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Periodic PM Policy for Repairable System with RCW or NCW

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Abstract. This paper suggests the optimal periodic preventive maintenance policies after the combination warranty is expired. After the combination warranty is expired, a repairable system undergoes PM periodically and is minimally repaired at each failure. And also the system is replaced by a new system at the N th PM. In this case, we derive the mathematical formula for the expected cost rate per unit time. The optimal number and period for the periodic PM that minimize the expected cost rate per unit time are obtained. Some numerical examples are presented for illustrate purpose.

Key Words : *preventive maintenance, renewing combination warranty, non-renewing combination warranty, hazard rate, minimal repair.*

1. INTRODUCTION

Among various types of warranty terms, two types of warranty policies are widely offered: renewing warranty and non-renewing warranty. Under a renewing warranty, the failed system during the warranty period is replaced with a new one and the warranty is renewed. Under a non-renewing warranty, the manufacturer guarantees a satisfactory service only during the original warranty period. The renewing warranty policy or non-renewing warranty policy can be further classified into three warranty arrangements: free-replacement warranty (FRW), pro-rata warranty (PRW) and combination warranty (CW). Under FRW, the manufacturer maintains the system during the warranty period at no charge to the user. The PRW charges the user a proportion of maintenance cost that is

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prorated to the age of the system when the failure occurs during the warranty period. Under CW, the manufacturer will replace any system that fails prior to w_1 from the time of purchase with an identical new system at no cost to the user. From time w_1 to w, any system that fails is replaced with an identical new system at pro-rata cost to the user.

Preventive maintenance (PM) is the action taken on a system while it is still operating, which is carried out in order to keep the system at the desired level of operation. PM of a repairable system attracts a great deal of interests among engineers and reliability analysts and is one of the most important and practical areas in reliability theory. Canfield (1986) considers a periodic PM model for which the PM slows the degradation process of the system, while the hazard rate keeps increasing monotonically. Nakagawa (1986) studies periodic and sequential PM policies for the system with minimal repair: the PM is done (i) at periodic times kx and (ii) at constant intervals $x_k\ (k=1,2,\cdots,N)$. Kim, Yum and Park (2000) extend the periodic PM model proposed by Canfield (1986) to the case when the system undergoes the PMs at different intervals.

The optimal PM Policies with warranty have been proposed in the literature. Chun (1992) determines the optimal number of periodic PM operations during the warranty period. Jack and Dagpunar (1994) obtain the optimal number of PM actions to carry out over a warranty period under a free maintenance warranty. Yeh and Lo (2001) determine the optimal number of PM actions, corresponding maintenance degrees, and the maintenance schedule when the length of warranty period was pre-specified. Sahin and Polatoglu (1996) consider two maintenance policies following the expiration of warranty: 1) the user applies minimal repair for a fixed length of time and replaces the system by a new one at the end of this period, and 2) the system is replaced by the user at first failure following the minimal repair period. They study the optimal maintenance policies for both models under the renewing warranty and non-renewing warranty of a fixed period, respectively. Jung, Lee and Park (2000) propose two types of periodic PM policies following the expiration of warranty: the renewing free-replacement warranty (RFRW) and the renewing pro-rata warranty (RPRW).

In this paper, we suggest the optimal periodic PM policies following the expiration of combination warranty: renewing combination warranty (RCW) and non-renewing combination warranty (NCW). After the combination warranty is expired, the system is maintained preventively at periodic times $w+kx$ and is replaced by a new system at the Nth PM, where $k=1,2,\cdots,N$. If the system fails between PMs, it undergoes only minimal repair. To obtain the optimal period and the optimal number for the periodic PM model following the expiration of warranty, the expected cost rate per unit time is derived. That is, we use the expected cost rate per unit time to determine the optimality of the PM. Section 2 describes the periodic PM model following the expiration of combination warranty. Section 3 considers periodic PM policy after RCW is expired. The expected cost rate per unit time under the periodic PM model following the expiration of RCW is obtained and then the optimal PM policy is considered. In Section 4, we propose the optimal PM policy for repairable system after NCW is expired. In Sections 3 and 4, we use the property of pseudo-convexity of the cost rate function to determine the optimal periodic PM policy.

2. ASSUMPTIONS AND NOTATION

We consider the optimal periodic PM policies after the combination warranty is expired. The followings are assumed:

1. A manufacturer provides RCW or NCW.
2. After the combination warranty is expired, the system is maintained preventively at periodic times $w+kx$, $k=1,2,\cdots,N$.
3. The PM slows the rate of system degradation.
4. If the system fails between PMs, it undergoes only minimal repair.
5. The system is replaced by a new system at the N th PM.
6. The times to conduct PM, minimal repair and replacement are negligible.

Notation

T	time to failure of a system
w_1	free-replacement warranty period $(0, w_1)$
w_2	pro-rata warranty period (w_1, w)
w	combination warranty period $(0, w)$, $w = w_1 + w_2$
$h(t)$	hazard rate without PM
$h_{RCW}(t)$	hazard rate after RCW is expired
$h_{NCW}(t)$	hazard rate after NCW is expired
x	period of PM after combination warranty is expired
N	number of PM after combination warranty is expired

3. PERIODIC PM POLICY AFTER RCW IS EXPIRED

Let T denote the time to failure of a system and let w and x be the lengths of warranty period and PM period, respectively. We use $F(t)$ and $f(t)$ to denote the life distribution function of T and its corresponding density function, respectively. The following definitions are frequently referred to in this paper

Definition 3.1 The hazard rate of a life distribution F is defined as

$$h(t) = f(t) / \overline{F}(t)$$

for t such that $\overline{F}(t) > 0$, where $\overline{F}(t) = 1 - F(t)$.

Definition 3.2 A life distribution F is IFR (DFR) if $h(t)$ is non-decreasing (non-increasing) in t.

Under RCW, the manufacturer will replace any system that fails prior to w_1 from the time of purchase with an identical new system at no cost to the user. From time w_1 to w, any system that fails is replaced with an identical new system at pro-rata cost to the user. Upon failure of any system, the warranty begins anew.

To obtain the expected maintenance cost under the periodic PM model following the expiration of RCW, we assume that Canfield's periodic PM model (1986). For Canfield's model, each PM reduces operational stress to that existing α time units previous to the PM intervention, where α is a restoration interval and is less than or equal to the PM intervention interval. Thus, the hazard rate after the expiration of RCW becomes

$$h_{RCW}(t)=\begin{cases} h(t), & for\ w\le t\le w+x \\ \sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+w))-h(i(x-\alpha)+w)\}+h(t-k\alpha), & \\ & for\ w+kx\langle t\le w+(k+1)x, k=1,2,\cdots. \end{cases} \quad (1)$$

3.1 Expected Cost Rate Per Unit Time

We first determine the expected cost rate per unit time under the periodic PM model following the expiration of RCW. To do so, we need the expected maintenance cost and the expected cycle length. The expected maintenance cost for running the periodic PM following the expiration of RCW during the one cycle can be obtained as sum of the expected warranty cost $E(C_{w_1,w})$, the expected minimal repair cost $E(C_m)$, the expected periodic PM cost $E(C_{pm})$, the expected failure cost $E(C_f)$ and the expected replacement cost $E(C_r)$. If the system fails during the FRW period $(0, w_1)$, the failed system is replaced free of charge to the user by the manufacturer. And if the system fails during the PRW period (w_1, w), the failed system is replaced with a new one at a pro-rata cost to the user. Thus, the expected cost of replacement during the RCW period is given by

$$E(C_{w_1,w})=\frac{c_r}{w}\left(I(w)-I(w_1)\right),$$

where $I(s)=\int_0^s tf(t)dt$ and c_r is the unit cost of replacement. Using (1) and the results of Boland (1982), the expected cost of minimal repair during the PM period $(w, w+Nx)$ can be easily obtained as follows.

$$E(C_m)=c_m\overline{F}(w)\left[\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+w))-h(i(x-\alpha)+w)\}x + \sum_{k=0}^{N-1}\int_{kx+w}^{(k+1)x+w}h(t-k\alpha)dt\right],$$

where c_m is the unit cost of minimal repair. Since the system undergoes the PM periodically and is replaced by a new one at the N th PM, we have

$$E(C_{pm})=c_{pm}\overline{F}(w)(N-1),$$

$$E(C_r)=c_r\overline{F}(w),$$

where c_{pm} is the unit cost of PM. If a cycle ends with a warranty replacement, the user incurs a failure cost. And if a cycle contains a PM period, the user incurs a failure cost each time the system is minimal repaired in this period. Thus, the expected failure cost is given by

$$E(C_f) = c_{f,w}F(w) + c_{f,m}\overline{F}(w)\left[\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+w)) - h(i(x-\alpha)+w)\}x + \sum_{k=0}^{N-1}\int_{kx+w}^{(k+1)x+w} h(t-k\alpha)\,dt\right],$$

where $c_{f,w}$ is the unit failure cost during the warranty period and $c_{f,m}$ is the unit failure cost during the PM period.

Therefore, the expected maintenance cost under the periodic PM model following the expiration of RCW is given

$$EC_{RCW}(x,N) = A + B(c_m + c_{f,m}),$$

where

$$A = \frac{c_r}{w}\big(I(w) - I(w_1)\big) + \overline{F}(w)(c_{pm}(N-1) + c_r) + c_{f,w}F(w),$$

$$B = \overline{F}(w)\left[\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+w)) - h(i(x-\alpha)+w)\}x + \sum_{k=0}^{N-1}\int_{kx+w}^{(k+1)x+w} h(t-k\alpha)\,dt\right].$$

Now, we consider the expected cycle length of the system when the periodic PM policy is applied following the expiration of RCW. A cycle of the system begins with the installation of a new system. If the system fails during its warranty period $(T \rangle w)$, it is replaced by a new one under the same warranty terms and the cycle ends. Under the RCW, the cycle length is defined as the lifelength of the new system installed initially. If the system survives to age w $(T \rangle w)$, then the cycle is extended by a fixed number of periodic PM's with each PM having a period of fixed length x. Thus, if the system is replaced at the N th PM, the cycle is extended by Nx. Consequently, the expected cycle length can be represented as

$$EL_{RCW}(x,N) = \int_0^w tf(t)\,dt + (w+Nx)\overline{F}(w).$$

From the expected maintenance cost $EC_{RCW}(x,N)$ and the expected cycle length $EL_{RCW}(x,N)$, the expected cost rate per unit time under the periodic PM model following the expiration of RCW is given by

$$C_{RCW}(x,N) = \frac{A + B(c_m + c_{f,m})}{\int_0^w tf(t)\,dt + (w+Nx)\overline{F}(w)}. \tag{2}$$

3.2 Optimal Periodic PM Policy

This section considers the optimal periodic PM policy following the expiration of RCW. To determine the values of N and x which minimize $C_{RCW}(x,N)$, given in (2), we first find an optimal period x^* for a fixed N. Differentiating $C_{RCW}(x,N)$ with respect to x and setting it equal to 0, we obtain

$$a_1(a_2 + xa_3 + a_4) + N\overline{F}(w)(x^2 a_3 + xa_4 - a_5) = \frac{NA}{c_m + c_{f,m}}, \tag{3}$$

where

$$a_1 = I(w) + w\overline{F}(w),$$
$$a_2 = \sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+w)) - h(i(x-\alpha)+w)\},$$
$$a_3 = \sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h'((i-1)(x-\alpha)+(x+w))i - h'(i(x-\alpha)+w)i\},$$
$$a_4 = \sum_{k=0}^{N-1}\{(k+1)h((k+1)x+w-k\alpha) - kh(kx+w-k\alpha)\},$$
$$a_5 = \sum_{k=0}^{N-1}\int_{kx+w}^{(k+1)x+w} h(t-k\alpha)\,dt.$$

Theorem 3.1. Suppose that F is an IFR distribution with strictly increasing and convex hazard rate. Let $N \geq 1$ be given. Then, $C_{RCW}(x,N)$, given in (2), is pseudo-convex in x. Furthermore, $x^* = 0$ if and only if $a_1 h(w) \geq A/(c_m + c_{f,m})$ and $0 \langle x^* \langle \infty$ is the unique solution of (3) if and only if $a_1 h(w) \langle A/(c_m + c_{f,m})$.

Next, we consider the problem of finding the optimal period, x^*, and the optimal number of PM, N^*, prior to the replacement of the system, assuming that neither x nor N is fixed. To solve the problem, we first determine x_N as a function of N satisfying (3). If the conditions of Theorem 3.1 is satisfied, then x_N exists and is uniquely determined. Thus, the expected maintenance cost rate per unit time during one cycle can be expressed as a function of N as follows.

$$C_{RCW}(x_N, N) = \frac{A + B_N(c_m + c_{f,m})}{\int_0^w tf(t)\,dt + (w + Nx_N)\overline{F}(w)}, \tag{4}$$

where

$$B_N = \overline{F}(w)\left[\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x_N-\alpha)+(x_N+w)) - h(i(x_N-\alpha)+w)\}x_N + \sum_{k=0}^{N-1}\int_{kx_N+w}^{(k+1)x_N+w} h(t-k\alpha)\,dt\right].$$

Since the formula (4) is a function of N alone, N^* can be determined by

$$N^* = \min_N C_{RCW}(x_N, N), \quad N = 1, 2, \cdots. \tag{5}$$

Theorem 3.2. Suppose that F is an IFR distribution with strictly increasing and convex hazard rate and let $N \geq 1$ be given. Then, there exists a x_N which satisfies (3) and thus, the value of N^* satisfying (5) is the optimal number of PM which minimizes the expected cost rate per unit time, given in (2).

3.3 Numerical Example

Suppose that the failure time of a system follows a Weibull distribution i.e., $h(t)=\beta\lambda^{\beta}t^{\beta-1}$ for $\beta>2$ and $t\geq 0$. We investigate the pattern changes of x^*, N^* and $C_{RCW}(x^*,N^*)$ for various choices of w_1 and c_r when β, w, λ, c_m, c_{pm}, $c_{f,w}$ and $c_{f,m}$ are fixed. Table 3.1 gives the values of x^*, N^* and $C_{RCW}(x^*,N^*)$ for various choices of w_1 and c_r. The pair (x^*,N^*) is determined so that the expected cost rate per unit time under the periodic PM model following the expiration of RCW is minimized for the given values. Table 1 shows that as the value of c_r increases for a fixed w_1, the values of N^* and $C_{RCW}(x^*,N^*)$ increase. It is also observed that when w_1 becomes longer for fixed c_r, $C_{RCW}(x^*,N^*)$ decreases.

Table 3.1 Optimal periodic PM policies after RCW is expired
($\alpha=x,\beta=3,w=0.5$, $\lambda=1$, $c_m=c_{pm}=1$, $c_{f,w}=c_{f,m}=1$)

		w_1			
		0.1	0.2	0.3	0.4
$c_r=5$	x^*	0.74935	0.74835	0.74404	0.73262
	N^*	1	1	1	1
	$C_{RCW}(x^*,N^*)$	6.08741	6.07764	6.03577	5.92551
$c_r=10$	x^*	1.08637	1.08512	1.07973	1.06546
	N^*	1	1	1	1
	$C_{RCW}(x^*,N^*)$	9.81462	9.79911	9.73262	9.55754
$c_r=15$	x^*	0.57085	0.76815	0.76471	0.75562
	N^*	3	2	2	2
	$C_{RCW}(x^*,N^*)$	12.73750	12.71960	12.6414	12.43560

4. PERIODIC PM POLICY AFTER NCW IS EXPIRED

This section considers the periodic PM policy following the expiration of NCW. Under the NCW, the manufacturer replaces any system that fails prior to w_1 from the time of purchase with an identical new system at no cost to the user. From w_1 to w, any system that fails is replaced by an identical new system at a pro-rata cost to the user. All replaced systems assume remaining time and terms of the original warranty.

Under the RCW, the age of the system surviving to the end of a warranty period is always w. However, under the NCW, the age of the system surviving to the end of a warranty period could be anywhere between 0 and w. Usually, this age should be available to the user at the end of the warranty period. Let y and k denote the age of the system in use at the end of a NCW period and the number of replacements during a NCW period. If y equals to w, then k is zero and if k is zero, then y equals to w.

As assumed in Section 3, to obtain the expected maintenance cost under the periodic PM model following the expiration of NCW, we assume that Canfield's periodic PM model (1986). Thus, the hazard rate after the expiration of NCW becomes

$$h_{NCW}(t) = \begin{cases} h(t), & \text{for } y \le t \le y + x \\ \sum_{i=1}^{k} \{h((i-1)(x-\alpha)+(x+y)) - h(i(x-\alpha)+y)\} + h(t-k\alpha), & \\ & \text{for } y + kx \langle t \le y + (k+1)x, k = 1, 2, \cdots. \end{cases} \quad (6)$$

4.1 Expected Cost Rate Per Unit Time

To obtain the expected maintenance cost under the periodic PM model following the expiration of NCW, we need the expected maintenance cost and the expected cycle length. Because the all replaced systems assume remaining time and terms of the original warranty, the expected cycle length can be easily obtained as follows.

$$EL_{NCW}(x, N) = w + Nx.$$

The expected maintenance cost under the periodic PM model following the expiration of NCW can be obtained as sum of the expected warranty cost $E(C_{w_1,w})$, the expected minimal repair cost $E(C_m)$, the expected periodic PM cost $E(C_{pm})$, the expected failure cost $E(C_f)$ and the expected replacement cost $E(C_r)$. If the system fails during the FRW period $(0, w_1)$, the failed system is replaced free of charge to the user by the manufacturer. And if the system fails during the PRW period (w_1, w), the failed system is replaced with a new one at a pro-rata cost to the user. Also, the age of the system in use at the end of a NCW period is y and the all replaced systems assume remaining time and terms of the original warranty. Thus, the expected cost of replacement during the NCW period is given by

$$E(C_{w_1,w}) = \begin{cases} c_r\left(\dfrac{(w-w_1)-y}{w-w_1}\right), & 0 \le y \langle w - w_1 \\ 0, & y \ge w - w_1. \end{cases}$$

Using (6) and the results of Boland (1982), the expected cost of minimal repair during the PM period $(y, y+Nx)$ can be obtained as follows.

$$E(C_m) = c_m\left[\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+y)) - h(i(x-\alpha)+y)\}x + \sum_{k=0}^{N-1}\int_{kx+y}^{(k+1)x+y} h(t-k\alpha)\,dt\right]$$

Since the system undergoes the PM periodically and is replaced by a new one at the N th PM, we have

$$E(C_{pm}) = c_{pm}(N-1),$$

$$E(C_r) = c_r.$$

If a cycle ends with a warranty replacement, the user incurs a failure cost. And if a cycle contains a PM period, the user incurs a failure cost each time the system is minimal repaired in this period. Since k is the number of replacements during the NCW period, the expected failure cost is given by

$$E(C_f) = k\,c_{f,w} + c_{f,m}\left[\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+y)) - h(i(x-\alpha)+y)\}x + \sum_{k=0}^{N-1}\int_{kx+y}^{(k+1)x+y} h(t-k\alpha)\,dt\right].$$

Thus, the expected maintenance cost under the periodic PM model following the expiration of NCW is as follows.

$$EC_{NCW}(x,N) = C + D(c_m + c_{f,m}),$$

where

$$C = \begin{cases} c_r\left(\dfrac{(w-w_1)-y}{w-w_1}\right) + (N-1)c_{pm} + c_r + k\,c_{f,w}, & 0 \le y \langle w-w_1 \\ (N-1)c_{pm} + c_r + k\,c_{f,w}, & y \ge w-w_1, \end{cases}$$

$$D = \left[\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+y)) - h(i(x-\alpha)+y)\}x + \sum_{k=0}^{N-1}\int_{kx+y}^{(k+1)x+y} h(t-k\alpha)\,dt\right]$$

From the expected maintenance cost $EC_{NCW}(x,N)$ and the expected cycle length $EL_{NCW}(x,N)$, the expected cost rate per unit time under the periodic PM model following the expiration of NCW given by

$$C_{NCW}(x,N) = \frac{C + D(c_m + c_{f,m})}{w + Nx}. \qquad (7)$$

4.2 Optimal Periodic PM Policy

To determine the values of N and x which minimize $C_{NCW}(x,N)$, given in (7), we first find an optimal PM period x^* for a repairable system with NCW when N is assumed to be fixed. Differentiating $C_{NCW}(x,N)$ with respect to x and setting it equal to 0, we obtain

$$w(b_1 + xb_2 + b_3) + N(x^2 b_2 + xb_3 - b_4) = \frac{NC}{c_m + c_{f,m}}, \qquad (8)$$

where

$$b_1 = \sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x-\alpha)+(x+y))-h(i(x-\alpha)+y)\},$$
$$b_2 = \sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h'((i-1)(x-\alpha)+(x+y))i-h'(i(x-\alpha)+y)i\},$$
$$b_3 = \sum_{k=0}^{N-1}\{(k+1)h((k+1)x+y-k\alpha)-kh(kx+y-k\alpha)\},$$
$$b_4 = \sum_{k=0}^{N-1}\int_{kx+y}^{(k+1)x+y} h(t-k\alpha)\,dt.$$

Theorem 4.1. Suppose that F is an IFR distribution with strictly increasing and convex hazard rate. Let $N \geq 1$ be given. Then, $C_{NCW}(x,N)$, given in (7), is pseudo-convex in x. Furthermore, $x^* = 0$ if and only if $wh(y) \geq C/(c_m + c_{f,m})$ and $0 \langle x^* \langle \infty$ is the unique solution of (8) if and only if $wh(y) \langle C/(c_m + c_{f,m})$.

When neither x nor N is assumed to be known, we first need to find the value of x as a function of N satisfying (8), denoting it by x_N. Such a value is uniquely determined under the conditions given in Theorem 4.1. Replacing x in the expression for $C_{NCW}(x,N)$ of (7) by x_N, we obtain

$$C_{NCW}(x_N,N) = \frac{C + D_N(c_m + c_{f,m})}{w + Nx_N}, \tag{9}$$

where

$$D_N = \left[\sum_{k=1}^{N-1}\sum_{i=1}^{k}\{h((i-1)(x_N-\alpha)+(x_N+y))-h(i(x_N-\alpha)+y)\}x_N + \sum_{k=0}^{N-1}\int_{kx_N+y}^{(k+1)x_N+y} h(t-k\alpha)\,dt\right].$$

Since the formula (9) is a function of N alone, N^* can be determined by

$$N^* = \min_N C_{NCW}(x_N,N),\ N = 1,2,\cdots. \tag{10}$$

Theorem 4.2. Suppose that F is an IFR distribution with strictly increasing and convex hazard rate and let $N \geq 1$ be given. Then, there exists a x_N which satisfies (8) and thus, the value of N^* satisfying (10) is the optimal number of PM which minimizes the expected cost rate per unit time, given in (7).

Once the value of N^* is determined, the optimal period x^* can be computed by replacing N of (8) by N^* and by solving the resulting equation.

4.3 Numerical Example

Suppose that the failure time of a system follows a Weibull distribution i.e., $h(t) = \beta\lambda^{\beta} t^{\beta-1}$ for $\beta > 2$ and $t \geq 0$. We investigate the pattern changes of x^*, N^* and $C_{NCW}(x^*, N^*)$ for various choices of y and c_r when β, w, w_1, c_m, c_{pm}, $c_{f,w}$ and $c_{f,m}$ are fixed. Table 4.1 gives the values of x^*, N^* and $C_{NCW}(x^*, N^*)$ for various choices of y and c_r. Table 4.1 shows that as the value of c_r increases for a fixed w_1, the values of N^* and $C_{NCW}(x^*, N^*)$ increase. The total waiting time for the next replacement by user, which is the product of x^* and N^*, is shown to be decreasing as y increases for a fixed c_r.

Table 4.1 Optimal periodic PM policies after NCW is expired
($\alpha = x$, $\beta = 3, w = 0.5, w_1 = 0.2$, $\lambda = 1$, $c_m = c_{pm} = 1$, $c_{f,w} = c_{f,m} = 1$, $k = 1$)

		y			
		0.1	0.15	0.2	0.25
$c_r = 5$	x^*	0.74767	1.13652	1.05330	0.96574
	N^*	2	1	1	1
	$C_{NCW}(x^*, N^*)$	6.65572	6.45503	6.12598	5.76431
$c_r = 10$	x^*	0.43120	0.49339	0.52825	0.58894
	N^*	7	5	4	3
	$C_{NCW}(x^*, N^*)$	9.64457	9.61984	9.43794	9.09980
$c_r = 15$	x^*	0.36913	0.41728	0.46544	0.48488
	N^*	11	8	6	5
	$C_{NCW}(x^*, N^*)$	11.70860	11.80280	11.6939	11.38990

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