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# Block Error Performance Evaluation of DS-CDMA System with Combined Techniques in Mobile Communication Channel

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## ABSTRACT

In this paper, we analyze the block error performance of the CDMA system, whose information data are spread spectrum with PN sequences, and the chip signals are modulated with BPSK signal, combining MRC(Maximum Ratio Combine) diversity techniques with repetition transmission, FEC(Forward Error Correction) code in mobile communication channel which is characterized by Nakagami fading.

As a results of study, the coding techniques provide more efficient improvement than a diversity techniques, but coding techniques are required the adding bandwidth as many coding rate. Also, when the system is combined MRC diversity techniques with coding techniques, the amount of improvement is dramatically increased.

## 요 약

본 논문에서는 Nakagami 페이딩 채널 특성의 이동 통신 환경에서 정보 데이터를 PN 시퀀스로 대역 확산시켜 BPSK 변조를 취한 CDMA 시스템에 MRC 다이버시티와 반복 전송(다수결 선택)이나 부호화 기법을 다양하게 결합하였을 때의 블록 오류 성능 향상에 대하여 분석한다. 분석 결과로써, 부호화는 다이버시티 기법보다 더 효과적인 성능 향상을 보였다. 그러나 부호화는 더 넓은 주파수 대역을 필요로 하게 된다. 또한 MRC 다이버시티와 부호화 기법을 결합하였을 때 성능 개선이 크게 향상되었다.

## Keywords

Block error, MRC diversity, BCH, RS, Golay, Repetition transmission, Nakagami fading

## 1. INTRODUCTION

The Direct-Sequence(DS) CDMA in wireless and cellular mobile communications is attractive to the view point of random access capability and resistance against multipath fading [1].

The transmission of information over radio channels with multiple changing propagation paths is subject to fading, i.e., random time variations of the receiver signal strength. For digital

transmission over a fading channel, time variation causes a changing error probability with the effect of clustering errors at the receiver output.

The present work was motivated by the need to evaluate block data transmission performance under fading conditions for application to sequential polling of vehicles in a fleet. These systems commonly employ some form of error control, such as error detection block coding, so that evaluation of performance requires the

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analysis of the probabilities distribution of the number of errors in code blocks transmitted under fading conditions. In data communication applications, the expressions of error probabilities are important in evaluating system performance.

Previous work on error probabilities have been conducted considering mobile radio channels with Rayleigh [2]-[4], Ricain [5] and Nakagami fading [6] characteristics. The Nakagami fading is chosen to characterize the fading channel because it takes the Rayleigh distribution as a special case, approximates the Rician distribution well, models fading conditions which are more or less severe than those of Rayleigh, and more importantly, fits experimental data better than Rayleigh or Ricain distributions [7],[8].

In this paper, we analyze the error performance of the DS-CDMA BPSK system combining with repetition transmission, FEC code, and MRC diversity techniques in mobile communication channel which is characterized by Nakagami fading.

## II. PERFORMANCE OF DS-CDMA BPSK SYSTEM IN NAKAGAMI FADING CHANNEL

This section is concerned with the calculation of the error probability of the DS-CDMA communications in a multipath faded channel that is modeled by a discrete set of  $m$ -distribution faded paths. The system, which is illustrated in Fig. 1, consists of  $K$  users. Each user is assumed to be used BPSK along with DS spread-spectrum modulation. The  $k$ th user first generates data bits at a rate of  $1/T$  bits per second. Its binary data signal  $b_k(t)$  and signature sequence signal  $a_k(t)$  are given by [9]

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_i^{(k)} p_T(t-iT) \quad (1)$$

$$a_k(t) = \sum_{i=-\infty}^{\infty} a_i^{(k)} p_{T_c}(t-iT_c) \quad (2)$$

where  $p_\tau(t)$  is a unit rectangular pulse on  $[0, \tau)$ ,  $b_i^{(k)}$  is one symbol of the  $k$ th transmitted signal and  $a_i^{(k)}$  is the  $k$ th user's code sequence. Both of these symbols take on values in  $\{-1, 1\}$  and we have  $a_i^{(k)} = a_{i+N}^{(k)}$  for all  $i$  and  $k$  and for some integer  $N = T/T_c$  where  $T$  is the bit interval duration and  $T_c$  is the chip length.

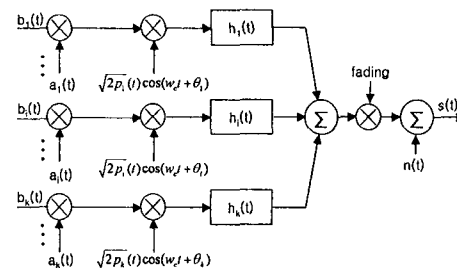


Fig. 1. DS-CDMA system model.

The modulated BPSK signals of  $k$  users are transmitted to channel, and then they are distorted by fading which is characterize a Nakagami fading.

A Nakagami fading characterizes channels with different fading depths through a parameter called amount of fading. The signal envelope,  $R$  is a random variable with a Nakagami fading probability density function (pdf) [10] i.e.,

$$p(R) = \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{mR^2}{\Omega}\right) \quad (3)$$

where  $\Gamma(\cdot)$  is the Gamma function,  $\Omega/2 = \overline{R^2}/2$  is the mean power of the enveloped signal by fading, and  $m$  is fading index ( $m = \Omega^2 / (\overline{R^2} - \Omega) \geq 1/2$ ). if we are represented to equation (3) by  $\gamma (= \frac{R^2}{2N})$ , the pdf of  $\gamma$  is found to be

$$p(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(m) \gamma_o} \exp\left(-\frac{m\gamma}{\gamma_o}\right) \quad (4)$$

where  $\gamma_o = \Omega/2N$  is average signal to noise power ratio. Note that (3) and (4) is represented to Rayleigh and AWGN pdf, when  $m$  is 1, and  $m$  is infinity, respectively.

The pdf of Nakagami fading which changes equation (4) into average signal to noise ratio of DS-CDMA system, is [2].

$$p_{cdma}(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(m) \Gamma'} \exp\left(-\frac{m\gamma}{\Gamma'}\right) \quad (5)$$

where  $\Gamma'$  is average signal to noise power ratio in DS-CDMA system and is given by

$$\Gamma' = \frac{1}{\frac{2(L \cdot K - 1)}{3PG} + \frac{N_o}{2E_b}} \quad (6)$$

where  $L$  is the multipath number of channel,  $K$  is the number of multiple access user, and  $PG$  is processing gain.

The block error probability of BPSK with a signal to noise power ratio (SNR)  $\gamma$  is

$$P_{SM}(\gamma) = \frac{1}{2} \operatorname{erfc}(\sqrt{\gamma}) \quad (7)$$

The block error probability for BPSK signaling over a Nakagami fading can be found by averaging the bit error probability of (7) with respect to the pdf of fading.

$$P_{SMP} = \int_0^\infty P_{SM}(\gamma) p_{cdma}(\gamma) d\gamma \quad (8)$$

### III. PERFORMANCE OF MRC DIVERSITY AND CHANNEL CODING IN NAKAGAMI FADING

#### A. MRC Diversity Techniques

Assuming a maximal ratio combining approach with  $L$  identical branches, the diversity effect is examined as follows. The output signal to noise power ratio after maximal ratio combining is equal to the sum of the signal to noise power ratio of the various combining branches. It can be shown that pdf of the resulted signal to noise power ratio is

$$p_{mrc}(\gamma) = \frac{\gamma^{mL-1}}{\Gamma(mL)} \left(\frac{m}{\Gamma'}\right)^{mL} \exp\left(-\frac{m\gamma}{\Gamma'}\right) \quad (9)$$

The average error probability of BPSK signal over the Nakagami fading channel with MRC diversity reception is

$$P_{MRC} = \int_0^\infty P_{SM} p_{mrc}(\gamma) d\gamma \quad (10)$$

#### B. Repetition Transmission

In this techniques each message is sent an odd number of times and at the reception, a bit by bit majority decision is applied. If  $s$  is the number of repeats, a  $(s+1)/2$  out of  $s$  majority voting process is used to determine each valid bit in the message.

The probability of bit error,  $P_{TB}$ , after an  $(s+1)/2$  out of  $s$  majority voting is the probability of at least  $(s+1)/2$  errors occurring, i.e.,

$$P_{TB} = \sum_{i=(s+1)/2}^s \binom{s}{i} (P)^i (1-P)^{s-i} \quad (11)$$

where  $P_{TB}$  is the bit error probability of a single transmission. The probability of having exactly  $j$  corrupted bits in an  $N$ -bit message is

$$P_{TB}(N, j) = \binom{N}{j} (1 - P_{TB})^{N-j} (P_{TB})^j \quad (12)$$

when errors generate more than  $t$  bits, the probability of message error is

$$P_{eM} = 1 - \sum_{j=0}^t P_{TB}(N, j) \quad (13)$$

where  $P_{TB}(N, j)$  is given by equation (12).

Repetition transmission is also a type of code, known as repetition code. The repetition code is represented as  $(s, 1)$ , having a minimum distance equal to  $s$ . Therefore, it is able to correct up to  $(s - 1)/2$  errors in an  $s$ -bit message, having a rate  $R = 1/s$

### C. Error Correcting Code

A linear block code having  $k$  information bits and  $n - k$  redundancy bits is described as  $(n, k)$ . The ratio  $R = k/n$  is called the code rate. Moreover, if its minimum distance is  $d_{\min}$ , this code is able to correct up to  $t = (d_{\min} - 1)/2$  bits out of  $n$  bits.

It is easy to see that, for a code that can correct up to  $t$  bits, the probability of message error is

$$P_{eM} = 1 - \sum_{j=0}^t P(N, j) \quad (14)$$

$$P(N, j) = \binom{N}{j} (1 - P)^{N-j} P^j \quad (15)$$

where  $P(N, j)$  is the probability of having exactly  $j$  bits error in an  $N$  bit message, and  $P$  is error probability according to modulation technique. The bit error probability  $P(N, j)$  used in Equation (15) varies according to be the modulation technique. It is important to note that, in order to keep the same transmitted power, the SNR per bit in the encoded message is multiplied by the code rate,  $R$ . In other words, the average SNR per bit of the encoded message is

$$(k/N) \Gamma'$$

As we consider another error correction code, one special subclass of the BCH codes is the particularly useful nonbinary set called Reed-Solomon codes. RS codes achieve the largest possible code minimum distance for any linear code with same encoder input and output block lengths. RS codes are particularly useful for burst-error correction; that is, they are effective for channels that have memory. Also, they can be used efficiently on channels where the set of input symbols is large. For nonbinary codes, the distance between two code words is defined as the number of nonbinary symbols in which the sequences differ. For RS codes the code minimum distance is given by

$$d_{\min} = n - k + 1 \quad (16)$$

where  $k$  is the number of information symbols being encoded, and  $n$  is the total number of code symbols in the encoded block. The code is capable of correcting any combination of  $t$  or fewer symbol errors, as follows

$$t = \frac{d_{\min} - 1}{2} = \frac{n - k}{2} \quad (17)$$

A  $t$ -error-correcting RS code with an alphabet of  $2^m$  symbols has  $n = 2^m - 1$  and  $k = 2^m - 1 - 2t$ , where  $m = 2, 3, \dots$

The RS decoded symbol error probability,  $P_E$  [10], can be written in terms of the channel symbol error probability,  $p$

$$P_E = \frac{1}{n} \sum_{j=t+1}^n j \binom{n}{j} p^j (1 - p)^{n-j} \quad (18)$$

The bit error probability can be upper bounded by the symbol error probability for specific

modulation types.

#### IV. RESULT AND DISCUSSION

In this paper, we now consider the performance of the DS-CDMA BPSK system in  $m$ -distribution fading channel. As a technique for the performance improvement, diversity, coding, and repetition transmission have been used, and their performance have been compared and analyzed.

To compare the performance of both coding and repetition transmission, we set the error correcting capability at the same  $t=3$  bits. Accordingly, we select 2 branch MRC diversity, (23,12) Golay code, (7,5) RS code, and (5,1) Majority voting.

Fig. 2 is shown the effects of diversity according to the number of multiple access user when  $m$  is equal to 3. As user number is increased, interval between curves is decreased by degrees. it is shown that, with no diversity, voice service is available to less than 5 users. With, however, diversity, voice service is reachable up to 20 users, even data service is possible with less than 5 users.

Fig. 3 show BER performance related to fading figure  $m$  and the number of multiple access user at  $E_b/N_o=18$  dB. As fading is deeper( $m$  decreases), improvement of diversity decreases. Also even though user increases, slope of curve is not large. But it is shown that in the case of  $m=3$ , improvement of diversity is large, and sensitive to user.

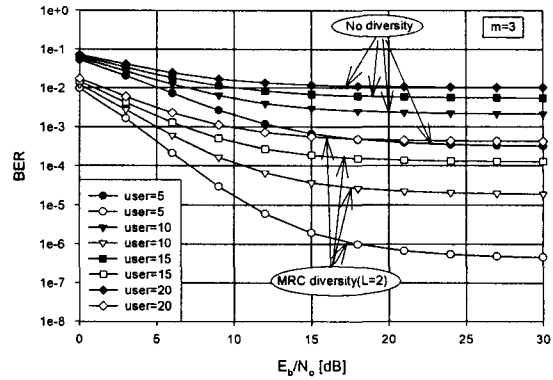


Fig. 2. Error rates for DS/CDMA BPSK system according to MRC diversity and user ( $L=2$ ,  $m=3$ ).

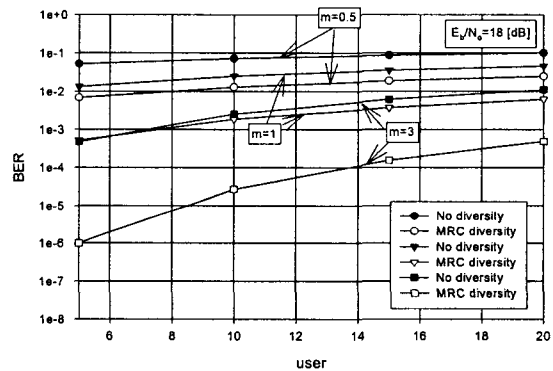


Fig. 3. Error rates for DS/CDMA BPSK signal not using and using maximal ratio diversity according to user and fading index  $m$  ( $L=2$ ).

In the case of user=10, and  $m=3$ , fig 4 represents degree of improvement according to improvement techniques(diversity, code, and repetition transmission). In order to compare coding techniques with repetition transmission, we adopt to be equal to error correcting ability( $t=3$ ). we are shown that when use only one of improvement techniques, at higher  $E_b/N_o$ , Golay code is better than the others, and when use diversity combined with codes, also Golay code is the best.

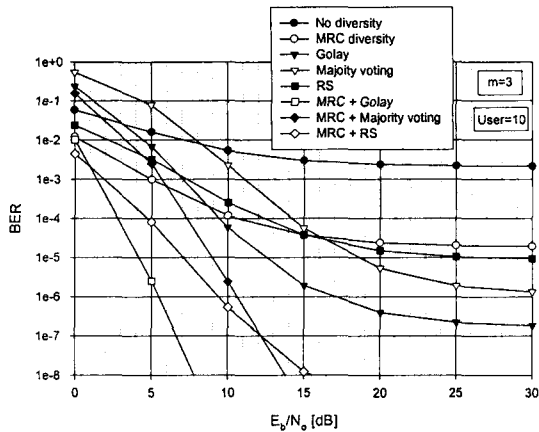


Fig. 4. Comparison of the techniques using diversity combined with coding and repetition transmission (user=10,  $m=3$ ).

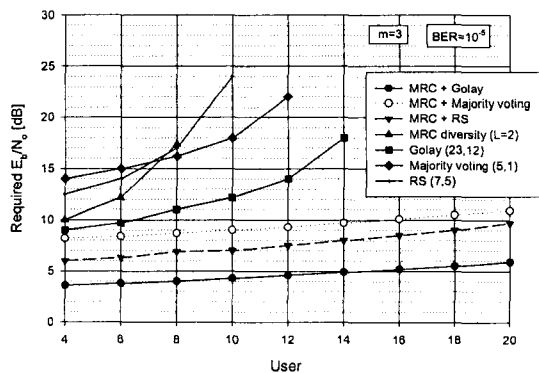


Fig. 5. Comparison of the techniques using diversity combined with coding and repetition transmission ( $BER=10^{-5}$ ,  $m=3$ ).

Fig. 5 shows relation of the required  $E_b/N_0$  according to the number of user to satisfy data service quality ( $BER=10^{-5}$ ). In the case of only diversity, user is able to up to 8. In the case of only coding or repetition transmission, it is shown that Golay, RS, and Majority voting is restricted within 14, 10, and 12 respectively. But diversity combined with codes techniques can be reduced restriction of users.

When use only Coding or diversity, coding is

better than diversity at high  $E_b/N_0$ , but the former has problem in aspect of increase of bandwidth. When it is combined MRC diversity techniques with coding techniques, the amount of improvement is increased. But Bit error performance improvement is accomplished at the expense of increasing system complexity and cost. Some techniques can yield better results than others, but can be more costly. Therefore, the decision for one or another technique depends on the analysis of cost versus effectiveness.

### V. CONCLUSIONS

In this paper, we analyze the block error performance improvement of the DS-CDMA BPSK signal with combining repetition transmission, FEC code and MRC diversity techniques in mobile communication channel which is characterized by  $m$ -distribution fading. The results that we have obtained in this paper are that the coding techniques provides more efficient improvement over diversity techniques, but coding techniques are required the adding bandwidth as many coding rate. When it is combined MRC diversity techniques with coding techniques, the amount of improvement is dramatically increased. Block error performance improvement is accomplished at the expense of increasing system complexity and cost. Some techniques can yield better results than others, but can be more costly. Therefore, the decision for one or another techniques depends on the analysis of cost versus effectiveness. In the following, we shall be going to investigate this trade-off.

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