das, Dean and Gupta(1998) and Mukerejee(1997) gave some PDC block designs. Ghosh and Divecha(1997) obtained partially balanced PDC and CDC block designs by forming all pairs of crosses between the treatment within each block of a conventional incomplete block design.

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Block designs for partial diallel crosses often require each cross to be replicated several times.

In this paper we consider the situation where the experiment is carried out in two phases in order to have fewer replications of the crosses. The object in the first phase is to select a few best lines from each group on the basis of their gca effects. The selected lines in one group are then compared with those of the other group in the second phase of the experiment. The purpose of this paper is to provide a class of block designs for the first phase of the experiment. In the literature PDC designs have been discussed for $n_c = ps/2$, $s < p-1$, distinct crosses where s is the constant number of other lines each line is crossed with. The PDC designs of this paper involve two distinct values of s. The designs given are especially useful as they require each cross to be replicated only once. A table of designs is also provided.

2. PRELIMINARIES

We consider the case where there are two groups of lines of sizes p_1 and p_2, with $p = p_1+p_2$. Let the lines in the ith group be denoted by $(i-1)p_1+j$, $j = 1, 2, \dots, p_i$, $i = 1, 2$. Consider a block design D_b involving $n_c = p_1p_2$ distinct crosses laid out in b blocks of k crosses each, with each line from group i contributing to n_c/p_i crosses, $i = 1, 2$. Following e.g. Gupta and Kageyama(1994), the model for the data is assumed to be

$$Y = \mu 1_n + \Delta_1 g + \Delta_2 \beta + \varepsilon, \tag{2.1}$$

where Y is the $n \times 1$ vector of responses, μ is the overall mean, 1_t is the $t \times 1$ vector of 1's, and $g = (g_1, g_2, \dots, g_p)'$ and $\beta = (\beta_1, \beta_2, \dots, \beta_b)'$ are the vectors of p gca effects and b block effects respectively; the rectangular matrices Δ_1, Δ_2 are the corresponding design matrices, and ε is the $n \times 1$ vector of independent random errors with zero expectations and constant variance σ^2. The information matrix C for estimating pairwise comparisons among the gca parameters is then given by

$$C = G - \tfrac{1}{k} N_b N_b', \tag{2.2}$$

where $G = (g_{ij})$ is a symmetric matrix, g_{ii} denotes the number of other lines line i is crossed with, and g_{ij} denotes the number of replications of the cross (i,j) for $i < j = 1, 2, \dots, p$. Consider two lines in each of the n crosses as the block contents of a design D_c with block size $k = 2$, and let N_c denote the $p \times n$ incidence matrix of the block design thus obtained. Then $G = N_c N_c'$. The matrix N_b is the $p \times b$ line versus block incidence matrix of the design. Thus N_b is the usual incidence matrix; in the present context, it is obtained by ignoring the crosses, and thus by considering $2k$ lines as the contents of a block. Note that $N_b 1_b = r 1_p$, $N_b' 1_p = 2k 1_b$.

For $\ell = 1, 2$, our interest lies in the the gca comparisons $g_i - g_j$, $i < j = (\ell-1)p_1+1, (\ell-1)p_1+2, \dots, (\ell-1)p_1+p_\ell$. That is, we are interested in comparing the lines within each group with respect to their gca effects. A class of designs for estimating these comparisons is given in the next section. For the designs of the next section, we assume that $g_{ij} = 1$ and $g_{ii} = n_c/p_\ell$, $i < j = (\ell-1)p_1+1$, $(\ell-1)p_1+2, \ldots, (\ell-1)p_1+p_\ell$, $\ell = 1,2$.

3. METHOD OF CONSTRUCTION

We now present a method of constructing block designs for comparisons within two groups of inbred lines. For this purpose, we construct block designs using Latin square idea. Let p_1, p_2 be the number of inbred lines in each group, and consider $p_1 = c_1p_2+c_2$ ($c_1, c_2 > 0$ integer). First, p_2 inbred lines are set out in an $p_2 \times p_2$ array such that each inbred line occurs once in each row and once in each column of the array. We represent the array as L and take any (p_1-p_2) rows of p_2 rows belonging to L and augment them to columns of L such that the number of columns of the augmented L is p_1. In order to make cross between two groups, we superimpose p_1 inbred lines belonging to 1st group on each row of the augmented L in turn. Then making cross on the superimposed and augmented L, we can construct a block design D with parameters

$$p = p_1+p_2, b = p_2, k = p_1, r_c = 1, n_c = p_1p_2, \lambda_1 = 0, \lambda_2 = 1,$$

where λ_l ($l = 1,2$) denote the number of replicated times of cross between lines i and j belonging to lth group. We explain method of construction using the following example.

Example. For $p = 8, p_1 = 5, p_2 = 3$, we have two groups of lines as following:

$$\text{Group1} : \{1,2,3,4,5\}, \quad \text{Group2} : \{6,7,8\}.$$

We can construct the following rectangular arrays in order to make cross between two groups.

$$\begin{bmatrix} 6 & 7 & 8 & 6 & 7 \\ 7 & 8 & 6 & 7 & 8 \\ 8 & 6 & 7 & 8 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

In our method, block design is obtained by making cross between the two groups.

Block 1: $(1,6),(2,7),(3,8),(4,6),(5,7)$
Block 2: $(1,7),(2,8),(3,6),(4,7),(5,8)$
Block 3: $(1,8),(2,6),(3,7),(4,8),(5,6)$

For the purpose of estimating gca effects within groups of inbred lines, we find C of equation (2.2) in design D_b. First, from our method of construction of D_b, it is easily verified that $g_{ii}(i = 1,2,\ldots,p)$ of G is given by

$$g_{ii} = \begin{cases} p_2, & if i = 1,2,\ldots,p_1 \\ p_1, & if i = p_1+1, p_1+2, \ldots, p_1+p_2. \end{cases}$$

Also, for non-diagonal elements of G, if two lines i and j belong to the same group,$g_{ij} = 0$, otherwise $g_{ij} = 1$. We now find the elements of NN' of equation(2.2). Note that for finding the number of within-block concurrences of two lines, the lines are taken as the contents of a block. First, we can see that the elements of NN' are p_2 when two lines i and j belong to 1st group, and p_1 when both lines i and j belong to the different group. Finally, find the elements of NN' when two lines i and j belong to 2nd group. In here, consider only 2nd group elements in which $p_1 = c_1p_2 + c_2$. Let d and λ^* be diagonal and non-diagonal element of NN' when two lines i and$j$ belong to 2nd group. For $c_2 = 0$, it can be verified that d and λ^* are given by

$$d = \lambda^* = p_2c_1^2.$$

For $c_2 = 1$, let α be a inbred line belonging to 2nd group. Since all p_2 inbred lines belonging to 2nd group are replicated c_1 times in each block of p_2 blocks and α appears in only one block among p_2 blocks, we can see that the d and λ^* are given by

$$d = (c_1 + 1)^2 + (p_2 - c_2)c_1^2 = (c_1 + 1)^2 + (p_2 - c_2)c_1^2,$$
$$\lambda^* = 2c_1(c_1 + 1) + c_1^2(p_2 - 2).$$

Next, we find d and λ^* for $c_2 \geq 2$. Similarly for $c_2 = 1$, all p_2 inbred lines are replicated c_1 times in each block of p_2 blocks, and c_2 inbred lines belonging to 2nd group are repeated in each block of p_2 blocks. We now restrict attention to the case in which the design constructed by the c_2 lines appearing in each block is a balanced incomplete block design with parameters

$$v = b = p_2, r = k = c_2, \lambda = \frac{c_2(c_2 - 1)}{p_2 - 1}.$$

Let α, β be two lines belonging to 2nd group and $\lambda_{\alpha,\beta}$ be the number of concurrences of the lines α and β in the balanced incomplete block design. For determining d and λ^*, it is helpful to consider the following 3 cases separately for α, β in the balanced incomplete block design.

(i) α, β occur together,
(ii) $\alpha(\beta)$ occurs with a line other than $\beta(\alpha)$,
(iii) α, β don't occur in any block among p_2 blocks.

For (i), α, β occur together in λ blocks and α, β are repeated $(c_1 + 1)$ times, respectively. So, contribution to $\lambda_{\alpha,\beta}$ from λ blocks is $\lambda(c_1 + 1)^2$. For (ii), appear to $c_2 - \lambda$ blocks without $\beta(\alpha)$ and $\alpha(\beta)$ is repeated $(c_1 + 1)$ times in these blocks. So, contribution to $\lambda_{\alpha,\beta}$ from $c_2 - \lambda$ blocks is $2(c_2 - \lambda)c_1(c_1 + 1)$. Finally, for (iii), since the number of remaining blocks is given by $p_2 - \lambda - (c_2 - \lambda) - (c_2 - \lambda)$ and both α and β in these blocks are repeated c_1 times, it can be seen that contribution to $\lambda_{\alpha,\beta}$ from remaining blocks is given by $c_1^2(p_2 - 2c_2 + \lambda)$.
Thus,

$$d = (c_1 + 1)^2 + (p_2 - c_2)c_1^2,$$

$$\lambda^* = \lambda(c_1+1)^2 + 2(c_2-\lambda)c_1(c_1+1) + c_1^2(p_2 - 2c_2 + \lambda).$$

For the purpose of calculating efficiency of block design D_b, we now explain how to calculate the variance of a contrast among gca parameters within two groups of inbred lines.

Theorem 3.1 Let σ_1^2, σ_2^2 be the variance of a contrast among gca parameters within each group of inbred lines. Then,

$$\sigma_1^2 = Var_1(\hat{g}_i - \hat{g}_j) = \frac{2\sigma^2}{p_2}, \quad i,j < 1,2,\ldots,p_1,$$

$$\sigma_2^2 = Var_2(\hat{g}_i - \hat{g}_j) = \frac{2p_1\sigma^2}{p_2\lambda^*}, \quad i,j < p_1+1, p_1+2,\ldots,p_1+p_2.$$

Proof. For block design D,

$$C = \begin{bmatrix} p_2 I_{p_1} & J_{p_1p_2} \\ J_{p_2p_1} & p_1 I_{p_2} \end{bmatrix} - \frac{1}{p_1}\begin{bmatrix} p_2 I_{p_1} & p_1 J_{p_1p_2} \\ p_1 J_{p_2p_1} & B \end{bmatrix}$$
$$= \frac{1}{p_1}\begin{bmatrix} p_1p_2 I_{p_1} - p_2 J_{p_1} & 0 \\ 0 & p_1^2 I_{p_2} - B \end{bmatrix},$$

where

$$B = \begin{bmatrix} d & \lambda^* & \cdots & \lambda^* \\ \lambda^* & d & \cdots & \lambda^* \\ \vdots & \vdots & \ddots & \vdots \\ \lambda^* & \lambda^* & \cdots & d \end{bmatrix}.$$

Using d and λ^* , C is written by

$$C = \frac{1}{p_1}\begin{bmatrix} p_1p_2 I_{p_1} - p_2 J_{p_1} & 0 \\ 0 & \lambda^*(p_2 I_{p_2} - J_{p_2}) \end{bmatrix}$$
$$= \frac{1}{p_1}\begin{bmatrix} p_1p_2(I_{p_1} - \frac{1}{p_1}J_{p_1}) & 0 \\ 0 & \lambda^* p_2(I_{p_2} - \frac{1}{p_2}J_{p_2}) \end{bmatrix}.$$

From the above C, we can see that C^- is given by

$$C^- = p_1\begin{bmatrix} \left(p_1p_2(I_{p_1} - \frac{1}{p_1}J_{p_1})\right)^- & 0 \\ 0 & \left(\lambda^* p_2(I_{p_2} - \frac{1}{p_2}J_{p_2})\right)^- \end{bmatrix}$$

$$= \frac{1}{p_2}\begin{bmatrix} I_{p_1} & 0 \\ 0 & \frac{p_1}{\lambda^*}I_{p_2} \end{bmatrix}.$$

From above C^-, we can see that $\sigma_1^2 = 2\sigma^2/p_2$ and $\sigma_2^2 = 2p_1\sigma^2/p_2\lambda^*$.

4. TABLE OF DESIGNS

In this section, we now calculate efficiency of block design obtained using our method of construction and give efficiencies of block designs.
Let e_1^*, e_2^* be the efficiency in each group of D_b, respectively. Then the efficiencies e_1^* and e_2^* are as followings:

$$e_1^* = \frac{2\sigma^2}{p-2}/Var(\hat{g}_i - \hat{g}_j) = \frac{2\sigma^2}{p-2}/\frac{2\sigma^2}{p_2} = \frac{p_2}{p-2},$$

$$e_2^* = \frac{2\sigma^2}{p-2}/Var(\hat{g}_i - \hat{g}_j) = \frac{2\sigma^2}{p-2}/\frac{2p_1\sigma^2}{p_2\lambda^*} = \frac{p_2\lambda^*}{p_1(p-2)}.$$

Also, from the above e_1^* and e_2^*, we can see that the effciencies e_1 and e_2 using equation (16) of Singh and Hinkelmann(1998) are given by

$$e_1(Adjusted\ e_1^*) = \left(\frac{p(p-1)}{2}/n_c\right)e_1^* = \frac{p(p-1)}{2p_1p_2}e_1^* = \frac{(p_1+p_2)(p_1+p_2-1)}{2p_1(p_1+p_2-2)},$$

$$e_2(Adjusted\ e_2^*) = \left(\frac{p(p-1)}{2}/n_c\right)e_2^* = \left(\frac{p(p-1)}{2p_1p_2}\right)e_2^* = \frac{p\lambda^*}{2p_1^2}\left(\frac{p-1}{p-2}\right).$$

For $p \leq 24$, some numerical values of the efficiency various combinations of p_1 and p_2 can be founded in Table 1. The efficiency factors equal 1 for $p_1 = p_2$ or $p_1 = p_2 + 1$.

Table 1. Efficiencies of block designs

p_1	p_2	b	k	e_1	e_2	p_1	p_2	b	k	e_1	e_2
3	2	2	3	1	1	9	3	3	9	0.733	1
3	3	3	3	1	1	9	4	4	9	0.788	1
4	2	2	4	0.938	1	9	5	5	9	0.843	1
4	3	3	4	1	1	9	6	6	9	0.897	1
4	4	4	4	1	1	9	8	8	9	1	1
5	2	2	5	0.84	1	9	9	9	9	1	1
5	3	3	5	0.933	1	10	2	2	10	0.66	1
5	4	4	5	1	1	10	3	3	10	0.709	1
5	5	5	5	1	1	10	5	5	10	0.808	1
6	2	2	6	0.778	1	10	6	6	10	0.857	1
6	3	3	6	0.857	1	10	7	7	10	0.907	1
6	5	5	6	1	1	10	9	9	10	1	1
6	6	6	6	1	1	10	10	10	10	1	1
7	2	2	7	0.735	1	11	2	2	11	0.645	1
7	3	3	7	0.804	1	11	3	3	11	0.689	1
7	4	4	7	0.873	1	11	4	4	11	0.734	1
7	5	5	7	0.943	1	11	5	5	11	0.779	1
7	6	6	7	1	1	11	6	6	11	0.824	1
7	7	7	7	1	1	11	7	7	11	0.869	1
8	2	2	8	0.703	1	11	10	10	11	1	1
8	3	3	8	0.764	1	11	11	11	11	1	1
8	4	4	8	0.825	1	12	2	2	12	0.632	1
8	5	5	8	0.886	1	12	3	3	12	0.673	1
8	6	6	8	0.948	1	12	4	4	12	0.714	1
8	7	7	8	1	1	12	6	6	12	0.797	1
8	8	8	8	1	1	12	11	11	12	1	1
9	2	2	9	0.679	1	12	12	12	12	1	1

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