A study on the Optimal Adaptive Data Association for Multi-Target Tracking

이양원*

다중표적을 위한 최적 데이터 결합기법 연구

Yang-weon Lee*

본 연구는 한국과학재단 목적기초연구(R05-2002-000500-0)지원으로 수행되었음

ABSTRACT

This paper proposed a scheme for finding an optimal adaptive data association for multi-target between measurements and tracks. First, we assume the relationships between measurements as Mrkov Random Field. Also assumed a priori of the associations as a Gibbs distribution. Based on these assumptions, it was possible to reduce the MAP estimate of the association matrix to the energy minimization problem. After then, we defined an energy function over the measurement space, that may incorporate most of the important natural constraints.

Through the experiments, we analyzed and compared this algorithm with other representative algorithms. The result is that it is stable, robust, fast enough for real timecomputation, as well as more accurate than other methods.

Keywords

Multi-target tracking, Data Association

Ⅰ. Introductoin

The primary purpose of a multi-target tracking(MTT) system is to provide an accurate estimate of the target position and velocity from the measurement data in a field of view. Naturally, the performance of this system is inherently limited by the measurement inaccuracy and source uncertainty which arises from the presence of missed detection, false alarm, emergence of new targets into the surveillance region and disappearance of old targets from the surveillance region. Therefore, it is difficult to determine precisely which target corresponds to each of the closely spaced measurements. Although trajectory estimation problems have been well studied in the past, much of this previous work assumes that the particular target corresponding to each observation is known. Recently, with the proliferation of surveillance systems and their increased sophistication, the tools for designing algorithms for data association have been announced.

Generally, there are three approaches in data association for MTT: non-Bayesian approach based on likelihood function [1], Bayesian approach [2, 3], and neural network approach [4, 5]. The major difference of the first two approaches is how to treat the false alarms. The non-Bayesian approach calculates all the

*호남대학교 정보통신학과

접수일자 : 2002. 9. 23

likelihood functions of all the possible tracks with given measurements and selects the track which gives the maximum value of the likelihood function. Meanwhile, the tracking filter using Bayesian approach predicts the location of interest using *a posteriori* probability. The two approaches are inadequate for real time applications because the computational complexity is overwhelming even for relatively large targets and measurements and yet a computationally efficient substitute based on a careful understanding of its properties is lacking.

In this paper, we derive the new model for data association which reflects the natural constraints of the MTT problem and convert the derived model into the minimization problem of energy function by MAP estimator [10]. The coefficients of energy function is calculated by Lagrange multiplier, [9] and local dual theory [8].

This paper is organized as follows. Ch2 gives a brief description of the MTT problem and explains that data association problem can be formulated as a constrained optimization problem. In ch3, as an optimal method for solving this problem, we propose the use of differential of the Lagrange multiplier. In ch4, some simulation results of the proposed algorithm are given.

II. Problem Formulation and Energy Function

1. Representing Measurement-Target Relationship

Let m and n be the number of measurements and targets, respectively, in a surveillance region. Then, the relationships between the targets and measurements are efficiently represented by the *validation matrix* ω [6]:

$$\omega = \omega_{jt} \; | j \in [1, m],\ t \in [0, n],$$

(1)

where the first column denotes clutter and always $\omega_{j0} = 1$. For the other columns, $\omega_{jt} = (j \in [1, m],\ t \in [1, n])$, if the validation gate of target t contains the measurement j and $\omega_{jt} = 0$, otherwise.

Based on the validation matrix, we must find *hypothesis matrix* [6] $\hat{\omega}(= \omega_{jt} | j \in [1, m], t \in [0, n])$ that must obey the data association hypothesis(or feasible events [6]):

$$\begin{cases} \sum_{j=1}^{m} \widehat{\omega_{jt}} = 1 \ \ \text{for}(t \in [1, n]), \\ \sum_{t=0}^{n} \widehat{\omega_{jt}} = 1 \ \ \text{for}(j \in [1, m]). \end{cases} \qquad (2)$$

Here, $\widehat{\omega_{jt}} = 1$ only if the measurement j is associated with clutter ($t = 0$) or target ($t \neq 0$). Generating all the hypothesis matrices leads to a combinatorial problem, where the number of data association hypothesis increases exponentially with the number of targets and measurements.

2. Constraining Target Trajectories

To reduce the search space further, one must take advantage of additional constraints. Let's consider a particular situation of radar scan like Fig. 1.

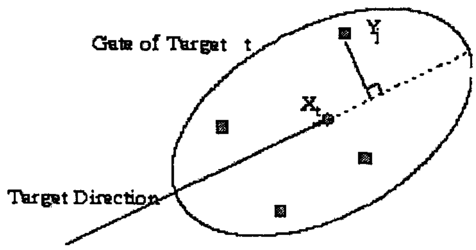

Fig1. Target trajectory and the measurements

In this figure, the position of the gate center of target t at time k is represented by $X_t(k)$. Also, $Y_j(k)$ means the coordinate of the measurement j at time k. Among the measurements included in this gate, at most one must be chosen as an actual target return. Note that the gate center is simply an estimate of this actual target position obtained by a Kalman filter [11].

Since the target must change its direction smoothly, a possible candidate must be positioned on the gate which is close to the trajectory as possible. As a measure of this distance, one can define the distance measure r_{jt} from the target trajectory line to the measurement point, $Y_j=(x_j, y_j)$ as

$$r^2_{jt}=\frac{(x_j d_{yt}-y_j d_{xt})^2}{(d^2_{xt}+d^2_{yt})} \quad (3)$$

here d_{x_t} and d_{y_t} are target's x,y axis directional distance between time k and $k+1$ which is estimated by prediction filter.

Notice that this is the normal from the the observation to the target trajectory.

Fig. 1 shows the distribution of the distance measure depending on the target direction within the target validation gate.

3. MAP Estimates for Data Association

The ultimate goal of this problem is to find the hypothesis matrix

$\hat{\omega}=\widehat{\omega}_{jt}|j\in[1,m], t\in[0,n]$, given the observation y and x, which must satisfy eq.2. From now on, let's associate the realizations-the gate center x, the measurement y, the validation matrix ω, and $\hat{\omega}$- to the random processes-X, Y, Ω, and $\hat{\Omega}$.

Next, consider that $\hat{\Omega}$ is a parameter space and ($\hat{\Omega}$,Y,X) is an observation space. Then, *a posteriori* can be derived by the Bayes rule:.

$$P(\hat{\omega}|\omega, y, x)=\frac{p(\omega|\hat{\omega})P(y, x|\hat{\omega})P(\hat{\omega})}{P(\omega, y, x)} \quad (5)$$

Here, we assumed that $P(\omega, y, x|\hat{\omega})=P(\omega|\hat{\omega})P(y, x|\hat{\omega})$, since the two variables ω and (x,y) are separately observed. This assumption makes the problem more tractable as we shall see later.

Given the parameter $\hat{\Omega}, \Omega$ and (X,Y) are observed. If the conditional probabilities describing the relationships between the parameter space and the observation spaces are available, one can obtain the MAP estimator:

$$\widehat{\omega}^*=\arg\max_{\hat{w}} P(\hat{\omega}|\omega, y, x) \quad (4)$$

4.R
epresenting Constraints by Energy Function

As a system model, we assume that the conditional probabilities are all Gibbs distributions:

$$\begin{cases} P(y, x|\hat{\omega})=\frac{1}{z_1}\exp-E(y, x|\hat{\omega}), \\ P(\omega|\hat{\omega})=\frac{1}{z_2}\exp-E(\omega|\hat{\omega}), \\ P(\hat{\omega})=\frac{1}{z_3}\exp-E(\hat{\omega}), \end{cases} \quad (5)$$

where $Z_s(s\in[1,2,3])$ is called partition function:

$$Z_s=\int_{\hat{w}\in\xi}\exp-E(\hat{\omega})d\hat{\omega}. \quad (6)$$

Here, E denotes the energy function. Substituting (6) into (4), (5) becomes

$$\hat{\omega}^*=\arg\min_{\hat{w}}[E(y, x|\hat{\omega})+E(\omega|\hat{\omega})+E(\hat{\omega})] \quad (7)$$

The energy functions are realizations of the constraints both for the target trajectories and the measurement-target relationships. For instance, the first term in (7) represents the distance between measurement and target and must be minimized using the constraints in (3). The second term intend to suppress the measurements which are uncorrelated with the valid measurements. The third term denotes constraints of the validation matrix and it can be designed to represent the two restrictions as shown in (2). The energy equations of each term are defined respectively:

$$\begin{cases} E(y,x|\hat{\omega}) = \sum_{t=1}^{n}\sum_{j=1}^{m}\frac{(x_j d_{yt}-y_j d_{xt})^2}{(d_{xt}^2+d_{yt}^2)}\,\widehat{\omega_{jt}} \\ E(\omega|\hat{\omega}) = \sum_{t=1}^{n}\sum_{j=1}^{m}(\hat{\omega}_{jt}-\omega_{jt})^2 \\ E(\hat{\omega}) = \sum_{t=1}^{n}(\sum_{j=1}^{m}\hat{\omega}_{jt}-1)+\sum_{j=1}^{m}(\sum_{t=0}^{n}\hat{\omega}_{jt}-1) \end{cases} \quad (8)$$

Putting (8) into (7), one gets

$$\hat{\omega}^* = \arg\min_{\hat{\omega}}[\alpha\sum_{t=1}^{n}\sum_{j=1}^{m}r_{jt}\hat{\omega}_{jt}+\frac{\beta}{2}\sum_{t=1}^{n}\sum_{j=1}^{m}(\hat{\omega}_{jt}-\omega_{jt})^2 + \sum_{t=1}^{n}(\sum_{j=1}^{m}\hat{\omega}_{jt}-1)+\sum_{j=1}^{m}(\sum_{t=0}^{n}\hat{\omega}_{jt}-1)] \quad (9)$$

where $r_{jt}=\frac{(x_j d_{yt}-y_j d_{xt})^2}{(d_{xt}^2+d_{yt}^2)}$, α and β are a coefficient of the weighted distance measure and the matching term respectively.

III. Finding Optimal Solution

1. Minimizing the Energy Function

The problem is to find $\widehat{\omega}^*$ such that $\widehat{\omega}^* = \arg\min_{\hat{\omega}\geq 0} L(\hat{\omega},\lambda,\varepsilon)$, where

$$L(\hat{\omega},\lambda,\varepsilon)=\alpha\sum_{t=1}^{n}\sum_{j=1}^{m}r_{jt}\widehat{\omega_{jt}}+\frac{\beta}{2}\sum_{t=1}^{n}\sum_{j=1}^{m}(\hat{\omega}_{jt}-\omega_{jt})^2 + \sum_{t=1}^{n}\lambda_t(\sum_{j=1}^{m}\hat{\omega}_{jt}-1)+\sum_{j=1}^{m}\varepsilon_j(\sum_{t=0}^{n}\hat{\omega}_{jt}-1) \quad (10)$$

Here, λ_t and ε_j are just Lagrange multipliers. Note that (10) includes the effect of the first column of the association matrix, which represents the clutter as well as newly appearing targets. In general setting, we assume $m > n$, since most of the multitarget problem is characterized by many confusing measurements that exceed far over the number of original targets.

Using the convex analysis for the local duality [8], the optimal solution can be obtained by

$$(\hat{w}^*,\lambda^*,\epsilon^*) = argmax_{\epsilon}max_{\lambda}max_{\hat{w}\geq 0}L(\hat{w},\lambda,\epsilon) \quad (11)$$

The necessary conditions[8] for achieving extreme in (10) are

$$\begin{cases} \nabla_{\widehat{\omega_{jt}}}L(\hat{\omega},\lambda,\varepsilon)=0 \\ \nabla_{\lambda_t}L(\hat{\omega},\lambda,\varepsilon)=0 \\ \nabla_{\varepsilon_j}L(\hat{\omega},\lambda,\varepsilon)=0 \end{cases} \quad (12)$$

and then, one obtain the solution:

$$\begin{aligned} \epsilon_j^* &= \frac{m}{(1+n)(m-1)}[\frac{n+1}{m}\sum_{l=1 l\neq j}^{m}\epsilon_l \\ &-\frac{1}{m}\sum_{t=0}^{n}\sum_{l=1}^{m}\{\beta w_{lt}-\alpha r_{lt}(1-\delta_t)\} \\ &+\sum_{t=0}^{n}\{\beta w_{jt}-\alpha r_{jt}(1-\delta_t)\}] \\ &= \frac{m}{(m-1)}\tilde{\epsilon}_j \\ &+\frac{m}{(n+1)(m-1)}\sum_{t=0}^{n}\{\beta(w_{jt}-\overline{w_t}) \\ &-\alpha(1-\delta_t)(r_{jt}-\overline{r_t})\} \end{aligned} \quad (13)$$

where $\overline{r_t}$, $\overline{\omega_t}$ and $\overline{\varepsilon}$ are average value at t column and $\tilde{\varepsilon}_j$ is average value except the element of ε_j. Therefore, for a given ω_{jt} and r_{jt}, ε^* can be sought successively by

$$\varepsilon_j^{\tau+1} = \frac{m}{m-1}\tilde{\varepsilon}_j^{\tau}+u\frac{m}{(n+1)(m-1)} \sum_{t=0}^{n}\beta(\omega_{jt}-\overline{\omega}_t)-\alpha(1-\delta_t)(r_{jt}-\overline{r}_t) \quad (14)$$

where u is an updating constant and τ an iteration index. In (14), if we calculate the average value of ε including ε_j, $\frac{m}{m-1}$ will be omitted. so in that case, we get the following :

$$\varepsilon_j^{r+1} = \overline{\varepsilon}_j^{\,r} + u\frac{1}{n+1}\sum_{i=0}^{n}\beta(\omega_{ji}-\overline{\omega}_i)-\alpha(1-\delta_i)(r_{ji}-\overline{r}_i) \qquad (15)$$

These equations need only $O(mn)$ multiplications for each frame of radar scan.

IV. Experimental Results

In this section, we present some results of the experiments comparing the performance of the proposed MAP estimate optimal data association(MAPODA) with the Hopfield Neural PDA(HNPDA) of Sengupta and Iltis [4]. Even though the MAPODA has a good structure for a parallel hardware, currently the algorithm is simulated by a serial computer. The performance of the MAPODA is tested in two separate cases in the simulation. In the first case, we consider two crossing and parallel targets for testing the track maintenance and accuracy in view of clutter density. In the second case, all the targets as listed in Table 1 are used for testing the multi-target tracking performance.

The dynamic models for the targets have been digitized using sampling period T normalized to 1 s and the state vectors have been represented in 2-dimensional Cartesian coordinates. Furthermore, only position measurements have been assumed to be available. The surveillance region used in the simulation is a 20 km by 20 km square and the initial positions and velocities of 10 targets in 2-dimensional plane are given in Table1. The crossing and parallel targets whose initial parameters are taken from target 1,2,3,and 4, respectively in Table 1 are tested. In Fig. 2, sample of track estimation errors between HNPDA and MAPODA are shown. and the RMS error in position and velocity in clutter density, $C=0.2$, are given in Fig. 3. The RMS estimation errors and track maintenance capability from the filtering based on the crossing and parallel targets are listed in Table 3. We note that an obvious trend in the results is making harder to maintain tracks by increasing the clutter density. We note also that, although we have simulated just two scenarios, the performance of the MAPODA is quite steady comparing with HNPDA in view of both tracking accuracy and maintenance.

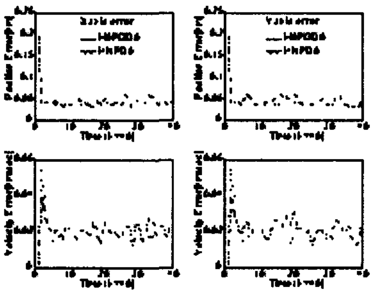

Fig.2 RMS Errors in position and velocity for crossing targets

Table 1: Initial Positions and Velocities of 10

Target i	Position(km)		Velocity(km/s)	
	x	y	$\dot{x}$	$\dot{y}$
1	-4.0	1.0	0.2	-0.05
2	-4.0	1.0	0.2	0.05
3	-6.0	-5.0	0.0	0.3
4	-5.5	-5.0	0.0	0.3
5	8.0	-7.0	-4.0	0.0
6	-8.0	-8.0	-0.4	0.0
7	-5.0	9.0	0.25	0.0
8	-0.5	8.9	0.25	0.0
9	0.5	-3.0	0.1	0.2
10	9.0	-9.0	0.01	0.2

In the second test, target's model which are maneuvering is simulated by the Singer model developed in [12]. Table 4 summarizes the rms position and velocity errors for each targets. The performance of the MAPODA is superior to that of HNPDA. The rms error of HNPDA for the target 8 has not been included since it loses track during the simulation.

Table 2: Maneuver Parameter of targert 8 and 9

Target i	Maneuvering type	Acceleration (m/s^2)	Turn period (sec)	Turn angle (deg)
8	Dog-leg	20	10	30
9	Constant Acceleration	10	15-35	0

Table 3: The track performance based on the crossing and parallel targets

Clutter density	Position error (km)		Velocity error (km/s)		Track maintenance(%)	
$C(/km^2)$	HNPDA	MAPODA	HNPDA	MAPODA	HNPDA	MAPODA
0.2	0.47	0.13	0.46	0.03	84	84
0.4	0.62	0.16	0.69	0.04	52	86
0.6	0.79	0.17	1.17	0.05	48	74
0.8	1.17	0.29	1.45	0.02	36	76
Average	0.76	0.19	0.94	0.04	55	80

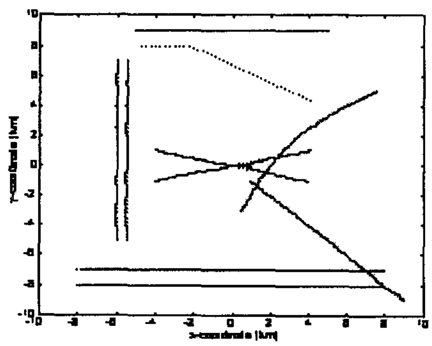

Fig. 3. Taracking ten targets

V. Conclusion

The purpose of this paper was to explore the MAP estimate optimal data association method as a tool for applying the multi-target tracking. It was shown that it always yields consistent data association, in contrast to the Hopfield Neural PDA, and that these associated data measurements are very effective for multi-target filter. Although the coefficients of the HNPDA were selected by the trial and error method, the MAPODA find the coefficients automatically. So, MAPODA is a general method about the solving the data association problems in multi-target tracking. A feature of our algorithm is that it requires only $O(m)$ storage, where m is the number of candidate measurement associations, compared to some branch and bound techniques, where the memory requirements grow exponentially with the number of targets. The experimental results show that the MAPODA is superior to the HNPDA in terms of error rate by 5.2%. This algorithm has several applications and can be effectively used in radar target tracking system.

Acknowledgements

This work was supported by a grant No. R05-2002-000-00500-0 from Korea Science & Engineering Foundation.

References

[1] Alspach D. L., ''A Gaussian sum approach to multi-target identification tracking problem,'' Automatica, Vol. 11, pp. 285-296, May 1975

[2] Reid D. B., ''An algorithm for tracking multiple targets,'' IEEE Trans. on Automat. Contr., Vol. 24, ,pp. 843-854,

Dec. 1979

[3] Bar-Shalom Y., ''Extension of probabilistic data associatiation filter in multitarget tracking,'' in Proc. 5th Symp. NonlinearEstimation Theory and its Application, pp. 16-21, Sept. 1974

[4] Sengupta, D., and Iltis, R. A.,''Neural solution to the multitarget tracking data association problem,'' IEEE Trans. on AES, AES-25, pp. 96-108, Jan. 1989

[5] Kuczewski R., ''Neural network approaches to multitarget tracking,'' In proceedings of the IEEE ICNN conference, 1987

[6] Fortmann T. E., Bar-Shalom Y., and Scheffe M.,"Sonar Tracking of Multiple Targets Using Joint Probabilistic Data Association," IEEE J. Oceanic Engineering, Vol. OE-8, pp.173-184, July 1983

[7] Fortmann T. E., Bar-Shalom Y., Tracking and Data Association. Orland Acdemic Press, Inc. p224, 1988

[8] Hiriart-Urruty J. B. and Lemarrecchal C. Convex Analysis and Minimization Algorithms I, Springer-Verlag, 1993.

[9] Luenberger D. G., Linear and Nonlinear Programming, Addition-wesley Publishing Co., 1984.

[10] Lee Y. W., and Jeong H. ,''A Neural Network Approach to the Optimal Data Association in Multi-Target Tracking,'' Proc. of WCNN'95, INNS Press.

[11] Kalman R.E.,''A new approach to linear filtering and prediction problems,'' Trans. ASME, (J. Basic Eng.,) Vol.82, pp34-45, Mar. 1960

[12] Singer, R.A., ''Estimating optimal tracking filter performance for manned maneuvering targets,'' IEEE Transactions on Aerospace and Electronic Systems, Vol. 6, , pp. 473-483, July 1970

[13] Won C.S. and Derin H.,''Unsupervised segmentation of noisy and textured images using Markov random fields,'' CVGIP: Graphical Models and Image Processing, Vol. 54, No. 4, pp. 308-328, July, 1992.

저자소개

이양원(Yang-Weon Lee)

1992년 서울대학교 제어계측공학과 졸업(공학석사)

1996년 포항공과대학교 전자전기공학과졸업(공학박사)

1996~현재 호남대학교 정보통신공학과 교수

1982~1996년 국방과학연구소 선임연구원

※관심분야 : 레이더신호처리,인터넷 응용기술