■ 論 文 ■

An Equality-Based Model for Real-Time Application of A Dynamic Traffic Assignment Model

동적통행배정모형의 실시간 적용을 위한 변동등식의 응용

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요 약

본 연구에서 변동등식에 근거한 동적경로선택조건을 도출하여 동적통행배정모형을 제안한다. 동적경로선택조건은 운전자에 의해 이용된 경로, 링크 그리고 출발지의 출발시간을 고려하여 도출되며 이 조건을 동적통행배정모형으로 전환하는 과정에서 모형의 변동등식문제로 압축된다. 등식이론에 근거한 모델의 이론적 배경을 입증하기 위해 제안된 동적통행배정모형이 필요충분조건을 만족함이 증명되었다. 해법으로서 기존의 제안된 네트워크의 시간과 공간확장기법을 채택하지 않고 물리적 네트워크가 직접 알고리즘에 반영되도록 하기위해 각 링크의 시간대별 통행량과 방출통행량을 링크진입통행으로 표현하여 시간종속 통행시간함수를 단일변수로 처리하여 대각화알고리즘에 반영하였다.

소규모 비대칭 네트워크 적용결과 사용자 동적최적경로선택조건이 만족됨을 입증하였는데 단위시간간격이 적을수록 개선된 효과를 보여준다. I-394네트워크 실험결과로서 기존의 변동부등식에 근거한 알고리즘에 비해 제안된 알고리즘이 최소한93%이상의 컴퓨터연산 속도의 개선효과를 가져왔다. 등식이론에 근거한 모델개발의 장점으로서는 제안된 모델의 최적해의 계산시간이 전체시간의 증가에 전혀 영향을 받지 않는 다는 것으로 이는 동적통행배정모형에 적용될 네트워크의 규모가 커질수록 제안된 알고리즘의 계산 효율성은 더욱 증가하는 것을 의미한다. 따라서 제안된 동적통행배정모형은 대규모 시간종속적 교통망에서 교통상황의 변화에 민감하게 반응할 수 있는 실시간첨단교통제어의 핵심기능으로서 역할수행이 기대된다.

Ⅰ. Introduction

Dynamic Traffic Assignment(DTA), as an embedded technology in the Advanced Transportation Management and Information System(ATMIS), provides a realistic representation of the traffic flow and routing behavior by considering traffic variations in real-time. DTA has become a focus of research for the last decade and is gradually maturing. Most DTA models can be classified into two categories: macroscopic models which are flow-based and microscopic models which are vehicle-based. The macroscopic approach uses flow, density, and speed as the basic variables, and is usually in the form of analytical models. The microscopic approach, on the other hand, uses the characteristics of individual drivers and/or vehicles as variables and usually uses simulation technology to formulate and solve the problem.

INTEGRATION(Van Aerde, 1994), DYNASMART (Mahmassani et al, 1993), and MIT model(Ben-Akiva, et al, 1997) are well known models in the category of simulation-based models. In these models, each vehicle is modeled in detail with its own capability and restrictions. Thus, the simulation models are able to offer very detailed dynamic representation of vehicle movements and driver characteristics, such as acceleration and car-following behavior, as well as emulate and evaluate the impact of different control strategies.

Within the analytical model category, three major approaches are developed to solve the problem, including the mathematical programming approach, optimal control theory and the variational inequality (VI) approach. The category of the mathematical programming approach includes some of the earliest papers in the DTA field those by Merchant and Nemhauser(1978), in which a discrete time model was presented for dynamic traffic assignment with the objective of minimizing the total cost. Ho(1980) and Carey(1987) provided some improvements and solutions for the M-N model. Jason(1991) presented a mathematical programming formulation of the dynamic user equilibrium problem(DUE) and described a dynamic traffic assignment procedure that can be applied to large networks to generate approximate solutions. Han 2000, 2002) explained dynamic network loading methods in a DTA model by proposing deterministic queue concept in a link performance function.

The application of optimal control theory to dynamic transportation network modeling becomes attractive because of the time-dependent characteristics of transportation networks. Luque and Friesz (1980) formulated the first dynamic system optimal (DSO) problem using optimal control theory. Later, researchers such as Ran and Shimazaki(1989), Friesz et al(1989), and Wie et al(1990) also employed optimal control theory to study the dynamic system-optimal and dynamic user-optimal problems.

Since the DTA problem is usually asymmetric and cannot be formulated as a mathematical programming problem, the variational inequality approach is becoming a powerful tool in the DTA field. Within this category, the DTA problem can be expressed as a VI formulation, and the solution methods for the VI problems, such as relaxation method and the projection method, are employed to solve the problems. Research using a VI formulation can be further classified into two groups: route-based models and link-based models. Since route-based models require the enumeration of all the possible routes in the network, they are not applicable to large networks. Ran and Boyce(1996) have developed a set of link-based VI models and solution algorithms.

For the purpose of real-time application of the DTA models, large computational time requirements of the models pose major challenges. However, these VI-based DTA models have not been successfully implemented since the computational times are too long to respond to time-varying traffic conditions and to permit of distribution of appropriate information. The main reason of huge computational time is that the link-based VI models require taking into considerations whole dimension of link and time in the formulation and solution algorithm. This computational speed problem is most serious for large networks because the size of multi-dimensional data structure adopted in the DTA models increases exponentially with linear increasing in the size of network.

To increase the computational performance of the DTA models, in this paper, a variational equality (VE) based route choice condition is derived. In the VE-based route choice condition, only used sets in terms of paths, links, and times are considered in the formulation in order to reduce the size of feasible solution set compared with the VI-based approach, which has to deal with the full size of link and time sets. Through this method, a link-based analytical DTA model is formulated and the solution algorithm is proposed to test and verify the model. The unique feature of the VE-based route choice condition is that the model can be designed to consider only used links and entry times of inflows into those links. A used link is a link that has traffic visited to it at certain time. Consequently, the size of the feasible solution sets is significantly reduced by considerably shortening the computational time needed to reach an optimal solution.

This paper is organized as follows. The second section briefly reviews on the VI-based route choice conditions. The VE-based link-based route choice condition is derived in the third section. A VE-based link-based DTA model is formulated in the forth section. A solution algorithm in terms of physical network approach is described in the fifth section. Numerical comparison of the VI-and VE-based DTA model is presented in the sixth section. This paper is completed with some conclusions.

II. Ideal Dynamic User-Optimal Route Choice Condition

1. Notation

Notation used in this paper is shown in ⟨Table 1⟩.

⟨Table 1⟩ Notations

$a,\hat{a},\tilde{a}$	L $\hat{a}$ a $\tilde{a}$, used link ink a, unused link
$p,\hat{p},\tilde{p}$	R $\hat{p}$, $\tilde{p}$ used route oute p, unused route
$A,\hat{A},\tilde{A}$	L A, $\hat{A}$, $\tilde{A}$ $A=\hat{A}\cup\tilde{A}$): (used link set unused link set ink set
$t,\hat{t},\tilde{t}$	C $\hat{t}$, $\tilde{t}$ used time ontinuous time t, unused time
$T,\hat{T},\tilde{T}$	C $\hat{T}$, $\tilde{T}$: $T=\hat{T}\cup\tilde{T}$) (used time horizon set ontinuous time horizon set T, unused time horizon set
$K,\hat{K},\tilde{K}$	D $\hat{K}$, $\tilde{K}$: $K=\hat{K}\cup\tilde{K}$) (used time horizon set iscrete time horizon set K, unused time horizon set
n	Discrete departure time interval n
k	Discrete time interval k
rs	Origination and destination pair
$f^{rs}(t)$	Departure flow rate from origin r to destination s at time t
$u_{ap}^{rs}(t)$	Inflow rate on link a at time t
$v_{ap}^{rs}(t)$	Exit flow rate on link a at time t
$x_{ap}^{rs}(t)$	Number of vehicles on link a at time t
$E^{rs}(t)$	Cumulative number of vehicles arriving at destination s from origin r by time t
$p_{ap}^{rs}(k)$	Inflow rate on link a at the beginning of time interval k
$q_{ap}^{rs}(k)$	Exit flow rate on link a at the beginning of time interval k
$y_{ap}^{rs}(k)$	Number of vehicles on link a at the beginning of time interval k
$\overline{E}^{rs}(k)$	Cumulative number of vehicles arriving at destination s from origin r by the beginning of time interval k
$A(j)$ $B(j)$	Set of links whose tail node is j Set of links whose head node is j
$e^{rs}(t)$	Arrival flow rate from origin r toward destination s at time t
$\tau_a(t)$	Mean actual travel time over link a for flows entering link a at time t
$\bar{\tau}_a(t)$	Estimated mean actual travel time over link a for flows entering link A at time t

〈Table 1〉 Notations (continue)

$\eta_p^{rs}(t)$	Mean actual travel time for route p between (r, s) for flows departing origin r at time t
$\Omega_a^{rs}(k)$	Travel cost of link at the beginning of time interval k.
$\pi^{rs}(t)$	Minimal mean actual route travel time between (r, s) for flows departing origin r at time t

2. Ideal Dynamic User-Optimal DUO Route Choice Condition

The ideal dynamic user-optimal DUO route choice problem is to determine the dynamic trajectories of link states and inflow and exit flow control variables at each instant of time resulting from drivers using minimal-time routes, given the network, the link travel time functions and the time-dependent O-D departure rate requirements.

The dynamic user-optimal(DUO) principle is defined as the temporal generalization of Wardrops first principle:

For each O-D pair at each interval of time, if the actual travel times experienced by travelers departing at the same time are equal and minimal, the dynamic traffic flow over the network is in a travel-time based ideal dynamic user-optimal state.

The above definitions can also be called a predictive or anticipatory DUO model, since the actual route time is predicted using the corresponding route choice model. This model assumes each traveler will have perfect information about the future network conditions and will comply with the guidance instructions based on ideal DUO route choice conditions. Travelers will not regret what decisions they made before their journeys.

The mathematical description of the ideal route-based DUO can be written as the extension of the steady state user optimal condition as below:

$$\eta^{rs}(t) - \pi^{rs}(t) \geq 0, \quad \forall r,s \tag{1}$$

$$f^{rs}(t) * \left| \eta^{rs}(t) - \pi^{rs}(t) \right| = 0, \quad \forall r,s \tag{2}$$

$$f^{rs}(t) \geq 0, \quad \forall r,s \tag{3}$$

The above condition can be expressed as link-based ideal DUO route choice conditions as follows:

$$\pi^{ri*}(t) + \tau_a[t + \pi^{ri*}(t)] - \pi^{rj*}(t) \geq 0, \forall a = (i,j), r, s \tag{4}$$

$$u_a^{rs*}([t + \pi^{ri*}(t)]\{\pi^{ri*}(t) + \tau_a[t + \pi^{ri*}(t)] - \pi^{rj*}(t)\} = 0, \quad \forall a = (i,j), r, s \tag{5}$$

$$u_a^{rs}([t + \pi^{ri*}(t)] \geq 0, \forall a = (i,j), r, s \tag{6}$$

Equations (5) show that the used links by traveler are on the shortest paths in DUO status. The link-based route choice condition is formulated as a link-based dynamic user optimal route choice model.

$$\int_0^T \sum_{rs} \sum_a \{\pi^{ri*}(t) + \tau_a[t + \pi^{ri*}(t)] - \pi^{rj*}(t)\} \cdot \{u_a^{rs}[t + \pi^{ri*}(t)] - u_a^{rs*}[t + \pi^{ri*}(t)]\} dt \geq 0 \tag{7}$$

Ⅲ. Formulation of VE-Based Link-Based Route Choice Condition

1. Variational Inequality(VI) and Equality(VE)

The inequality problem is a general problem formulation that encompasses a set of mathematical problems, including nonlinear equations, optimization problems, complementarity problems and fixed point problems. Variational inequalities(VI) were originally developed as a tool for the study of certain classes of partial differential equations such as those that arise in mechanics. Variational equalities(VE) can be considered subsets of VI in the level of feasible solution set with the same objective value of VI problem. In this section, we present the definitions of a variational inequality(VI) and a variational equality(VE) problems.

Here we are dealing with a vector of decision variables $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ and a vector of cost functions $\mathbf{f(x)} = [f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_n(\mathbf{x})]$. Define G as a given closed convex set of the decision variables $\mathbf{x}$; f is a vector of given continuous functions defined on R^n. In the dynamic problem, we are concerned with a vector of control variable $\mathbf{u}(t) = [u_1(t), u_2(t), \cdots, u_n(t)]$ and state variable $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$ and their dynamic processes

$$\dot{x}(t) = h[x(t), u(t)] \tag{8}$$

where the state variables $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$ and state equations $\mathbf{h} = [h_1(t), h_2(t), \cdots, h_n(t)]$. Associated with the dynamic processes, there is a vector of cost functions $\mathbf{F}(t) = [F_1(t), F_2(t), \cdots, F_n(t)]$. Each element of the cost function vector is a function of state and control variables, i.e.,

$$F_i(t) = F_i[\mathbf{x}(t), \mathbf{u}(t)] \quad i = 1, 2, \cdots, m \tag{9}$$

Since the state variables $\mathbf{x}(t)$ can be determined by the state equations when the control variables $\mathbf{u}(t)$ are given, the vector of cost functions can be further simplified as $\mathbf{F}(t) = [\mathbf{u}(t)]$. Define G(t) as a given closed convex set of the control variables $\mathbf{u}(t)$. We assume $\mathbf{F}(t)$ is a set of given continuous functions from G(t) to $R^n(t)$. Then, we give the following definition of the dynamic variational inequality problem.

[Definition 1]

The infinite-dimensional variation inequality problem is to determine a control vector $\mathbf{u}(t)^* \in G(t) \subset R^n(t)$, such that

$$\mathbf{F}[\mathbf{u}^*(t)] \cdot [\mathbf{u}(t) - \mathbf{u}(t)^*] \geq 0, \qquad \forall \mathbf{u}(t) \in G(t) \tag{10}$$

Equation (4) expresses the fact that from the minimum point $\mathbf{u}(t)^*$ the function does not decrease in any direction into the set $G(t)$. Moreover, if the minimum point is an interior point of $G(t)$, then we obtain the "variable equality" $\mathbf{F}[\mathbf{u}(t)] = 0$, a functional equation for the (gradient) operator $\mathbf{F}$.

[Definition 2]

The infinite-dimensional variational equality problem is to determine a control vector $\mathbf{u}(t)^* \in G(t) \subset R^n(t)$, such that

$$\mathbf{F}[\mathbf{u}^*(t)] = 0 \qquad \forall \mathbf{u}^*(t) \in G(t) \tag{11}$$

However, the following definition is also useful where continuous time problems need to be transformed to discrete time problems and comparisons need to be made with static problems.

[Definition 3]

The infinite-dimensional variational inequality problem is to determine a vector $\mathbf{x}^* \in G \subset R^n_+$ such that

$$\int_0^T \mathbf{F}^T[\mathbf{u}^*(t)] \cdot [\mathbf{u}(t) - \mathbf{u}(t)^*] dt \geq 0, \qquad \forall \mathbf{u}(t) \in G(t) \tag{12}$$

[Definition 4]

The infinite-dimensional variational equality problem is to determine a control vector $\mathbf{u}(t)^* \in G(t) \subset R^n(t)$, such that

$$\int_0^T \mathbf{F}^T[\mathbf{u}^*(t)] dt = 0, \qquad \forall \mathbf{u}(t) \in G(t) \tag{13}$$

2. VE-based Link-Based Route Choice Condition

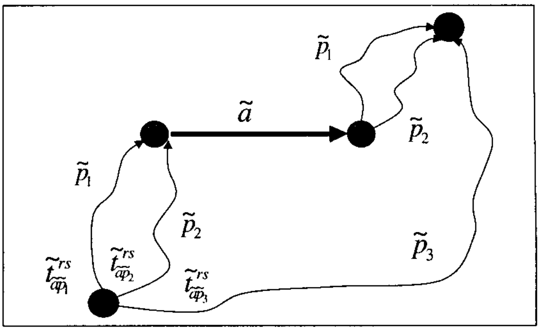
New link-based route choice condition is derived. This route choice condition can be basis for the formulation of time-dependent variation equality(VE) model of link-based dynamic traffic assignment. For this purpose, the set reduction scheme(SRS) of time interval and link is applied to represent a VI-based model as a VE-based model.

The definition of a used link is a link that has inflows, which depart from any origination at certain times. In the following, subscript, $\tilde{a}$ denotes a used link a, $\tilde{t}$ denotes departure t from any origin, and and $\tilde{p}$ a used route p. Then, $\tilde{t}^{rs}_{\tilde{a}\tilde{p}}$ indicates a departure time t from origination r toward used link $\tilde{a}$ using route $\tilde{p}$ connecting origin r and destination s. As known from the DUO route choice condition, when it is visited, the link $\tilde{a}$ must be on any shortest path $\tilde{p}$. Furthermore, the departure time of inflows $u^{rs}_{\tilde{a}\tilde{p}}$ to the link $\tilde{a}$ is $\tilde{t}^{rs}_{\tilde{a}\tilde{p}}$. Note that this departure time has a unique value because the inflows $u^{rs}_{\tilde{a}\tilde{p}}$ visit the link $\tilde{a}$ in the path $\tilde{p}$ only once.

Based on the above notation, all departure times from origin to link $\tilde{a}$ can be represented as a set. Denote $\tilde{t}_{\tilde{a}}$ as a used departure time set to link $\tilde{a}$. Then, conceptually, it can be recognized that the set $\tilde{t}_{\tilde{a}}$ embraces $\tilde{t}^{rs}_{\tilde{a}\tilde{p}}$ as a possible element. Equation (14) shows that two cases of mapping process from each departure time $\tilde{t}^{rs}_{\tilde{a}\tilde{p}}$ into $\tilde{t}_{\tilde{a}}$ based on the assumption that plural used routes pass through the link $\tilde{a}$. The first one is for the case when each $u^{rs}_{\tilde{a}\tilde{p}}$ departs to the link $\tilde{a}$ at different time $\tilde{t}^{rs}_{\tilde{a}\tilde{p}}$. In this case, each different $\tilde{t}^{rs}_{\tilde{a}\tilde{p}}$ is mapped to $\tilde{t}_{\tilde{a}}$. The second is for when some inflows $u^{rs}_{\tilde{a}\tilde{p}}$, which have different origins or destinations, depart to the link $\tilde{a}$ at the same time $\tilde{t}^{rs}_{\tilde{a}\tilde{p}}$. In this case, only one arrival time $\tilde{t}^{rs}_{\tilde{a}\tilde{p}}$ is mapped to $\tilde{t}_{\tilde{a}}$.

$$\begin{aligned}
&\tilde{t}^{rs}_{\tilde{a}\tilde{p}_i} \in \tilde{t}_{\tilde{a}},\quad \tilde{t}^{rs}_{\tilde{a}\tilde{p}_j} \in \tilde{t}_{\tilde{a}} && \text{if } \tilde{t}^{rs}_{\tilde{a}\tilde{p}_i} \neq \tilde{t}^{rs}_{\tilde{a}\tilde{p}_j} && \forall r,s,i \neq j \\
&\tilde{t}^{rs}_{\tilde{a}\tilde{p}_i} \in \tilde{t}_{\tilde{a}} && \text{if } \tilde{t}^{rs}_{\tilde{a}\tilde{p}_i} = \tilde{t}^{rs}_{\tilde{a}\tilde{p}_j} && \forall r,s,i \neq j
\end{aligned} \tag{14}$$

⟨Figure 1⟩ illustrates the detailed relationship of used paths, used links, and used departure time from origin to link $\tilde{a}$ based on one OD pair. Between the OD pair, there are three searched used paths, $\tilde{p}_1$, $\tilde{p}_2$, and $\tilde{p}_3$. Among these paths, two paths pass through link $\tilde{a}$ at two different departure times $\tilde{t}^{rs}_{\tilde{a}\tilde{p}_1}$ and $\tilde{t}^{rs}_{\tilde{a}\tilde{p}_2}$. Thus, $\tilde{t}^{rs}_{\tilde{a}\tilde{p}_1}$ and $\tilde{t}^{rs}_{\tilde{a}\tilde{p}_2}$ are elements of the set $\tilde{t}_{\tilde{a}}$.



⟨Figure 1⟩ Relationship of Used Path, Used Links, and Departure Times

Based on combining two terms $\tilde{a}$ and $\tilde{t}_{\tilde{a}}$, the new link-based ideal DUO route choice conditions can be derived. It follows :

$$\{\pi^{ri*}(\tilde{t}_{\tilde{a}}) + \tau_{\tilde{a}}[\tilde{t}_{\tilde{a}} + \pi^{ri*}(\tilde{t}_{\tilde{a}})] - \pi^{rj*}(\tilde{t}_{\tilde{a}})\} = 0, \quad \forall \tilde{a} = (i,j), r, s \tag{15}$$

$$u^{rs*}_{\tilde{a}}([\tilde{t}_{\tilde{a}} + \pi^{ri*}(\tilde{t}_{\tilde{a}})]\{\pi^{ri*}(\tilde{t}_{\tilde{a}}) + \tau_{\tilde{a}}[\tilde{t}_{\tilde{a}} + \pi^{ri*}(\tilde{t}_{\tilde{a}})] - \pi^{rj*}(\tilde{t}_{\tilde{a}})\} = 0, \forall \tilde{a} = (i,j), r, s \tag{16}$$

$$u^{rs}_{\tilde{a}}([\tilde{t}_{\tilde{a}} + \pi^{ri*}(\tilde{t}_{\tilde{a}})]) > 0, \quad \forall \tilde{a} = (i,j), r, s \tag{17}$$

Equation (9) shows that a used link $\tilde{a}$ at time $\tilde{t}_{\tilde{a}} + \pi^{ri*}(\tilde{t}_{\tilde{a}})$ is always on any shortest route departing from origin r in DUO status. Thus this route choice condition includes only the equality sign. Note that equation (4) in the previously proposed DUO route choice condition includes both equality and inequality signs in order to take both used and unused links into considerations. Equation (17) specifies that when a link $\tilde{a}$ is used, an inflow always has a positive value. Also, note that the Equation (6) in the previously proposed DUO route choice condition includes both equality and inequality signs to take account of inflows of link $\tilde{a}$ for entire departure times.

[Theorem 1]

Link-time-based ideal DUO route choice condition

(15)-(17) imply route-time-based ideal DUO route choice conditions with positive inflow $f_{\tilde{p}}^{rs}(\tilde{t}_{\tilde{p}})>0$.

If we consider end node j of link any used link as a destination s, then link-time-based ideal DUO route choice conditions (15)-(17) apply to O-D pair rs.

If there is a flow on link $\tilde{a}$, i.e. $u_{\tilde{a}}^{rs}(\tilde{t}_{\tilde{a}}+\pi^{ri}(\tilde{t}_{\tilde{a}}))>0$ from the definition of route choice condition (15) we have

$$\{\pi^{ri*}(\tilde{t}_{\tilde{a}})+\tau_{\tilde{a}}[\tilde{t}_{\tilde{a}}+\pi^{ri*}(\tilde{t}_{\tilde{a}})]-\pi^{rs*}(\tilde{t}_{\tilde{a}})\}=0 \qquad (18)$$

Specify $\tilde{p}$ as the used route via link $\tilde{a}$ and the minimal ideal travel time subroute from origin r to node i. Thus, the flow $f_{\tilde{p}}^{rs*}(\tilde{t}_{\tilde{a}})$ on route p is positive at time $\tilde{t}_{\tilde{a}}$ and we have

$$\eta_{\tilde{p}}^{rs*}(\tilde{t}_{\tilde{a}})=\pi^{ri*}(\tilde{t}_{\tilde{a}})+\tau_{\tilde{a}}[\tilde{t}_{\tilde{a}}+\pi^{ri*}(\tilde{t}_{\tilde{a}})] \qquad (19)$$

In other words, we have

$$f_{\tilde{p}}^{rs*}(\tilde{t}_{\tilde{a}})\left[\eta_{\tilde{p}}^{rs*}(\tilde{t}_{\tilde{a}})-\pi^{rs*}(\tilde{t}_{\tilde{a}})\right]=0 \qquad (20)$$

Note that the above equation applies to any positive route inflow $f_{\tilde{p}}^{rs*}(\tilde{t}_{\tilde{a}})>0$ and $\tilde{t}_{\tilde{a}}=\tilde{t}_{\tilde{p}}$. Thus, $\eta_{\tilde{p}}^{rs}(\tilde{t}_{\tilde{p}})-\pi^{rs}(\tilde{t}_{\tilde{p}})=0$. Thus, equation (20) and $f_{\tilde{p}}^{rs*}(\tilde{t}_{\tilde{p}})>0$ imply the route-time-based ideal DUO route choice conditions (21)-(23).

$$\eta_{\tilde{p}}^{rs}(\tilde{t}_{\tilde{p}})-\pi^{rs}(\tilde{t}_{\tilde{p}})=0, \quad \forall r,s,\tilde{p} \qquad (21)$$

$$f_{\tilde{p}}^{rs}(\tilde{t}_{\tilde{p}})*\left[\eta_{\tilde{p}}^{rs}(\tilde{t}_{\tilde{p}})-\pi^{rs}(\tilde{t}_{\tilde{p}})\right]=0, \quad \forall r,s,\tilde{p} \qquad (22)$$

$$f_{\tilde{p}}^{rs}(\tilde{t}_{\tilde{p}})>0, \quad \forall r,s,\tilde{p} \qquad (23)$$

The proof is complete.

Ⅳ. Variational Equality Model Formulation

In this section, set reduction scheme(SRS) is applied to reduce the VI-based problem as a VE-based problem. The link-based DTA model is formulated based on the derived VE-based route choice condition.

The equivalent variational equality(VE) formulation of the dynamic user optimal route choice conditions (15)-(17) can be stated as follows:

$$\sum_{rs}\sum_{\tilde{a}}\int_0^{\tilde{T}_{\tilde{a}}}\{\pi^{ri*}(\tilde{t}_{\tilde{a}})+\tau_{\tilde{a}}[\tilde{t}_{\tilde{a}}+\pi^{ri*}(\tilde{t}_{\tilde{a}})]-\pi^{rj*}(\tilde{t}_{\tilde{a}})\}dt=0 \quad (24)$$

where $\tilde{T}_{\tilde{a}}$ is the maximum size of $\tilde{t}_{\tilde{a}}$.

[Theorem 2]

Variational inequality (7) can be reduced as variational equality problem (24) by using set reduction scheme.

Proof

Let $\Omega_a^{rj*}(t)$ and $U_a^{rs*}(t)$ be as follows:

$$\Omega_a^{rj*}(t)=\pi^{ri*}(t)+\tau_a[t+\pi^{ri*}(t)]-\pi^{rj*}(t) \qquad (25)$$

$$U_a^{rs*}(t)=u_a^{rs}[t+\pi^{ri*}(t)]-u_a^{rs*}[t+\pi^{ri*}(t)] \qquad (26)$$

By simplifying equation (4) using equations (25) and (26), we have

$$\int_0^T\sum_{rs}\sum_a\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)\}dt \qquad (27)$$

Since time horizon T is summation set of unused time horizon $\hat{T}$ and used time horizon $\tilde{T}$, i.e., $T=\hat{T}\cup\tilde{T}$. It follows that

$$\begin{aligned}&\int_0^T\sum_{rs}\sum_a\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)\}dt\\&=\int_0^{\hat{T}}\sum_{rs}\sum_a\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)\}dt\\&+\int_0^{\tilde{T}}\sum_{rs}\sum_a\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)\}dt\end{aligned} \qquad (28)$$

For unused time horizon $\hat{T}$, the first terms does not affect the object value, i.e. $\int_0^{\hat{T}}\sum_{rs}\sum_{a}\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)$. Thus, equation (27) can can be reduced as solely used time horizon $\hat{T}$. It follow that

$$\int_0^{T}\sum_{rs}\sum_{a}\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)\}dt = \int_0^{\tilde{T}}\sum_{rs}\sum_{a}\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)\}dt \quad (29)$$

Link set A is also summation set of unused link set $\hat{A}$ and used link set $\tilde{A}$,$\tilde{T}$, i.e., $A=\hat{A}\cup\tilde{A}$. It follows that

$$\int_0^{\tilde{T}}\sum_{rs}\sum_{a}\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)\}dt = \int_0^{\tilde{T}}\sum_{rs}\sum_{\hat{a}\in\hat{A}}\{\Omega_{\hat{a}}^{rj*}(t)\cdot U_{\hat{a}}^{rs*}(t)\}dt + \int_0^{\tilde{T}}\sum_{rs}\sum_{\tilde{a}\in\tilde{A}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt \quad (30)$$

For used time, unused links among all links, $\int_0^{\tilde{T}}\sum_{rs}\sum_{\hat{a}\in\hat{A}}\{\Omega_{\hat{a}}^{rj*}(t)\cdot U_{\hat{a}}^{rs*}(t)\}dt=0$. Thus, equation (27) can be expressed as solely used link set $\tilde{A}$ and used time horizon set $\tilde{T}$.

$$\int_0^{\tilde{T}}\sum_{rs}\sum_{a}\{\Omega_a^{rj*}(t)\cdot U_a^{rs*}(t)\}dt = \int_0^{\tilde{T}}\sum_{rs}\sum_{\tilde{a}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt\,\forall\tilde{a}\in\tilde{A}, t\in\tilde{T} \quad (31)$$

If used time is compressed for each used link $\tilde{a}$ integral sign $\int_0^{\tilde{T}}$ can be switched into next to $\sum$ by set definition of $\tilde{T}_{\tilde{a}}$ from $\tilde{T}=\tilde{T}_{\tilde{a}_1}\cup\tilde{T}_{\tilde{a}_2}\cup\cdots\cup\tilde{T}_{\tilde{a}_n}^{\tilde{a}}$ and $\tilde{a}_1,\tilde{a}_2,\cdots\tilde{a}_n\in\tilde{A}$.

$$\int_0^{\tilde{T}}\sum_{rs}\sum_{\tilde{a}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt = \sum_{rs}\sum_{\tilde{a}}\int_0^{\hat{T}_{\tilde{a}}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt + \sum_{rs}\sum_{\tilde{a}}\int_0^{\tilde{T}_{\tilde{a}}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt \quad (32)$$

For unused time in each used link $\tilde{a}$, $\sum_{rs}\sum_{\tilde{a}}\int_0^{\hat{T}_{\tilde{a}}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt=0$. It follows that

$$\int_0^{\tilde{T}}\sum_{rs}\sum_{\tilde{a}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt = \sum_{rs}\sum_{\tilde{a}}\int_0^{\tilde{T}_{\tilde{a}}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt\ \forall\tilde{a}\in\tilde{A}, t\in\tilde{T}_{\tilde{a}} \quad (33)$$

Since $t=\tilde{t}_{\tilde{a}}$,

$$\sum_{rs}\sum_{\tilde{a}}\int_0^{\tilde{T}_{\tilde{a}}}\{\Omega_{\tilde{a}}^{rj*}(t)\cdot U_{\tilde{a}}^{rs*}(t)\}dt = \sum_{rs}\sum_{\tilde{a}}\int_0^{\tilde{T}_{\tilde{a}}}\{\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\cdot U_{\tilde{a}}^{rs*}(\tilde{t}_{\tilde{a}})\}dt$$

$$\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\cdot U_{\tilde{a}}^{rs*}(\tilde{t}_{\tilde{a}}) = \Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\cdot u_{\tilde{a}}^{rs}([\tilde{t}_{\tilde{a}}+\pi^{ri*}(\tilde{t}_{\tilde{a}})]) - \Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\cdot u_{\tilde{a}}^{rs*}([\tilde{t}_{\tilde{a}}+\pi^{ri*}(\tilde{t}_{\tilde{a}})])=0 \quad (34)$$

From the definition of the route choice condition (10), $\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\cdot u_{\tilde{a}}^{rs*}([\tilde{t}_{\tilde{a}}+\pi^{ri*}(\tilde{t}_{\tilde{a}})])=0$. Thus we have

$$\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\cdot U_{\tilde{a}}^{rs*}(\tilde{t}_{\tilde{a}}) = \Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\cdot u_{\tilde{a}}^{rs}([\tilde{t}_{\tilde{a}}+\pi^{ri*}(\tilde{t}_{\tilde{a}})])=0 \quad (35)$$

From equation (11), $u_{\tilde{a}}^{rs}([\tilde{t}_{\tilde{a}}+\pi^{ri*}(\tilde{t}_{\tilde{a}})])$. Thus we have

$$\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\cdot U_{\tilde{a}}^{rs*}(\tilde{t}_{\tilde{a}}) = \Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})=0 \quad (36)$$

Integrating the above equation from 0 to $\tilde{T}_{\tilde{a}}$ for each link $\tilde{a}$, it follows that

$$\int_0^{\tilde{T}_{\tilde{a}}}\{\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\}dt=0 \qquad \text{where } \tilde{a}=(i,j) \quad (37)$$

Summing above equation for all links $\tilde{a}$ and all origins r, we obtain variational equality

$$\sum_{rs}\sum_{\tilde{a}}\int_0^{\tilde{T}_{\tilde{a}}}\{\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\}dt=0 \quad (38)$$

The proof is complete.

[Theorem 3]

The dynamic traffic flow satisfying constraints (15)~(17) is in a ideal DUO route-based choice state if and only if it satisfies variational equality problem (24).

Proof of necessity

We need to prove that link-time-based ideal DUO route choice condition (15)~(17) imply variational equality (24). For any used link $\tilde{a}$, a feasible inflow at time $\tilde{t}_{\tilde{a}}$ is

$$u_{\tilde{a}}^{r*}(\tilde{t}_{\tilde{a}}) > 0 \qquad \forall \tilde{a} = (i,j), r \tag{39}$$

By definition of route choice conditions (15)-(17) we have

$$\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}}) = 0 \qquad \forall \tilde{a}, r; \tilde{a} = (i,j) \tag{40}$$

Integrating the above equation from 0 to $\tilde{T}_{\tilde{a}}$ for each link $\tilde{a}$, it follows that

$$\int_0^{\tilde{T}_{\tilde{a}}} \{\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\} dt = 0 \qquad \text{where } \tilde{a} = (i,j) \tag{41}$$

Summing above equation for all links $\tilde{a}$ and all origins r, we obtain variational equality

$$\sum_{rs}\sum_{\tilde{a}} \int_0^{\tilde{T}_{\tilde{a}}} \{\Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}})\} dt = 0 \tag{42}$$

Proof of sufficiency

We need to prove that any solution $u_{\tilde{a}}^{r*}(\tilde{t}_{\tilde{a}})$ to variational equality (24) satisfies link-time-based ideal DUO route choice condition (15)~(17) . We know that the first and third ideal DUO route choice conditions (15) and (17) hold by definition. Thus, we need to prove that the second ideal DUO route choice condition (16) also holds.

Assume that the second ideal DUO route choice condition (16) does not hold only for a link $\tilde{b} = (l,m)$ for an origin n during a time interval $[d-\delta, d+\delta] \in [0, \tilde{T}_{\tilde{b}}]$, i.e.,

$$u_{\tilde{b}}^{r}(\tilde{t}_{\tilde{b}}) > 0 \text{ and } \Omega_{\tilde{b}}^{rj*}(\tilde{t}_{\tilde{b}}) > 0 \quad \tilde{t}_{\tilde{b}} \in [d-\delta, d+\delta] \tag{43}$$

where

$$\Omega_{\tilde{b}}^{rm*}(\tilde{t}_{\tilde{b}}) = \pi^{rl*}(\tilde{t}_{\tilde{b}}) + \tau_{\tilde{b}}[\tilde{t}_{\tilde{b}} + \pi^{rl*}(\tilde{t}_{\tilde{b}})] - \pi^{rm*}(\tilde{t}_{\tilde{b}}) > 0 \qquad where \quad \tilde{b} = (l,m).$$

Note that the second ideal DUO route choice condition (16) holds for all links other than $\tilde{b} = (l,m)$ for origin n at time t. Equation (16) also holds for link $\tilde{b} = (l,m)$ at time for origins $r \neq n$ $\tilde{t}_{\tilde{b}} \in [d-\delta, d+\delta]$. It follows that

$$\sum_{rs}\sum_{\tilde{a}} \int_0^{\tilde{T}_{\tilde{a}}} \Omega_{\tilde{a}}^{rj*}(\tilde{t}_{\tilde{a}}) dt = \sum_{rs}\sum_{\tilde{b}} \int_{d-\delta}^{d+\delta} \Omega_{\tilde{b}}^{rj*}(\tilde{t}_{\tilde{b}}) dt > 0 \tag{44}$$

The above equation contradicts variational equality (24). Therefore, any optimal solution $\{u_{\tilde{a}}^{r*}(\tilde{t}_{\tilde{a}})\}$ to variational equality (24) satisfies the second ideal DUO route choice condition (16). Since we proved the necessity and sufficiency of the equivalence of variational equality (24) to link-time-based ideal DUO route choice condition (15)~(17), the proof is complete.

V. Solution Algorithm

In the DTA model, we have three time-dependent variables(inflow, flow and exit flow) in the cost function to consider. To handle these variables, the time-space network(TSN) approach has been most well-known technique(Driss-Kaitouni, 1993; Ran and Boyce, 1996; Yang and Meng, 1998). The TSN approach expands original network in proportion to time and space dimension of original problem. Thus, it explodes the problem size of original network. In this section, the physical network

(PN) approach is employed to solve the DTA problem. In the PN approach, flow and exit flow can be expressed solely by inflow, thus not only is the time-space expansion not required, but the problem size can be reduced. The PN algorithm was first proposed by Chen and Hsueh(1998). They introduce heuristic solution algorithm using MSA and Diagonalization techniques. This section introduces how to solve the proposed DTA problem using Flank-Wolfe algorithm and Diagonalization techniques.

1. Physical Network Approach

In the outer iteration step of diagonalization algorithm, the following terms are temporarily fixed (Ran and Boyce, 1996):

- Actual t $\tau_{\tilde{a}}(\tilde{n}_{\tilde{a}})$ $\bar{\tau}_{\tilde{a}}(\tilde{n}_{\tilde{a}})$. in the link flow propagation constraints as ravel time
- Actual travel time $\tau_{\tilde{a}}[\tilde{n}_{\tilde{a}}+\pi^{ri*}(\tilde{n}_{\tilde{a}})]$ $\Omega_{\tilde{a}}^{ri*}(\tilde{n}_{\tilde{a}})$ $\bar{\tau}_{\tilde{a}}[\tilde{n}_{\tilde{a}}+\bar{\pi}^{ri*}(\tilde{n}_{\tilde{a}})]$. as in the VE cost term
- Minimal travel times $\pi^{ri*}(\tilde{n}_{\tilde{a}})$ $\bar{\pi}^{ri*}(\tilde{n}_{\tilde{a}})$ $\pi^{rj*}(\tilde{n}_{\tilde{a}})$ $\bar{\pi}^{rj*}(\tilde{n}_{\tilde{a}})$ for each link and each origin node.as and as

Thus, the VE cost term becomes:

$$\bar{\Omega}_{\tilde{a}}^{rj*}(\tilde{n}_{\tilde{a}})=\bar{\pi}^{ri*}(\tilde{n}_{\tilde{a}})+\tau_{\tilde{a}}[\tilde{n}_{\tilde{a}}+\bar{\pi}^{ri*}(\tilde{n}_{\tilde{a}})]-\bar{\pi}^{rj*}(\tilde{n}_{\tilde{a}}) \quad (45)$$

For each relaxation iteration, the minimization problem that is equivalent to the VE problem can be expressed as:

$$\begin{aligned} \min Z &= \sum_{rs}\sum_{\tilde{a}}\sum_{\tilde{n}_{\tilde{a}}}\int_0^{u_{\tilde{a}}^{rs}[\tilde{n}_{\tilde{a}}+\bar{\pi}^{ri*}(\tilde{n}_{\tilde{a}})]}\bar{\Omega}_{\tilde{a}}^{rj*}(\tilde{n}_{\tilde{a}})d\omega \\ &= \sum_{\tilde{a}}\sum_{\tilde{k}_{\tilde{a}}}\{\int_0^{u_{\tilde{a}}(\tilde{k}_{\tilde{a}})}[\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})]d\omega \\ &\quad +\sum_r u_{\tilde{a}}^r(\tilde{k}_{\tilde{a}})[\bar{\pi}^{ri*}(\xi_{\tilde{a}}^r)-\bar{\pi}^{rj*}(\xi_{\tilde{a}}^r)]\} \end{aligned} \quad (46)$$

where $\tilde{k}_{\tilde{a}}=\xi_{\tilde{a}}^r+\bar{\pi}^{ri}(\xi_{\tilde{a}}^r)$ and $\xi_{\tilde{a}}^r=\tilde{n}_{\tilde{a}}$ is the departure time interval.

This minimization problem can be verified by taking the partial derivative of the objective function Z with respect to the inflow variable u.

The link travel time $\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ in the objective function (46) can be expressed as a function of inflow, flow and exit flow. Therefore, the objective function (46) is as follows:

$$\begin{aligned} \min Z &= \sum_{\tilde{a}}\sum_{\tilde{k}_{\tilde{a}}}\{\int_0^{u_{\tilde{a}}(\tilde{k}_{\tilde{a}})}[\tau_{\tilde{a}}[u_{\tilde{a}}(\tilde{k}_{\tilde{a}}),v_{\tilde{a}}(\tilde{k}_{\tilde{a}}),x_{\tilde{a}}(\tilde{k}_{\tilde{a}})]]d\omega \\ &\quad +\sum_r [u_{\tilde{a}}^r(\tilde{k}_{\tilde{a}})]^{l+1}[\bar{\pi}^{ri*}(\xi_{\tilde{a}}^r)-\bar{\pi}^{rj*}(\xi_{\tilde{a}}^r)]\} \end{aligned} \quad (47)$$

s.t. Dynamic Route Assignment Constraints

$$\sum_{\tilde{a}\in A(r)}\sum_{\tilde{p}}u_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}})=f^{rs}(\tilde{k}_{\tilde{a}}),\forall r,s \quad (48)$$

Relationship between State and Control Variables

$$x_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}}+1)=x_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}})+u_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}})-v_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}}),\forall\tilde{a},\tilde{p},r,s \quad (49)$$

$$E^{rs}(\tilde{k}_{\tilde{a}}+1)=E^{rs}(\tilde{k}_{\tilde{a}})+\sum_{\tilde{a}\in\tilde{B}(s)}\sum_{\tilde{p}}v_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}}),\forall r,s\neq r \quad (50)$$

Flow Conservation Constraints

$$\sum_{\tilde{a}\in\tilde{B}(j)}v_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}})=\sum_{\tilde{a}\in\tilde{A}(j)}u_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}),\forall j,\tilde{p},r,s,j\neq r,s \quad (51)$$

$$\sum_{\tilde{a}\in\tilde{B}(s)}\sum_{\tilde{p}}v_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}})=e_{\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}}),\forall r,s,s\neq r \quad (52)$$

Link Flow Propagation Constraints

$$\begin{aligned} x_{\tilde{a}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}}) &= \sum_{\tilde{b}\in\tilde{p}}\{x_{\tilde{b}\tilde{p}}^{rs}[\tilde{k}_{\tilde{a}}+\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})]-x_{\tilde{b}\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}})\} \\ &\quad +\{E_{\tilde{p}}^{rs}[\tilde{k}_{\tilde{a}}+\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})]-E_{\tilde{p}}^{rs}(\tilde{k}_{\tilde{a}})\}, \\ &\quad \forall\tilde{a}\in\tilde{B}(j),\tilde{p},r,s,j\neq r \end{aligned} \quad (53)$$

Definitional Constraints

$$\sum_{rs\tilde{p}} u^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}}) = u_{\tilde{a}}(\tilde{k}_{\tilde{a}}), \sum_{rs\tilde{p}} v^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}}) = v_{\tilde{a}}(\tilde{k}_{\tilde{a}}), \sum_{rs\tilde{p}} x^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}}) = x_{\tilde{a}}(\tilde{k}_{\tilde{a}}), \forall \tilde{a} \quad (54)$$

Nonnegative Conditions

$$x^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}}+1) \geq 0, u^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}}) > 0, v^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}}) \geq 0, \forall \tilde{a}, \tilde{p}, r, s \quad (55)$$

$$e^{rs}_{\tilde{p}}(\tilde{k}_{\tilde{a}}) \geq 0, E^{rs}_{\tilde{p}}(\tilde{k}_{\tilde{a}}+1) \geq 0, \forall \tilde{p}, r, s \quad (56)$$

Boundary Conditions

$$E^{rs}_{\tilde{p}}(1) = 0, \forall \tilde{p}, r, s \quad (57)$$

$$x^{rs}_{\tilde{a}\tilde{p}}(1) = 0, \forall \tilde{a}, \tilde{p}, r, s \quad (58)$$

The minimization problem (47) can be verified by taking the partial derivative of the objective function Z with respect to the inflow variable u. Without considering flow dispersion and compression on a link, the discrete flow propagation constraint (53) can be expressed as follows:

$$u^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}}) = v^{rs}_{\tilde{a}\tilde{p}}[\tilde{k}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})] \quad (59)$$

That is, the exit flow that leaves link $\tilde{a}$ during time interval should be equal to the inflow that enters link during time interval $\tilde{k}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ $\tilde{a}$ $\tilde{k}_{\tilde{a}}$. The length of the time interval can be shortened so that equation (59) will hold for each interval in case flow dispersion and compression are considered.

For any link $\tilde{a}$, if inflow $u^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}})$ is determined, then exit flow $v^{rs}_{\tilde{a}\tilde{p}}[\tilde{k}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})]$ can be calculated by flow propagation constraint (59). Moreover, the number of vehicles $x_{\tilde{a}}(\tilde{t}_{\tilde{a}})$ can be expressed by $u_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ as follows:

$$x_{\tilde{a}}(\tilde{k}_{\tilde{a}}) = \sum_{\tilde{i}_{\tilde{a}}} u_{\tilde{a}}(\tilde{i}_{\tilde{a}}) \delta^{\tilde{i}_{\tilde{a}}}_{\tilde{a}2}(\tilde{k}_{\tilde{a}}) \quad (60)$$

where

$\delta^{\tilde{i}_{\tilde{a}}}_{\tilde{a}2}(\tilde{k}_{\tilde{a}}) = 1$, if $\tilde{k}_{\tilde{a}} < \tilde{i}_{\tilde{a}} \leq \tilde{k}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ and $\tilde{i}_{\tilde{a}}, \tilde{k}_{\tilde{a}} \in \tilde{K}_{\tilde{a}}$;

$\delta^{\tilde{i}_{\tilde{a}}}_{\tilde{a}2}(\tilde{k}_{\tilde{a}}) = 0$, otherwise.

Equation (60) is another application of the flow propagation constraint (59), which states that the inflow vehicles $u_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ will stay on link $\tilde{a}$ between its entry interval $\tilde{k}_{\tilde{a}}$ and its exiting interval $\tilde{k}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})$. Subsequently, the exit flow $v_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ can be expressed by $u_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ as follows:

$$v_{\tilde{a}}(\tilde{k}_{\tilde{a}}) = \sum_{\tilde{i}_{\tilde{a}}} u_{\tilde{a}}(\tilde{i}_{\tilde{a}}) \delta^{\tilde{i}_{\tilde{a}}}_{\tilde{a}1}(\tilde{k}_{\tilde{a}}) \quad (61)$$

where

$\delta^{\tilde{i}_{\tilde{a}}}_{\tilde{a}1}(\tilde{k}_{\tilde{a}}) = 1$, if $\tilde{i}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{i}_{\tilde{a}}) = \tilde{k}_{\tilde{a}}$ and $\tilde{i}_{\tilde{a}}, \tilde{k}_{\tilde{a}} \in \tilde{K}_{\tilde{a}}$;

$\delta^{\tilde{i}_{\tilde{a}}}_{\tilde{a}1}(\tilde{k}_{\tilde{a}}) = 0$, otherwise.

Equation (60) is also an application of the flow propagation constraint (59), which states that the inflow $u_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ will exit link $\tilde{a}$ after the link travel time $\tilde{k}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})$. Thus, by substituting equation (60) and (61), link travel time $\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ in the objective function (47) can be expressed as a function of inflow only. Therefore, the objective function (47) is revised as:

$$\min Z = \sum_{\tilde{a}} \sum_{\tilde{k}_{\tilde{a}}} \{ \int_0^{u_{\tilde{a}}(\tilde{k}_{\tilde{a}})} [\tau_{\tilde{a}}[u_{\tilde{a}}(\tilde{g}_{\tilde{a}}), u_{\tilde{a}}(\tilde{k}_{\tilde{a}})]] d\omega + \sum_r [u^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})] \cdot [\overline{\pi}^{ri^*}(\xi^r_{\tilde{a}}) - \overline{\pi}^{rj^*}(\xi^r_{\tilde{a}})] \} \quad (62)$$

where the impacts of $x_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ and $v_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ on $\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ are reflected by inflow variables $u_{\tilde{a}}(\tilde{g}_{\tilde{a}})$. As part of the relaxation procedure, these impacts are temporarily fixed during each relaxation iteration, i.e. $u_{\tilde{a}}(\tilde{g}_{\tilde{a}})$ is temporarily set as $\tau_{\tilde{a}}[\overline{u}_{\tilde{a}}(1), \overline{u}_{\tilde{a}}(2), ..., \overline{u}_{\tilde{a}}(\tilde{k}_{\tilde{a}}-1), u_a(\tilde{k}_{\tilde{a}})]$ or $\tau_{\tilde{a}}[u_{\tilde{a}}(\tilde{k}_{\tilde{a}}), \overline{v}_{\tilde{a}}(\tilde{k}_{\tilde{a}}), \overline{x}_{\tilde{a}}(\tilde{k}_{\tilde{a}})]$ during each relaxation iteration. Thus, the objective function (62) can be revised as:

$$\min Z = \sum_{\tilde{a}}\sum_{\tilde{k}_{\tilde{a}}}\{\int_0^{u_{\tilde{a}}(\tilde{k}_{\tilde{a}})}[\tau_{\tilde{a}}[\bar{u}_{\tilde{a}}(\tilde{g}_{\tilde{a}}), u_{\tilde{a}}(\tilde{k}_{\tilde{a}})]]d\omega + \sum_r [u^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})]^{l+1}[\bar{\pi}^{ri*}(\xi^r_{\tilde{a}}) - \bar{\pi}^{rj*}(\xi^r_{\tilde{a}})]\} \quad (63)$$

where

$\bar{u}_{\tilde{a}}(\tilde{g}_{\tilde{a}}) = \tau_{\tilde{a}}[\bar{u}_{\tilde{a}}(1)\cdot\delta^1_{\tilde{a}}, \bar{u}_{\tilde{a}}(2)\cdot\delta^2_{\tilde{a}}, ..., \bar{u}_{\tilde{a}}(\tilde{k}_{\tilde{a}}-1)\cdot\delta^{\tilde{k}_{\tilde{a}}-1}_{\tilde{a}}]$

where $\delta^m_{\tilde{a}l} = 1$, if $m \in \tilde{K}_{\tilde{a}}$; $\delta^m_{\tilde{a}l} = 0$, otherwise.

This problem is solvable using the Frank-Wolfe method by considering only used link element $\tilde{a}$ used time set $\tilde{k}_{\tilde{a}}$. Furthermore, the constraints (48)~(53) can be expressed in terms of and $u_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ only.

2. Frank-Wolfe Method

Applying the Frank-Wolfe algorithm to the minimization of the discretized DUO route choice problem requires a solution of the following linear program at each iteration. As discussed in the above, the NLP problem has only one variable $u_{\tilde{a}}(\tilde{k}_{\tilde{a}})$. Thus, the LP subproblem can be simplified as:

$$\min_p \hat{Z} = \nabla_u Z(u) p^T \quad (64)$$

The objective function (64) can be further formulated as:

$$\min \hat{Z} = \sum_{rs}\sum_{\tilde{a}\tilde{p}}\sum_{\tilde{k}_{\tilde{a}}}[\frac{\partial Z}{\partial u^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})} p^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}})] = \sum_{rs}\sum_{\tilde{a}\tilde{p}}\sum_{\tilde{k}_{\tilde{a}}} p^{rs}_{\tilde{a}\tilde{p}}(\tilde{k}_{\tilde{a}})\cdot\bar{\Omega}^r_{\tilde{a}}(\xi^r_{\tilde{a}}) \quad (65)$$

Note that the cost term $\bar{\Omega}^r_{\tilde{a}}(\xi^r_{\tilde{a}})$ under relaxation is defined as:

$$\bar{\Omega}^r_{\tilde{a}}(\xi^r_{\tilde{a}}) = \frac{\partial Z}{\partial u^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})} = \tau_{\tilde{a}}[\bar{u}_{\tilde{a}}(1)\cdot\delta^1_{\tilde{a}}, \bar{u}_{\tilde{a}}(2)\cdot\delta^2_{\tilde{a}}, ..., \bar{u}_{\tilde{a}}(\tilde{k}_{\tilde{a}}-1)\cdot\delta^{\tilde{k}_{\tilde{a}}-1}_{\tilde{a}}, u_{\tilde{a}}(\tilde{k}_{\tilde{a}})] + \bar{\pi}^{ri}(\xi^r_{\tilde{a}}) - \bar{\pi}^{rj}(\xi^r_{\tilde{a}}) \quad (66)$$

where $\delta^m_{\tilde{a}l} = 1$, if $m \in \tilde{K}_{\tilde{a}}$; $\delta^m_{\tilde{a}l} = 0$, otherwise.

In this combined algorithm, the relaxation procedure is defined as the outer iteration and the F-W procedure is defined as the inner iteration. The one-dimensional search procedure that looks for the optimal step size α^l is:

$$\min_{0\le\alpha^l\le 1}\sum_{\tilde{a}}\sum_{\tilde{k}_{\tilde{a}}}\{\int_0^{u^{l+1}_{\tilde{a}}(\tilde{k}_{\tilde{a}})}\tau_{\tilde{a}}[\bar{u}_{\tilde{a}}(1)\cdot\delta^1_{\tilde{a}}, \bar{u}_{\tilde{a}}(2)\cdot\delta^2_{\tilde{a}}, ..., \bar{u}_{\tilde{a}}(\tilde{k}_{\tilde{a}}-1)\cdot\delta^{\tilde{k}_{\tilde{a}}-1}_{\tilde{a}}, u_{\tilde{a}}(\tilde{k}_{\tilde{a}})]d\omega + \sum_r [u^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})]^{l+1}[\bar{\pi}^{ri}(\xi^r_{\tilde{a}}) - \bar{\pi}^{rj}(\xi^r_{\tilde{a}})]\} \quad (67)$$

where $\delta^m_{\tilde{a}1} = 1$, if $m \in \tilde{K}_{\tilde{a}}$; $\delta^m_{\tilde{a}1} = 0$, otherwise.

Denote the new solution at inner F-W iteration (l+1) as $u^{l+1}_a(k)$. It follows that

$$u^{l+1}_{\tilde{a}}(\tilde{k}_{\tilde{a}}) = u^l_{\tilde{a}}(\tilde{k}_{\tilde{a}}) + \alpha^l[u^l_{\tilde{a}}(\tilde{k}_{\tilde{a}}) - p^l_a(\tilde{k}_{\tilde{a}})] \quad (68)$$

where α^l is the result of the optimal step size searching.

3. Problem Size of VE-Based DTA Model

It was proved in the previous sections that the VE-Based DTA model has less feasible solution sets compared to the VI-Based DTA model using set reduction scheme(SRS) and physical network (PN) approaches. This section shows that the VE-based model has the same objective value as the VI-based model and requires less computational cost compared to the VI-based model in the Frank-Wolfe algorithm step.

[Lemma 1]

The objective value of the VE-based DTA model has the same objective value as the VI-based DTA model in every Frank-Wolfe step.

It is required that the objective function of the VE-based model has the same objective value as the VI-based model in each Frank-Wolfe algorithm step.

Proof.

Since discrete time horizon K is summation set of unused time horizon $\hat{K}$ and used time horizon $\tilde{K}$, i.e., $K = \hat{K} \cup \tilde{K}$. It follows that

$$\sum_{k}\sum_{a}\int_{0}^{u_a(k)}[\tau_a(k_a)]d\omega = \sum_{\hat{k}}\sum_{a}\int_{0}^{u_a(\hat{k})}[\tau_a(\hat{k})]d\omega + \sum_{\tilde{k}}\sum_{a}\int_{0}^{u_a(\tilde{k})}[\tau_a(\tilde{k})]d\omega \quad (69)$$

For unused time horizon $\hat{K}$, the first terms does not affect the object value, i.e. $\sum_{\hat{k}}\sum_{a}\int_{0}^{u_a(\hat{k})}[\tau_a(\hat{k})]d\omega = 0$. Thus, equation (69) can be reduced as solely used time horizon $\tilde{K}$. It follows that

$$\sum_{k}\sum_{a}\int_{0}^{u_a(k)}[\tau_a(k_a)]d\omega = \sum_{\tilde{k}}\sum_{a}\int_{0}^{u_a(\tilde{k})}[\tau_a(\tilde{k})]d\omega \quad (70)$$

By definition of $\tilde{K} = \tilde{K}_{\hat{a}} \cup \tilde{K}_{\tilde{a}}, \forall \hat{a} \in \hat{A}, \tilde{a} \in \tilde{A}$, we have

$$\sum_{\tilde{k}}\sum_{a}\int_{0}^{u_a(\tilde{k})}[\tau_a(\tilde{k})]d\omega = \sum_{\hat{a}}\sum_{\tilde{k}_{\hat{a}}}\int_{0}^{u_{\hat{a}}(\tilde{k}_{\hat{a}})}[\tau_{\hat{a}}(\tilde{k}_{\hat{a}})]d\omega + \sum_{\tilde{a}}\sum_{\tilde{k}_{\tilde{a}}}\int_{0}^{u_{\tilde{a}}(\tilde{k}_{\tilde{a}})}[\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})]d\omega \quad (71)$$

Since during the portion of time, $\tilde{K}_{\hat{a}}$ the first terms does not affect the object value, i.e. $\sum_{\hat{a}}\sum_{\tilde{k}_{\hat{a}}}\int_{0}^{u_{\hat{a}}(\tilde{k}_{\hat{a}})}[\tau_{\hat{a}}(\tilde{k}_{\hat{a}})]d\omega = 0$. Thus, equation (71) can be reduced as solely used time by used links $\tilde{K}_{\tilde{a}}$. It follows that

$$\sum_{\tilde{k}}\sum_{a}\int_{0}^{u_a(\tilde{k})}[\tau_a(\tilde{k})]d\omega = \sum_{\tilde{a}}\sum_{\tilde{k}_{\tilde{a}}}\int_{0}^{u_{\tilde{a}}(\tilde{k}_{\tilde{a}})}[\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})]d\omega \quad (72)$$

The proof is complete.

The computational costs of both the VE- and VI-based models are demonstrated in the example 1 below.

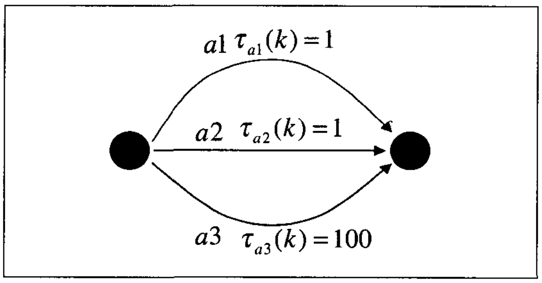
[Example 1]

An employed sample network consists of three paths(links) between one OD pair rs. ⟨Figure 2⟩ illustrates the network with link and link cost function on each link.

Employed assumption for the case study is as follows:

- Time Horizon : 100K
- Link a2 is broken down from time interval 4
- 20 inflow rate at time intervals 1,3,5 and 0 for other time intervals

⟨Table 2⟩ shows the result of the sample network study in terms of results of the VI-based objective function.



⟨Figure 2⟩ A sample network

⟨Table 2⟩ Result of minimization problem

t^{rs}	$f^{rs}_{\tilde{p}_1} = u_{\tilde{a}_1} = u^r_{\tilde{a}_1}$	$f^{rs}_{\tilde{p}_2} = u_{\tilde{a}_2} = u^r_{\tilde{a}_2}$	$f^{rs}_{\tilde{p}_3} = u_{\tilde{a}_3} = u^r_{\tilde{a}_3}$
1	10	10	0
2	0	0	0
3	10	10	0
4	0	0	0
5	20	0	0
6	0	0	0
⋮	⋮	⋮	⋮
100	0	0	0

Based on the ⟨Table 2⟩, the derived sets are as follows:

- $T=\{1,2,3,\cdots,100\}$, $\hat{T}=\{2,4,6,7,\cdots,100\}$, $\tilde{T}=\{1,3,5\}$
- $A=\{a_1,a_2,a_3\}$, $\tilde{A}=\{a_1,a_2\}$, $\hat{A}=\{a_3\}$
- $\tilde{T}^{rs}_{\tilde{p}_1}=\tilde{T}_{\tilde{a}_1}=\{1,3\}$, $\tilde{T}^{rs}_{\tilde{p}_2}=\tilde{T}_{\tilde{a}_2}=\{1,3,5\}$, $\tilde{T}^{rs}_{\tilde{p}_3}=\tilde{T}_{\tilde{a}_3}=\{\}$
- $\bar{\pi}^{rr}(1\sim 100)=0$, $\bar{\pi}^{rs}(1\sim 100)=1$
- $\hat{T}^{rs}_{\hat{p}_1}=\hat{T}_{\tilde{a}_1}=\{2,4,5,6,\cdots,100\}$,
  $\hat{T}^{rs}_{\tilde{p}_2}=\hat{T}_{\tilde{a}_2}=\{2,4,6,\cdots,100\}$, $\hat{T}^{rs}_{\hat{a}_3}=\hat{T}_{\hat{a}_3}=\{1,\cdots,100\}$

The VI- and VE-based objective values can be evaluated as follows: as can be seen, it is noticed that two objective values are equal to zero and the VE-based method uses much reduced calculation steps than the VI-based method in each Frank-Wolfe step.

VI-based evaluation

$$\min Z=\sum_k\left\{\sum_a\int_0^{u_a(k)}[\tau_a(k)]d\omega+\sum_r u_a^r(k+\bar{\pi}^{rr^*}(k))\left[\bar{\pi}^{rr^*}(k)-\bar{\pi}^{rs^*}(k)\right]\right\}$$
$$=\begin{pmatrix}1*\left[\sum_{k=1}^{100}u_{a_1}(k)\right]+1*\left[\sum_{k=1}^{100}u_{a_2}(k)\right]+100*\left[\sum_{k=1}^{100}u_{a_3}(k)\right]+\\(-1)*\left[\sum_{k=1}^{100}u_{a_1}(k)\right]+(-1)*\left[\sum_{k=1}^{100}u_{a_2}(k)\right]+(-1)*\left[\sum_{k=1}^{100}u_{a_3}(k)\right]\end{pmatrix}=0$$

VE-based evaluation

$$\min Z=\sum_{\tilde{a}}\left\{\sum_{\tilde{k}_{\tilde{a}}}\int_0^{u_{\tilde{a}}(\tilde{k}_{\tilde{a}})}[\tau_{\tilde{a}}(\tilde{k}_{\tilde{a}})]d\omega+\sum_r u_{\tilde{a}}^r(\tilde{k}_{\tilde{a}}+\bar{\pi}^{rr^*}(\tilde{k}_{\tilde{a}}))\left[\bar{\pi}^{rr^*}(\tilde{k}_{\tilde{a}})-\bar{\pi}^{rs^*}(\tilde{k}_{\tilde{a}})\right]\right\}$$
$$=\begin{pmatrix}1*\left[u_{\tilde{a}_1}(1)+u_{\tilde{a}_1}(3)+u_{\tilde{a}_1}(5)\right]+1*\left[u_{\tilde{a}_2}(1)+u_{\tilde{a}_2}(3)\right]+\\(-1)*\left[u^r_{\tilde{a}_1}(1)+u^r_{\tilde{a}_1}(3)+u^r_{\tilde{a}_1}(5)\right]+(-1)*\left[u^r_{\tilde{a}_2}(1)+u^r_{\tilde{a}_2}(3)\right]+\end{pmatrix}=0$$

Note that since this step is evaluated in every iteration of Frank-Wolfe algorithm, it can save *computational cost.*

4. Summary of Solution Algorithm

A heuristic algorithm is proposed by combining diagonalization techniques, Frank-Wolfe algorithm, and physical network approach. The summary of the algorithm is as follows:

[Step 0] Initialization

Initialize every link a as an empty set, as an unused link, $\hat{A}=A$, and used link set $\tilde{A}=\{\ \}$, and it means $\tilde{K}_{\tilde{a}}=\{\ \}$ and $\tilde{k}_{\tilde{a}}=\{\ \}$. Initialize all link flows $\{x_a^{(0)}(k)\},\{u_a^{(0)}(k)\},\{v_a^{(0)}(k)\}$ to zero and calculate initial time estimates $\tau_a^{(1)}(k)=\tau_a(u_a(k),v_a(k),x_a(k))$. Set the outer iteration counter l=1.

[Step 1] Relaxation

Set the inner iteration counter n=1. Find a new approximation of actual link travel times in terms of $\tilde{a}\in\tilde{A}$ and $\tilde{k}_{\tilde{a}}:\bar{\tau}^{(n)}_{\tilde{a}}(\tilde{k}_{\tilde{a}})=\tau_{\tilde{a}}(u^{(*)}_{\tilde{a}}(\tilde{k}_{\tilde{a}}),v^{(*)}_{\tilde{a}}(\tilde{k}_{\tilde{a}}),x^{(*)}_{\tilde{a}}(\tilde{k}_{\tilde{a}}))$, where (*) denotes the final solution obtained from the most recent inner iteration. Update the value of $\delta^{\tilde{k}_{\tilde{a}}}_{\tilde{a}1}(\tilde{i}_{\tilde{a}}),\delta^{\tilde{k}_{\tilde{a}}}_{\tilde{a}2}(\tilde{i}_{\tilde{a}})$ according to equations (60) and (61) by using $\bar{\tau}^{(n)}_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ Solve the route choice problem.

[Step 1.1] Update

Calculate $\tau_{\tilde{a}}(\bar{u}_{\tilde{a}}(\tilde{1}_{\tilde{a}}),\bar{u}_{\tilde{a}}(\tilde{2}_{\tilde{a}}),\ldots,\bar{u}_{\tilde{a}}((\widetilde{k-1})_{\tilde{a}}),u_{\tilde{a}}(\tilde{k}_{\tilde{a}}))$ using the travel time function.

[Step 1.2] Direction Finding and Update of Used Link and Time Sets

Based on $\tau_a(\bar{u}_a(1),\bar{u}_a(2),\ldots,\bar{u}_a(k-1),u_a(k))$, search *search the minimal-cost route over the physical* network. Perform an all-or-nothing assignment, yielding subproblem solution $p^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})$. Calculate $q^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ and $y^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ by using $\delta^{\tilde{k}_{\tilde{a}}}_{\tilde{a}1}(\tilde{i}_{\tilde{a}}),\delta^{\tilde{k}_{\tilde{a}}}_{\tilde{a}2}(\tilde{i}_{\tilde{a}})$. Update $\tilde{A}=\tilde{A}\cup\{\tilde{a}\}$, $\tilde{k}_{\tilde{a}}=\tilde{k}_{\tilde{a}}\cup\tilde{k}^r_{\tilde{a}}$ and $\tilde{K}_{\tilde{a}}=\tilde{K}_{\tilde{a}}\cup\tilde{k}_{\tilde{a}}$.

[Step 1.3] Line Search

Find the optimal step size that solves the one dimensional search problem using a standard line search procedure.

[Step 1.4] Move

Find a new solution by combining $u^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ *and* $p^r_{\tilde{a}}(\tilde{k}_{\tilde{a}})$ using the optimal step size.

[Step 1.5] Convergence Test for Inner Iteration

If n equals a pre-specified number, go to step 2; otherwise, set n=n+1, go to step 1.1.

[Step 2] Convergence Test for Outer Iteration

If $\bar{\tau}_{\tilde{a}}^{(l)}(\tilde{k}_{\tilde{a}}) \cong \bar{\tau}_{\tilde{a}}^{(l-1)}(\tilde{k}_{\tilde{a}})$, stop. The current solution $\{x_{\tilde{a}}(\tilde{k}_{\tilde{a}})\}\{u_{\tilde{a}}(\tilde{k}_{\tilde{a}})\}\{v_{\tilde{a}}(\tilde{k}_{\tilde{a}})\}$ is in a near optimal state; otherwise, set $l=l+1$, go to step 1.

Ⅵ. Computational Experience

The test is designed to demonstrate that the new model and algorithm built on VE is efficient in terms of computational efforts, and produces results that are consistent with the principles of DUO.

1. Link Travel Time Function

Since the relationship between travel time and traffic density is monotonic, a modified Greenshields formula is used in the new algorithm to determine the speed on a freeway link.

$$\text{If } k \le k_j,\ u = u_{min} + (u_{max} - u_{min})(1 - \frac{k}{k_j});$$
$$\text{If } k > k_j,\ u = u_{min}. \qquad (73)$$

where,

u : speed,

u_{min} : minimum speed at jam density,

u_{max} : free flow speed,

k : density,

k_j : jam density.

Thus, the travel time along a freeway link will be:

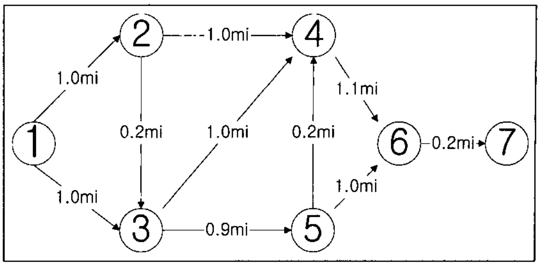
$$\text{If } k \le k_j,\ \tau_a(t) = \frac{L_a}{u_{min} + (u_{max} - u_{min})(1 - \frac{k}{k_j})};$$
$$\text{If } k > k_j,\ \tau_a(t) = \frac{L_a}{u_{min}} \qquad (74)$$

where,

La : length of link a,

Numerical Example

A 7-node, 10-link asymmetric network is constructed as the test network, as shown in 〈Figure 3〉.



〈Figure 3〉 A Test Network

〈Table 3〉 Path Travel Time Experienced by Vehicles Departing at Used Time Interval

Used Times ($\tilde{t}_{\tilde{p}}$)	Used Routes ($\tilde{p}$)	Ideal Route Travel Time (Time Interval)
1	1 2 4 6 7	4.2 + 4.2 + 4.6 + 2.0 = 15.0
	1 3 5 6 7	4.4 + 4.0 + 4.4 + 1.6 = 14.4
2	1 2 4 6 7	4.5 + 4.5 + 5.0 + 9.6 = 23.6
	1 3 4 6 7	4.9 + 4.0 + 5.0 + 9.6 = 23.5
	1 3 5 6 7	4.9 + 4.5 + 4.9 + 9.6 = 23.9
3	1 2 3 5 6 7	4.8 + 1.1 + 4.9 + 5.2 + 3.5 = 19.5
	1 3 5 6 7	5.6 + 4.9 + 5.2 + 3.5 = 19.2
4	1 2 4 6 7	5.4 + 4.7 + 5.4 + 3.5 = 19.0
	1 3 4 6 7	6.2 + 4.0 + 5.4 + 3.5 = 19.1
5	1 2 4 6 7	5.6 + 4.9 + 5.9 + 9.6 = 26.0
	1 2 3 4 6 7	5.6 + 1.0 + 4.3 + 5.9 + 9.6 = 26.4
	1 2 3 5 6 7	5.6 + 1.0 + 5.3 + 4.6 + 9.6 = 26.1
	1 3 4 6 7	5.9 + 4.3 + 5.8 + 9.6 = 25.6
	1 3 5 6 7	5.9 + 4.6 + 5.0 + 9.6 = 25. 1
6	1 2 4 6 7	5.8 + 5.1 + 5.9 + 9.6 = 26.4
	1 2 3 4 6 7	5.8 + 0.9 + 4.3 + 5.9 + 9.6 = 26.5
	1 2 3 5 6 7	5.8 + 0.9 + 4.8 + 5.0 + 9.6 = 26. 1
	1 3 4 6 7	6.6 + 4.3 + 5.9 + 9.6 = 26.4
7	1 2 4 6 7	6.3 + 4.7 + 5.8 + 9.6 = 26.4
	1 2 3 4 6 7	6.3 + 1.0 + 4.3 + 5.8 + 9.6 = 27.0
	1 2 3 5 6 7	6.3 + 1.0 + 5.0 + 5.0 + 9.6 = 26.9
	1 3 4 6 7	6.7 + 4.3 + 5.8 + 9.6 = 26.4
	1 3 5 6 7	6.7 + 5.0 + 5.0 + 9.6 = 26.3
8	1 2 4 6 7	6.6 + 4.7 + 5.6 + 9.6 = 26.5
	1 2 3 4 6 7	6.6 + 0.9 + 4.3 + 5.6 + 9.6 = 27.0
	1 2 3 5 6 7	6.6 + 0.9 + 5.6 + 5.4 + 9.6 = 28.1
	1 3 4 6 7	7.5 + 4.3 + 5.6 + 9.6 = 27.0
	1 3 5 6 7	7.5 + 5.6 + 5.4 + 9.6 = 28.1
9	1 2 4 6 7	6.6 + 4.8 + 5.7 + 9.6 = 26.7
	1 3 4 6 7	7.1 + 4.3 + 5.6 + 9.6 = 26.6
	1 3 5 6 7	7.1 + 5.6 + 5.4 + 9.6 = 27.7
10	1 2 4 6 7	6.6 + 4.8 + 5.6 + 9.6 = 26.6
	1 3 5 6 7	7.9 + 4.7 + 6.0 + 9.6 = 28.2

The characteristics of the network are:

- Origin is node 1 and destination is node 7.
- The O-D flows are 40 vehicles per each time

interval of the first 10 time intervals
- Each time interval has 15 seconds.
- Each link is a freeway segment with a capacity of 2200vph.
- Each link has 1 lane.
- Free flow speed is 30 mph.

The actual path travel times experienced by the 10 groups of vehicles are shown in ⟨Table 3⟩. In the ⟨table 3⟩, model results are summarized based on the route travel time experienced by the used paths. The experienced ideal route travel time is calculated based on the following equation.

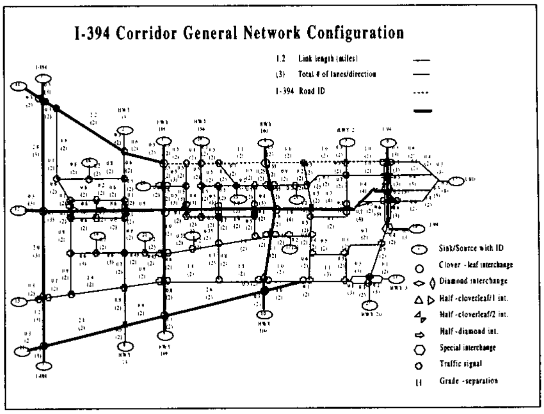
As shown in the ⟨table 3⟩, the actual travel times for all used paths are almost equal with small marginal errors. This satisfies the ideal DUO principle proposed in this paper. The results show that the solution from new algorithm is compatible with the DUO condition. It is noteworthy that when the size of time interval is shortened, a more accurate estimation can be achieved.

2. Computational Experience on I-394 Network

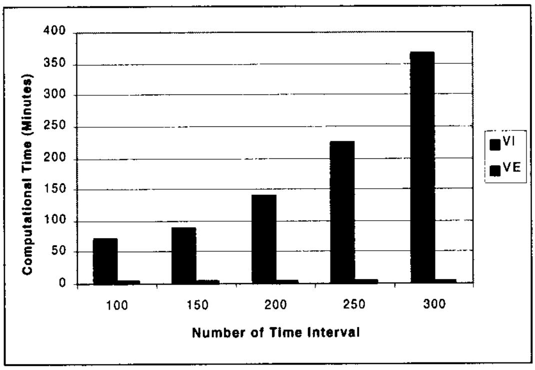
The I-394 network ⟨Figure 4⟩, consisting of 130-nodes and 347-links with 27*27 O-D pairs, was employed to estimate the computational time of the proposed model. Two programs were run on NOVA workstations of 200MHz frequency. The comparison of computational times calculated by the new algorithm(VE approach) and the previous algorithm(VI approach) is presented in ⟨Figure 5⟩. The conditions for the comparison are 4 outer iterations and 5 inner iterations.

While the previous algorithm converges to a solution after 72.93 minutes, the new algorithm converges after 5.35 minutes. After the first convergence, the computational time of VI increases exponentially up to 366.3 minutes. However, the solution algorithm based on the VE almost does not increase. As the network and problem sizes are increased, the VE algorithm will require considerably less time than VI algorithm.

⟨Figure 6⟩ shows a comparison of the two algorithms in terms of problem sizes represented by number of used links and entry time intervals. For the VE-based algorithm, 267 links are used and average number of entry time intervals for each used link is 26 times. Since this used links and entry times are independent of the number of time intervals, the problem size does not change.



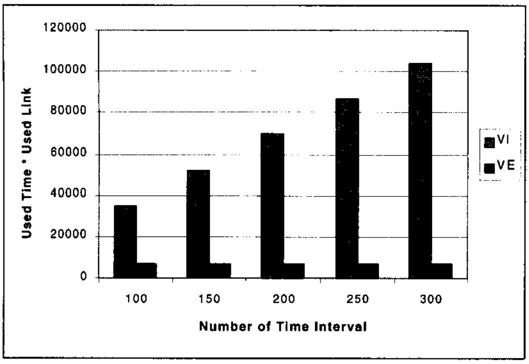
⟨Figure 4⟩ I-394 Network



⟨Figure 4⟩ Comparison of the Performance of the VE and VI Algorithms

⟨Table 4⟩ Comparison of VI and VE in terms of Computational Times

Time Interval (K)	VI (Seconds)	VE (Seconds)	VE/VI*100 (%)
100	4376	321	7.3
150	5438	321	5.9
200	8434	322	3.8
250	13456	323	2.4
300	21980	324	1.5



⟨Figure 6⟩ Comparison of the Performance of the VE and VI Algorithms

However, for the VI-based algorithm solves the problem in terms of the full solution set size for each time interval. Thus, the problem size increases linearly as number of time intervals increase. Since dynamic problems involve 3 dimensional arrays (i.e., number of times, number of links, and number of origins), the linear increase of problem size results in an exponential increase in computational time.

Ⅶ. Conclusion

Variational equality route choice condition is proposed and applied for formulating and solving a link-based analytical DTA model. In VE-based condition, the model is designed to reduce the size of solution sets by considering only used links and entry time intervals of inflows to those links based on the definition that all used links are on the shortest path at entry time intervals of some positive inflows. The unique feature of VE is that compared with VI-based route choice conditions, equality sign in nonnegative inflow variable and inequality sign in route choice condition can be dropped. Thus, compared with VI-based route choice model, feasible solution set size can be reduced dramatically without affecting objective value. In addition, the length of the horizon time does not have any effect on the computational performance of the model.

From the results implemented on an asymmetric toy network, it is shown that the algorithm produces a solution satisfying the ideal DUO condition. The computational experience on the I-394 network demonstrates that the proposed model can save more than 93% of computational time compared to the previous VI-based approach. Most importantly, the computational speed of the algorithm is not affected by the horizon time or the number of time intervals. Since only used time intervals of each used link are considered, the length of the horizon time does not have any effect on the computational performance of the proposed model. In conclusion, the model is more applicable for real-time applications because of the faster computational speed.

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