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Calculation of Effective Angular Correlation in the HPGe Spectroscopy of Co-60 γ-rays

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Abstract

The angular correlation effect was investigated for Co-60 γ-ray spectroscopy by using HPGe detector and the effective angular correlation was theoretically calculated by considering the finite detector solid angle. For the calculation of effective angular correlation, the detection efficiency as a function of γ-ray incident direction was obtained by using Monte Carlo method and the first interaction model. The results and the methods used in the calculation are discussed.

Key Words : angular correlation effect, Co-60, HPGe detector, effective angular correlation, Monte Carlo method, the first interaction model

1. Introduction

In general, γ-rays are emitted isotropically from radioisotopes because the spin direction of decaying nucleus is distributed randomly in space. In case of cascade γ-rays from a nucleus, however, the angular distribution of the direction of following γ-ray with respect to that of the preceding γ-ray is non-isotropic due to the angular momenta couplings between the emitted γ-rays and the excited nucleus. This effect has been the study topic by numerous groups[1-3], and also been measured[4-11] to establish the nuclear structure information such as the nuclear level spin-parity and the γ-ray multi-polarity. For some nuclei, the measurement is still going on and reported[12]. Nevertheless, the angular correlation effect is typically not considered nor evaluated quantitatively in the field of radioactivity measurement. Here are some examples of the most sensitive to the angular correlation effect, coincidence summing correction[13-15] and sum peak method[16] by using coincidence counting or coincident sum peak counting. However, the effect is usually ignored because firstly, it is smaller than those due to other causes like dead time, pile-up, etc. and secondly, so much time and effort are involved to evaluate the effect. But both the correction of coincidence summing and the absolute activity determined by sum-peak method concern the accuracy level of less than a few % while in the case of Co-60, the widely used radioisotope in many applications, the angular correlation between the two cascade γ-rays is known very strong and, even further complicated, the angular correlation effect so much depends on the geometric condition of detection. Therefore it is necessary to evaluate properly the angular correlation effect for individual detection system and isotope in order to enhance the accuracy of counting measurement.

The theory for the angular correlation of cascade γ-rays is already established so well that it is possible to calculate the angular correlation of any cascade γ-rays[17]. And it is reported that the measurement agrees with the theory very well for some isotopes like Co-60[18] whose γ-rays have pure multi-polarity and are little interfered in the nucleus. The measurement of angular correlation uses a collimator with small opening angle in front of the detector to define the angle sharp and to improve the angular resolution. In the case of coincidence counting or coincidence sum peak counting, however, the full detector solid angle is used typically to increase the coincidence count rate. Then the measured effect of angular correlation is smeared significantly due to the geometric condition. Hence the effective angular correlation is considered by calculating the angular resolution in terms of the γ-ray detection efficiency, $\epsilon(\beta)$, as a function of incident direction of γ-rays[19-21]. The required angular detection efficiency $\epsilon(\beta)$ can be obtained by measurement or calculation. *Measuring $\epsilon(\beta)$ is a direct and obvious* way, but it involves a considerable work and is restricted to the available collimator geometry. Calculating $\epsilon(\beta)$ is also complicated but versatile since calculation is possible for any detection condition. The most accurate method of calculating $\epsilon(\beta)$ is so far by using Monte Carlo simulation of the γ-ray absorption process in the detector. The Monte Carlo method is not restricted by detection geometry but able to calculate the efficiency with sufficient accuracy at the expense of time. An alternative method of calculating $\epsilon(\beta)$ to Monte Carlo method is sought in this study, which is the first interaction model. The first interaction model is an approximate method but makes the calculation so fast and simple that it is the most practical method to use in assessing the effective angular correlation for any given γ-ray or detector type and geometry. The calculated $\epsilon(\beta)$ by using Monte Carlo method and by the first interaction model, and the calculated effective angular correlation in Co-60 γ-rays for a HPGe detector are discussed.

2. Effective Angular Correlation of Cascade γ-rays

The angular correlation $W(\theta)$ between cascade γ-rays γ_1, γ_2 emitted from the transition $I_i \xrightarrow{\gamma_1} I \xrightarrow{\gamma_2} I_f$ (I_i, I, I_f : level spins) is given by[17],

$$W(\theta) = \sum_{\text{even } k}^{k_{max}} A_{kk} P_k(\cos\theta), \qquad (1)$$

where θ is the angle between the γ-ray directions, P_k is the Legendre polynomial and A_{kk} is the angular correlation coefficient. By denoting L_1, L_1' and L_2, L_2' for the multi-polarities of γ_1, γ_2 respectively, k_{max} and A_{kk} are given by

$$k_{max} = \min(2I, L_1 + L_1', L_2 + L_2'), \qquad (2)$$

$$A_{kk}=A_k(L_1L_1'I_iI)A_k(L_2L_2'I_fI), \quad (3)$$

where A_k is

$$A_k(L_1L_1'I_iI)=\frac{F_k(L_1L_1I_iI)+2\delta_1(\gamma_1)F_k(L_1L_1'I_iI)+\delta_1^2(\gamma_1)F_k(L_1'L_1'I_iI)}{1+\delta_1^2(\gamma_1)}. \quad (4)$$

Here, $\delta_1(\gamma_1)$ is the multi-polarity mixing ratio of γ_1 given by

$$\delta_1(\gamma_1)\equiv\frac{\langle I\,\|\,L_1'\pi_1'\,\|\,I_i\rangle}{\langle I\,\|\,L_1\pi_1\,\|\,I_i\rangle}, \quad (5)$$

and π is the parity of the transition. The F-coefficient $F_k(LL'I_iI)$ is given,

$$F_k(LL'I_iI)=(-1)^{I_i+I-1}[(2L+1)(2L'+1)(2I+1)(2k+1)]^{\frac{1}{2}}\begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix}\begin{Bmatrix} L & L' & k \\ I & I & I_i \end{Bmatrix}, \quad (6)$$

where ( ) and { } are the 3-j and 6-j symbol, respectively. $A_k(L_2L_2'I_fI)$, $\delta_2(\gamma_2)$ and $F_k(LL'I_fI)$ are given in the same way for γ_2. The multi-polarity mixing ratio $\delta(\gamma)$ can be obtained by theoretical consideration on the nuclear level structure involved or by direct measurement of angular correlation.

The effective angular correlation $\overline{W(\theta)}$ is the experimentally observed quantity when the used detector has finite aperture. Fig.1 represents the experimental geometry of effective angular correlation of γ_1, γ_2 by the two detector system (a) and by a single detector system (b). For the two detector system, the effective angular correlation is given by[19-21]

$$\overline{W(\theta)}=\sum_{\text{even }k}^{k_{max}} A_{kk}P_k(\cos\theta)\,Q_k(1)Q_k(2). \quad (7)$$

Here, Q_k is the attenuation correction factor and 1 or 2 in the parenthesis denotes that it is related to γ_1 with detector 1 or γ_2 with detector 2, respectively. Q_k is given below,

$$Q_k=J_k/J_0, \quad (8)$$

$$J_k(1)=\int_0^{\beta_1^{max}} d\beta_1\sin(\beta_1)\,P_k(\cos\beta_1)\varepsilon_1(\beta_1), \quad (9)$$

$$J_k(2)=\int_0^{\beta_2^{max}} d\beta_2\sin(\beta_2)\,P_k(\cos\beta_2)\varepsilon_2(\beta_2), \quad (10)$$

where θ is the angle between the two detector axes, β is the γ-ray incidence angle with respect to the detector axis, β^{max} is the maximum angle of β and the subscript 1, 2 indicates that it is related to γ_1 with detector 1 and γ_2 with detector 2, respectively. $\varepsilon(\beta)$ is the detection efficiency for γ-rays incident on the detector within a unit solid angle around the incidence angle β and hence corresponds to the angular component of the

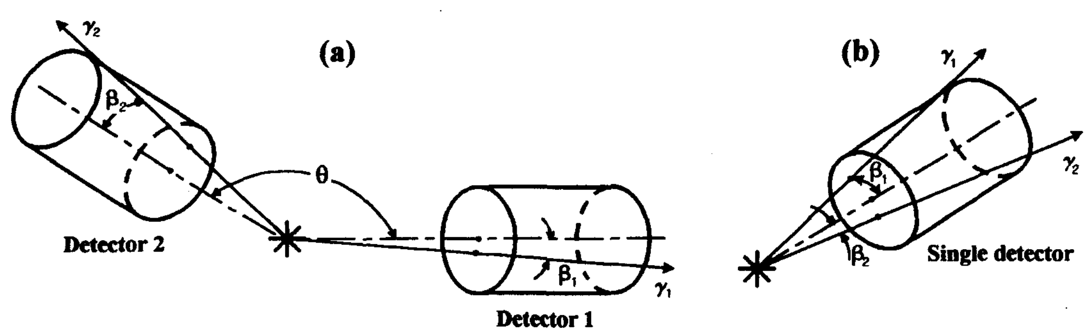

Fig. 1. Cascade γ-ray Detection System (a) with Two Detectors, (b) with a Single Detector.

total absorption peak efficiency in HPGe spectroscopy. When only a single detector is used for the measurment of cascade γ-rays as shown in Fig.1(b), the geometric condition is equivalent to the identical two detectors arranged at the same position with $\theta = 0$. Hence, the effective angular correlation for the single detector system is $\overline{W(0)}$.

3. Calculation

3.1. Total Absorption Peak Efficiency Depending on the γ-ray Incidence Angle

The total absorption peak efficiency depending on the γ-ray incidence angle, $\epsilon(\beta)$, for a HPGe detector is calculated for the 1.17 MeV and 1.33 MeV γ-rays from Co-60. In this calculation, the Monte Carlo method and the first interaction model are used with the detection geometry shown in Fig.2 and the calculation range of the source-to-detector distance is 0~30 cm. The HPGe detector modeled for the calculation is 72 cm^3 closed-ended coaxial type, the crystal diameter is 5.05 cm and the length is 3.65 cm. The detailed specification is given in ref.[22].

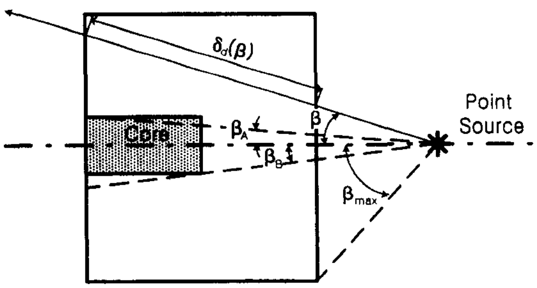

Fig. 2. The Geometrical Condition of Source and the HPGe Detector. The Three Angles β_A,β_B and β_{max} are Defined for the Three Distinct Directions Cutting the Edges of the Detector Core and Sensitive Volume, Respectively.

3.1.1. Monte Carlo Method

Monte Carlo method was used to simulate in detail the γ-ray absorption processes in the HPGe detector, and $\epsilon(\beta)$ was obtained from the result of the simulation. The Monte Carlo code used in this study is the modified version of that developed in the previous study[22] on the peak energy shift depending on the source position. The γ-ray generation part and the history recording part were rewritten to control the polar angle β of γ-rays emitted from the source and to record the interactions for each polar angle β. Then, $\epsilon(\beta)$ was calculated from the histories leading to full energy absorptions among those generated with polar angle β. The full energy absorption was defined as the history in which neither the secondary electrons nor K X-rays from Ge escape from the detector sensitive volume. The attenuation effect at detector end cap, IR window and the dead layer of Ge crystal was also considered in. The range of zero to the maximum polar angle β_{max} was divided into 200 bins with equal angular span, and more than 10,000 γ-rays were generated for each β bin. It took more than 10 hours to complete a calculation at a given source-to-detector distance for a single γ-ray energy on a Pentium PC platform. The results for three source-to-detector distances are presented in Fig.3.

3.1.2. First Interaction Model

Since the main mechanism of γ-ray energy deposition to the detector is photoelectric absorption, Compton scattering and pair production, $\epsilon(\beta)$ can be given by

$$\epsilon(\beta) = (1 - e^{-\mu_d \delta_d(\beta)}) \times N(\beta), \tag{11}$$

where μ_d is the total linear attenuation coefficient of Ge to the incident γ-ray, $\delta_d(\beta)$ is the extended

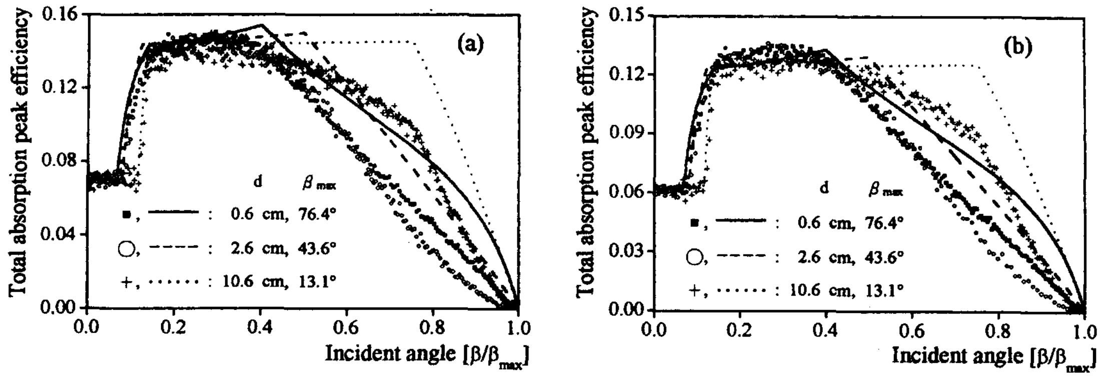

Fig. 3. Total Absorption Peak Efficiency $\epsilon(\beta)$ Obtained by Monte Carlo Method (symbols) and the First Interaction Model (lines) for (a) 1.17 MeV, (b) 1.33 MeV γ-rays at Several Source-to-detector Distances d. The Maximum Incidence Angle at Each Source-to-detector Distance is Normalized to 1 in the Abscissa.

path length through the detector sensitive volume in the direction of incidence angle β as illustrated in Fig.2, and $N(\beta)$ is the fraction of the first interactions which afterwards leads to total absorption by multiple interactions. For the γ-rays with energy larger than about 200 keV, the full energy absorption takes place, in most cases, after several interactions. For the HPGe detector used in this study, the full energy absorption of 1.33 MeV γ-ray occurs most probably (52%) by 3 or 4 multiple interactions[22]. $N(\beta)$ is dependent on the incidence angle and energy of γ-rays and also on the detection geometry while its analytic form is not known.

In the first interaction model, $N(\beta)$ is taken to be a constant, independent of the γ-ray incidence angle, but only depends on the γ-ray energy. Therefore $\epsilon(\beta)$ is approximated by

$$\epsilon(\beta) \approx N \times (1 - e^{-\mu_d \delta_d(\beta)}). \qquad (12)$$

Here N is a constant but an appropriate value for N is not required since it is canceled in calculating the effective angular correlation according to eqs. (8) ~ (10). The calculated $\epsilon(\beta)$ is shown in Fig.3, being compared with that obtained by using Monte Carlo method, while assuming N to be 0.223 and 0.208 for 1.17 and 1.33 MeV γ-rays, respectively.

3.2. Effective Angular Correlation

Effective angular correlation was calculated by using $\epsilon(\beta)$' s obtained with Monte Carlo method and those by the first interaction model. Due to the statistical nature of Monte Carlo method, the resultant $\epsilon(\beta)$ was further smoothened by three fitting curves before calculating the integration for the attenuation correction factor Q_k. By dividing the range of γ-ray incidence angle into three regions as shown in Fig.2, the trial fitting curves $f_\epsilon(\beta)$ used in this study were

$$f_\epsilon(\beta) = c \qquad (0 \leq \beta < \beta_A), \quad (13\text{-a})$$

$$f_\epsilon(\beta) = b_0 + b_1\beta \qquad (\beta_A \leq \beta < \beta_B), \quad (13\text{-b})$$

$$f_\epsilon(\beta) = (1 - e^{-\mu_d \delta_d(\beta)}) \times \sum_{n=0}^{4} a_n \beta^n \quad (\beta_B \leq \beta < \beta_{max}), \quad (13\text{-c})$$

where the coefficients a, b, c' s were determined by least squares fit to the Monte Carlo simulated $\epsilon(\beta)$. In the first interaction model, explicit form of

Table 1. The F-coefficients and A_{kk}' s Theoretically Calculated and Measured[18].

	F-coefficients		Theory	Measured[18]
k = 0	$F_0(2242) = 1$,	$F_0(2202) = 1$	$A_{00} = 1$	$A_{00} = 1$(by definition)
k = 2	$F_2(2242) = -0.1707$,	$F_2(2202) = -0.5976$	$A_{22} = 0.1020$	$A_{22} = 0.101 \pm 0.003$
k = 4	$F_4(2242) = -0.0085$,	$F_4(2202) = -1.069$	$A_{44} = 0.0091$	$A_{44} = 0.014 \pm 0.004$

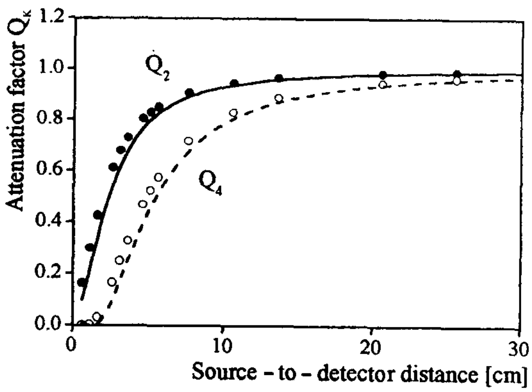

Fig. 4. The Attenuation Factor Q_k vs. Source-to-detector Distance. The Symbols and Lines are Calculated by Using Efficiency Data Obtained by Monte Carlo Method and the First Interaction Model, Respectively.

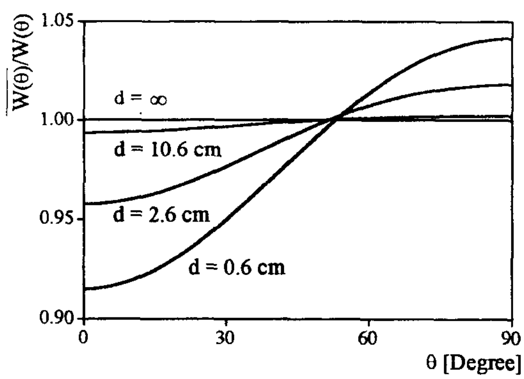

Fig. 5. The Relative Value of Effective Angular Correlation $\overline{W(\theta)}$, Obtained by Monte Carlo Method, to $W(\theta)$ for Several Source-to-detector Distances d.

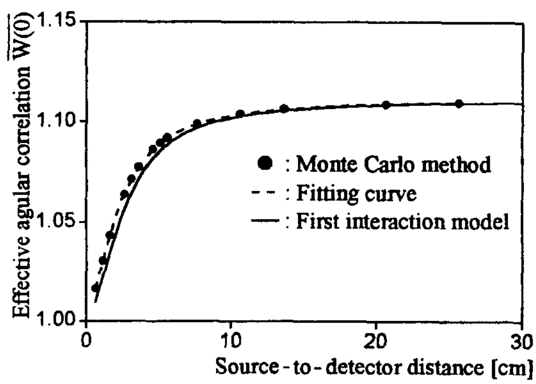

Fig. 6. The Effective Angular Correlation at Zero Degree $\overline{W(0)}$ vs. Source-to-detector Distance.

$\epsilon(\beta)$ was given and, hence, used in the integration directly. The obtained attenuation correction factors Q_k are presented in Fig.4.

The angular correlation coefficient A_{kk} for the cascade γ-rays from Co-60 is given by

$$A_{kk} = F_k(LLI_iI)F_k(LLI_fI), \qquad (14)$$

where L = 2 for both γ-rays have a pure multipolarity E2, and $I_i = 4$, $I = 2$, $I_f = 0$. The F-coefficients[17] and A_{kk}' s are shown in Table 1 with comparison to a measurement[18].

Finally, the effective angular correlation $\overline{W(\theta)}$ was presented at Fig.5 and Fig.6. In Fig. 5, the relative value of $\overline{W(\theta)}$ to $W(\theta)$ is shown to see the effect of angular smearing for several source-to-detector distances. In Fig. 6, the effective angular correlation at zero degree $\overline{W(0)}$ is shown for the source-to-detector distance up to 30 cm. The dotted line in Fig. 6 is the fitting curve $f_w(d)$ of $\overline{W(0)}$ to the discrete results obtained by Monte Carlo method and is given by

$$f_W(d) = \frac{a_1 - a_2}{1+(d/d_0)^p} + a_2, \quad (15)$$

where the parameters are; a_1 = 1.006, a_2 = 1.112, p = 1.662, and d_0 = 2.341 cm. The difference of effective angular correlation, $\overline{W(\theta)}$, from $W(\theta)$ is considerable for small source-to-detector distance, and particularly for special direction angles of zero and 90°. Hence simple substitution of $W(\theta)$ to $\overline{W(\theta)}$ is not valid. At small source-to-detector distance, the angular smearing effect is enhanced so much that the effective angular correlation at zero degree, $\overline{W(0)}$, becomes nearly 1. But at long source-to-detector distance $\overline{W(0)}$ approaches to 1.11 which corresponds to W(0) the sum of angular correlation coefficients.

4. Discussion and Conclusions

The results of attenuation correction factor Q_k and effective angular correlation $\overline{W(\theta)}$ calculated by the two methods of Monte Carlo and first interaction agreed to each other in less than a percent. However, $\epsilon(\beta)$ showed much difference between the two methods. The deviations are considerable in the region of $\beta/\beta_{max} > 0.4$, where the γ-ray incident paths are towards the edge of the detector sensitive volume. The difference indicates that the approximation of the first interaction model is crude. Fortunately the difference in $\epsilon(\beta)$ by the two methods was attenuated in calculating Q_k and $\overline{W(\theta)}$. It is because $\epsilon(\beta)$ is multiplied with Legendre polynomial, sinusoidal function and integrated over the detector solid angle in the calculation of Q_k and $\overline{W(\theta)}$. Noticeable difference in $\overline{W(0)}$ by the two methods is seen only for the source-to-detector distance less than 4 cm, but the magnitude of deviation is less than 0.8%. Even though Monte Carlo method provides detailed information for γ-ray absorption processes in the detector, the time and efforts are considerable to undertake. The current study shows that the first interaction model, even though an approximation, makes the calculation simple and fast with sufficient accuracy for the effective angular correlation. Therefore, the first interaction model can be used for various applications where the calculation of effective angular correlation is required.

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