Useful Control Equations for Practitioners on Dynamic Process Control

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Abstract

System identification and controller formulation are essential in dynamic process control. In system identification, data for system identification are obtained, and then they are analyzed so that the system model of the process is built, identified, and diagnosed. In controller formulation, the control equation is derived based on the result of the system identification. There has been much theoretical research on system identification and controller formulation. These theories are very useful when they are appropriately applied.

To our regret, however, these theories are not always effectively applied in practice because the engineers and the operators who manage the process often do not have the necessary understanding of required time series analysis methods. On the other hand, because of widespread use of statistical packages, system identification such as estimating ARMA models can be done with little understanding of time series analysis methods. Therefore, it might be said that the most theoretically difficult part in practice is the controller formulation.

In this paper, lists of control equations are proposed as a useful tool for practitioners to use. The tool supports bridging the gap between theory and practice in dynamic process control. Also, for some models, the generalized control equations are obtained.

Key words: ARMA model, time series analysis, minimum mean square error controller, system identification

1. Introduction

Many industrial products are produced by continuous processes. Alumina baking process is one example. Raw materials flow in a kiln and continuously react and become products. The output of the process, the average particle radius, which is an important process characteristic, is controlled by adjusting input of the process, the baking temperature, to achieve the target value. This is difficult to control accurately for two reasons: the *output does not* change *immediately* after adjusting the input; the output changes slowly after beginning to change. In such dynamic processes, few are accurately controlled.

To control a dynamic process effectively, there are two stages that should have been done. The first stage is the identification stage. Here the system model of the process is built, identified, and diagnosed. This is best done by thorough examination of the process and conduct of a designed experiment for identification. The second stage is the controller formulation stage. Here the control equation is calculated based on the result of the first stage.

There has been much research on system identification, namely by Box et al. (1994), Sage and Melsa (1971), Eykhoff (1974), and Mehra and Lainiotis (1976). There has also been a great deal of research on controller formulation, namely by Box et al. (1994), Aoki (1967), and Astrom (1970).

However, these theories are not always effectively applied in practice because the engineers and the operators who manage the process often do not have the necessary understanding of required time series analysis methods. In this paper, lists of control equations are proposed as a tool to support enabling and improving dynamic process control

2. Problems in Actual Processes

As described in the previous chapter, system identification and controller formulation are necessary for effective dynamic process control. The truth about actual dynamic processes is that not many processes are controlled sufficiently. One reason is the existence of huge and highly complicated processes. In those processes, many factors affect each other, and advanced and sophisticated methods, such as given in Akaike and Kitagawa (1994), are necessary. Another reason is the lack of understanding of time series analysis methods on the side of engineers and operators. Even if the process is relatively simple, there are cases where the process is controlled inefficiently because the engineers and the operators do not have the necessary academic techniques. This paper is focused on the latter case. We propose a tool that bridges the gap between theory and practice.

Time series analysis methods and control theories are generally mathematically difficult. It is their degree of difficulty that lowers the chances of those theories to be applied in practice. Recently, statistical software packages, such as SAS, SPSS, S-plus, Statistica, etc., were widely used anywhere. These packages could perform most time series analysis methods such as estimating

ARMA models and spectral analysis. As for system identification, these packages allowed us to perform system identification easily. However, as for controller formulation, deriving control equations were not included in the packages, and utilization of time series analysis methods is necessary. Therefore, the need for a tool that can help derive control equations is increasing.

A very popularly used approach is the MMSE (minimum mean square error) controller discussed by Box et al. (1994). MMSE controllers are designed so that the sum of squares of the deviations from the target value is minimized. In this paper, we have calculated the control equations, which is the mathematical expression of the MMSE controller. We have calculated them for models that cover a significant part of the actual dynamic processes. The result is presented in completely expanded form in order to be readily used by practitioners. Utilizing these formulae, the control equation can be obtained by directly substituting the values of the result from system identification.

3. Model and Method

3.1 Model

The model considered in this paper is described as equations (1),(2).

$$Y_t = \frac{L_2(B)}{L_1(B)} B^{f+1} X_t + N_t \tag{1}$$

where the noise Nt is a linear stochastic process expressed as equation (2).

$$\begin{aligned} N_t &= a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \psi_3 a_{t-3} + \cdots \\ &= \psi(B) a_t \end{aligned} \tag{2}$$

Y_t: Process characteristic, Output

The variable to control. Control this variable as close as possible to the target value, which in this paper is set to zero without losing generality.

X_t: Manipulated Variable, Input

This variable is adjusted in order to control the process output.

N_t: Process Noise

ARMA process with order p for AR and q for MA, and its values considered as given.

a_t: White Noise

The variance of the white noise is considered as given.

f : Dead Time

The lag between process input and the process output, which is considered as given.

B : Backshift operator

$L_2(B)/L_1(B)$: Transfer Function

This function expresses relation between input and output.

3.2 Controller Formulation

When the process model is expressed in equations (1) and (2), then the MMSE controller is derived as shown in Box et al. (1994). It is given as equations (3) and (4).

$$X_t = -\frac{L_1(B)L_3(B)}{L_2(B)L_4(B)}Y_t \qquad (3)$$

where

$$L_3(B) = \psi_{f+1} + \psi_{f+2}B + \psi_{f+3}B^2 + \cdots$$

$$L_4(B) = 1 + \psi_1 B + \psi_2 B^2 + \cdots + \psi_f B^f \qquad (4)$$

Here, polynomials $L_3(B)$ and $L_4(B)$ have relation shown below.

$$\Psi(B) = L_4(B) + L_3(B)B^{f+1} \qquad (5)$$

So, the polynomials are part of the noise function $\Psi(B)$. How these two polynomials are decomposed depends on the dead time f. The polynomials cannot be expressed simply by using coefficients of the ARMA model, because the terms of the polynomials increase combinationally when order of AR, order of MA, and/or the dead time are/is large.

Therefore it is necessary to fix the degree of the model and the value of the dead time.

4. Deriving Equations

4.1 Procedure

When deriving a control equation, the following component of the model should be fixed in advance:

1) transfer function
2) dead time
3) noise model

In actual processes these components are to be known before or at the system identification stage. The procedure of deriving control equations is as follows.

Control Equation Deriving Procedure

1) Transfer function, dead time, noise model are determined.
2) $L_3(B)$ and $L_4(B)$ are calculated.
3) $L_1(B)$ to $L_4(B)$ are substituted for equation (3).
4) The result is arranged in a ready-to-use format.

4.2 Generalized Form

Generalized form of control equations for $\Psi(B)$=ARMA(p,q) and f=0 or 1, are derived as shown as follows.

4.2.1 Formulae for ARMA(p,q) and f=0

The noise is expressed as below.

$$\Psi(B) = \frac{1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3 + \cdots + \theta_q B^q}{1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - \cdots - \phi_p B^p} \qquad (6)$$

Let

$$M = \theta_1 + \theta_2 B + \theta_3 B^2 + \cdots + \theta_q B^{q-1} = \sum_{j=1}^{q} \theta_j B^{j-1},$$

and

$$R = \phi_1 + \phi_2 B + \phi_3 B^2 + \cdots + \phi_p B^{p-1} = \sum_{i=1}^{p} \phi_i B^{i-1} \qquad (7)$$

Then we have

$$\Psi(B)=\left\{1+\frac{R}{1-RB}B\right\}\{1+MB\} \tag{8}$$

which also can be expressed as

$$\Psi(B)=1+\frac{R+M}{1-RB}B \tag{9}$$

Hence we have

$$L_3(B)=\frac{R+M}{1-RB},\ L_4(B)=1 \tag{10}$$

For transfer function $L_2(B)/L_1(B)=\beta$, the control equation is given as follows,

$$X_t=-\frac{M+R}{\beta(1-RB)}Y_t \tag{11}$$

with its expanded form as shown below.

$$\beta X_t=\beta\sum_{i=1}^{p}\phi_i B^i X_t-\left(\sum_{j=1}^{q}\theta_j B^{j-1}+\sum_{i=1}^{p}\phi_i B^{i-1}\right)Y_t \tag{12}$$

For transfer function $L_2(B)/L_1(B)=\beta/(1-\alpha B)$, the control equation is

$$X_t=-\frac{(M+R)(1-\alpha B)}{\beta(1-RB)}Y_t \tag{13}$$

whose expanded form is shown as follows.

$$\beta X_t=\beta\sum_{i=1}^{p}\phi_i B^i X_t-\left(\sum_{j=1}^{q}\theta_j B^{j-1}-\alpha\sum_{j=1}^{q}\theta_j B^j\right)Y_t$$
$$-\left(\sum_{i=1}^{p}\phi_i B^{i-1}-\alpha\sum_{i=1}^{p}\phi_i B^i\right)Y_t \tag{14}$$

4.2.2 Formulae for ARMA(*p,q*) and *f*=1

The noise can be expressed as follows.

$$\Psi(B)=\frac{1+\theta_1 B+\theta_2 B^2+\theta_3 B^3+\cdots+\theta_q B^q}{1-\phi_1 B-\phi_2 B^2-\phi_3 B^3-\cdots-\phi_p B^p} \tag{15}$$

Let

$$m=\theta_2+\theta_3 B+\theta_4 B^2+\cdots+\theta_q B^{q-2}$$
$$=\sum_{j=2}^{q}\theta_j B^{j-2},$$

and

$$r=\phi_2+\phi_3 B+\phi_4 B^2+\cdots+\phi_p B^{p-2}$$
$$=\sum_{i=2}^{p}\phi_i B^{i-2} \tag{16}$$

Then we have

$$\Psi(B)=\frac{1+\theta_1 B+mB^2}{1-\phi_1 B-rB^2} \tag{17}$$

which also can be expressed as

$$\Psi(B)=\left\{1+\phi_1 B+\frac{\phi_1^2+r(1+\phi_1 B)}{1-\phi_1 B-rB^2}B^2\right\}$$
$$\times\{1+\theta_1 B+mB^2\}$$
$$\Psi(B)=1+(\phi_1+\theta_1)B$$
$$+\frac{\phi_1(\phi_1+\theta_1)+r\{1+(\phi_1+\theta_1)B\}+m}{1-\phi_1 B-rB^2}B^2 \tag{18}$$

Hence we have

$$L_3(B)=\frac{\phi_1(\phi_1+\theta_1)+r\{1+(\phi_1+\theta_1)B\}+m}{1-\phi_1 B-rB^2}$$
$$L_4(B)=1+(\phi_1+\theta_1)B \tag{19}$$

For transfer function $L_2(B)/L_1(B)=\beta$, the control equation is given as follows

$$X_t=-\frac{\phi_1(\phi_1+\theta_1)+r\{1+(\phi_1+\theta_1)B\}+m}{\beta(1-\phi_1 B-rB^2)\{1+(\phi_1+\theta_1)B\}}Y_t \tag{20}$$

which can be expanded to

$$\beta X_t = -\theta_1 B X_t + \left(\phi_1(\phi_1+\theta_1)B^2 + \sum_{i=2}^{p} \phi_i B^i \right) X_t - \left\{ \phi_1(\phi_1+\theta_1) + \sum_{i=2}^{p} \phi_i B^{i-2} + \sum_{j=2}^{q} \theta_j B^{j-2} \right\} Y_t + (\phi_1+\theta_1)\sum_{i=2}^{p} \phi_i B^{i-1} Y_t + (\phi_1+\theta_1)\sum_{i=2}^{p} \phi_i B^{i+1} X_t \quad (21)$$

For transfer function $L_2(B)/L_1(B) = \beta/(1-\alpha B)$, the control equation is given as follows.

$$X_t = -\frac{\{\phi_1(\phi_1+\theta_1) + r + m + r(\phi_1+\theta_1)B\}(1-\alpha B)}{\beta\{1+\theta_1 B - (r+\phi_1(\phi_1+\theta_1))B^2 - r(\phi_1+\theta_1)B^3\}} Y_t \quad (22)$$

which can be expanded into

$$\beta X_t = -\theta_1 B X_t + \left(\phi_1(\phi_1+\theta_1)B^2 + \sum_{i=2}^{p} \phi_i B^i \right) X_t - (1-\alpha B)\left\{ \phi_1(\phi_1+\theta_1) + \sum_{i=2}^{p} \phi_i B^{i-2} + \sum_{j=2}^{q} \theta_j B^{j-2} \right\} Y_t + (\phi_1+\theta_1)\sum_{i=2}^{p} \phi_i B^{i+1} X_t + (1-\alpha B)(\phi_1+\theta_1)\sum_{i=2}^{p} \phi_i B^{i-1} Y_t \quad (23)$$

4.3 Control Equation Tables

We have taken up the following conditions and derived control equations. These conditions are determined based on authors' experience on analyses of actual processes. These conditions would cover enough actual processes.

Conditions

a) Transfer function

a1) $L_2(B)/L_1(B) = \beta$

a2) $L_2(B)/L_1(B) = \beta / (1 - \alpha B)$

b) Dead time

b1) $f=0$

b2) $f=1$

c) Noise model

ARMA(p, q) process is considered.

c1) $p+q \leqq 4$ for $L_2/L_1=\beta$ and $f=0$

c2) $p+q \leqq 3$ for other conditions.

For the conditions given above, control equations are calculated. The result is summarized as Figures 1 to 4.

4.4 Practical Example

A practical example of obtaining control equation by applying the proposed method is shown. The process is the alumina baking process where the product is manufactured by a kiln which has typical dynamic features. Raw materials flow into the kiln and are baked by high temperature, and they are turned into the product gradually and continuously. The products are sampled every day, and control adjustments are made considering the current output and the process history. The adjustments were up to the operators but the control equation was necessary.

Through an experiment, the transfer function and dead time were identified as follows, where Y_t represent the process output (average particle radius) and X_t represent the process input (baking temperature).

AR\MA	0	1	2	3	4
0	$X_t=0$	$X_t=-\frac{\theta}{\beta}Y_t$	$X_t=-\frac{\theta_1}{\beta}Y_t-\frac{\theta_2}{\beta}Y_{t-1}$	$X_t=-\frac{\theta_1}{\beta}Y_t-\frac{\theta_2}{\beta}Y_{t-1}-\frac{\theta_3}{\beta}Y_{t-2}$	$X_t=-\frac{\theta_1}{\beta}Y_t-\frac{\theta_2}{\beta}Y_{t-1}-\frac{\theta_3}{\beta}Y_{t-2}-\frac{\theta_4}{\beta}Y_{t-3}$
1	$X_t=-\frac{\phi}{\beta}Y_t+\phi X_{t-1}$	$X_t=-\frac{\theta+\phi}{\beta}Y_t+\phi X_{t-1}$	$X_t=-\frac{\theta_1+\phi}{\beta}Y_t-\frac{\theta_2}{\beta}Y_{t-1}+\phi_1 X_{t-1}$	$X_t=-\frac{\theta_1+\phi}{\beta}Y_t-\frac{\theta_2}{\beta}Y_{t-1}-\frac{\theta_3}{\beta}Y_{t-2}+\phi_1 X_{t-1}$	
2	$X_t=-\frac{\phi_1}{\beta}Y_t-\frac{\phi_2}{\beta}Y_{t-1}+\phi_1 X_{t-1}+\phi_2 X_{t-2}$	$X_t=-\frac{\theta+\phi_1}{\beta}Y_t-\frac{\phi_2}{\beta}Y_{t-1}+\phi_1 X_{t-1}+\phi_2 X_{t-2}$	$X_t=-\frac{\theta_1+\phi_1}{\beta}Y_t-\frac{\theta_2+\phi_2}{\beta}Y_{t-1}+\phi_1 X_{t-1}+\phi_2 X_{t-2}$		
3	$X_t=-\frac{\phi_1}{\beta}Y_t-\frac{\phi_2}{\beta}Y_{t-1}-\frac{\phi_3}{\beta}Y_{t-2}+\phi_1 X_{t-1}+\phi_2 X_{t-2}+\phi_3 X_{t-3}$	$X_t=-\frac{\theta+\phi_1}{\beta}Y_t-\frac{\phi_2}{\beta}Y_{t-1}-\frac{\phi_3}{\beta}Y_{t-2}+\phi_1 X_{t-1}+\phi_2 X_{t-2}+\phi_3 X_{t-3}$			
4	$X_t=-\frac{\phi_1}{\beta}Y_t-\frac{\phi_2}{\beta}Y_{t-1}-\frac{\phi_3}{\beta}Y_{t-2}-\frac{\phi_4}{\beta}Y_{t-3}+\phi_1 X_{t-1}+\phi_2 X_{t-2}+\phi_3 X_{t-3}+\phi_4 X_{t-4}$				

Figure 1. MMSE Controller for $L_2(B)/L_1(B)=\beta$, $f=0$

AR\MA	0	1	2	3
0	$X_t=0$	$X_t=0$	$X_t=-\frac{\theta_2}{\beta}Y_t-\theta_1 X_{t-1}$	$X_t=-\frac{\theta_2}{\beta}Y_t-\frac{\theta_3}{\beta}Y_{t-1}-\theta_1 X_{t-1}$
1	$X_t=-\frac{\phi^2}{\beta}Y_t+\phi^2 X_{t-2}$	$X_t=-\frac{(\theta+\phi)\phi}{\beta}Y_t-\theta X_{t-1}+(\theta+\phi)\phi X_{t-2}$	$X_t=-\frac{(\theta_1+\phi)\phi+\theta_2}{\beta}Y_t-\theta_1 X_{t-1}+(\theta_1+\phi)\phi X_{t-2}$	
2	$X_t=-\frac{\phi_1^2+\phi_2}{\beta}Y_t-\frac{\phi_1\phi_2}{\beta}Y_{t-1}+(\phi_1^2+\phi_2)X_{t-2}+\phi_1\phi_2 X_{t-3}$	$X_t=-\frac{(\theta+\phi_1)\phi_1+\phi_2}{\beta}Y_t-\frac{(\theta+\phi_1)\phi_2}{\beta}Y_{t-1}-\theta X_{t-1}+[(\theta+\phi_1)\phi_1+\phi_2]X_{t-2}+(\theta+\phi_1)\phi_2 X_{t-3}$		
3	$X_t=-\frac{\phi_1^2+\phi_2}{\beta}Y_t-\frac{\phi_1\phi_2}{\beta}Y_{t-1}-\frac{\phi_1\phi_3}{\beta}Y_{t-2}+(\phi_1^2+\phi_2)X_{t-2}+\phi_1\phi_2 X_{t-3}+\phi_1\phi_3 X_{t-4}$			

Figure 2. MMSE Controller for $L_2(B)/L_1(B)=\beta$, $f=1$

AR\MA	0	1	2	3
0	$X_t=0$	$X_t=-\frac{\theta}{\beta}Y_t+\frac{\alpha\theta}{\beta}Y_{t-1}$	$X_t=-\frac{\theta_1}{\beta}Y_t+\frac{(\alpha\theta_1-\theta_2)}{\beta}Y_{t-1}+\frac{\alpha\theta_2}{\beta}Y_{t-2}$	$X_t=-\frac{\theta_1}{\beta}Y_t+\frac{(\alpha\theta_1-\theta_2)}{\beta}Y_{t-1}+\frac{(\alpha\theta_2-\theta_3)}{\beta}Y_{t-2}+\frac{\alpha\theta_3}{\beta}Y_{t-3}$
1	$X_t=-\frac{\phi}{\beta}Y_t+\frac{\alpha\phi}{\beta}Y_{t-1}+\phi X_{t-1}$	$X_t=-\frac{\theta+\phi}{\beta}Y_t+\frac{\alpha(\theta+\phi)}{\beta}Y_{t-1}+\phi X_{t-1}$	$X_t=-\frac{\theta_1+\phi}{\beta}Y_t+\frac{(\alpha(\theta_1+\phi)-\theta_2)}{\beta}Y_{t-1}+\frac{\alpha\theta_2}{\beta}Y_{t-2}+\phi_1 X_{t-1}$	
2	$X_t=-\frac{\phi_1}{\beta}Y_t+\frac{(\alpha\phi_1-\phi_2)}{\beta}Y_{t-1}+\frac{\alpha\phi_2}{\beta}Y_{t-2}+\phi_1 X_{t-1}+\phi_2 X_{t-2}$	$X_t=-\frac{\theta+\phi_1}{\beta}Y_t+\frac{[\alpha(\theta+\phi_1)-\phi_2]}{\beta}Y_{t-1}+\frac{\alpha\phi_2}{\beta}Y_{t-1}+\phi_1 X_{t-1}+\phi_2 X_{t-2}$		
3	$X_t=-\frac{\phi_1}{\beta}Y_t+\frac{(\alpha\phi_1-\phi_2)}{\beta}Y_{t-1}+\frac{(\alpha\phi_2-\phi_3)}{\beta}Y_{t-2}+\frac{\alpha\phi_3}{\beta}Y_t+\phi_1 X_{t-1}+\phi_2 X_{t-2}+\phi_3 X_{t-3}$			

Figure 3. MMSE Controller for $L_2(B)/L_1(B)=\beta/(1-\alpha B)$, $f=0$

$$Y_t = 0.553X_{t-2} + N_t \tag{6}$$

From equation (6), we knew that transfer function $L_2(B)/L_1(B) = 0.553$, and dead time=1. By applying equation (6), values of N_t's were calculated by using daily operational data. The plot of the noise is given in Figure 5.

Analyzing this data by a statistical package gave the result given below.

AR＼MA	0	1	2	3
0	$X_t = 0$	$X_t = 0$	$X_t = -\frac{\theta_2}{\beta} Y_t + \frac{\alpha\theta_2}{\beta} Y_{t-1} - \theta_1 X_{t-1}$	$X_t = -\frac{\theta_2}{\beta} Y_t + \frac{(\alpha\theta_2 - \theta_3)}{\beta} Y_{t-1} + \frac{\alpha\theta_3}{\beta} Y_{t-2} + \theta_1 X_{t-1}$
1	$X_t = -\frac{\phi^2}{\beta} Y_t + \frac{\alpha\phi^2}{\beta} Y_{t-1} + \phi^2 X_{t-2}$	$X_t = -\frac{(\theta+\phi)\phi}{\beta} Y_t + \frac{\alpha(\theta+\phi)\phi}{\beta} Y_{t-1} - \theta X_{t-1} + (\theta+\phi)\phi X_{t-2}$	$X_t = -\frac{(\theta_1+\phi)\phi+\theta_2}{\beta} Y_t + \frac{\alpha(\theta_1+\phi)\phi + \alpha\theta_2}{\beta} Y_{t-1} - \theta_1 X_{t-1} + (\theta_1+\phi)\phi X_{t-2}$	
2	$X_t = -\frac{\phi_1^2+\phi_2}{\beta} Y_t + \frac{\alpha(\phi_1^2+\phi_2) - \phi_1\phi_2}{\beta} Y_{t-1} + \frac{\alpha\phi_1\phi_2}{\beta} Y_{t-2} + (\phi_1^2+\phi_2) X_{t-2} + \phi_1\phi_2 X_{t-3}$	$X_t = -\frac{(\theta+\phi_1)\phi_1+\phi_2}{\beta} Y_t - \frac{(\theta+\phi_1)\phi_2}{\beta} Y_{t-1} + \frac{\alpha(\theta+\phi_1)\phi_1 + \alpha\phi_2}{\beta} Y_{t-1} + \frac{\alpha(\theta+\phi_1)\phi_2}{\beta} Y_{t-2} - \theta X_{t-1} + [(\theta+\phi_1)\phi_1 + \phi_2] X_{t-2} + (\theta+\phi_1)\phi_2 X_{t-3}$		
3	$X_t = -\frac{\phi_1^2+\phi_2}{\beta} Y_t + \frac{\alpha(\phi_1^2+\phi_2) - \phi_1\phi_2}{\beta} Y_{t-1} + \frac{\alpha\phi_1\phi_2 - \phi_1\phi_3}{\beta} Y_{t-2} + \frac{\alpha\phi_1\phi_3}{\beta} Y_{t-3} + (\phi_1^2+\phi_2) X_{t-2} + \phi_1\phi_2 X_{t-3} + \phi_1\phi_3 X_{t-4}$			

Figure 4. MMSE Controller for $L_2(B)/L_1(B) = \beta/(1-\alpha B)$, $f=1$

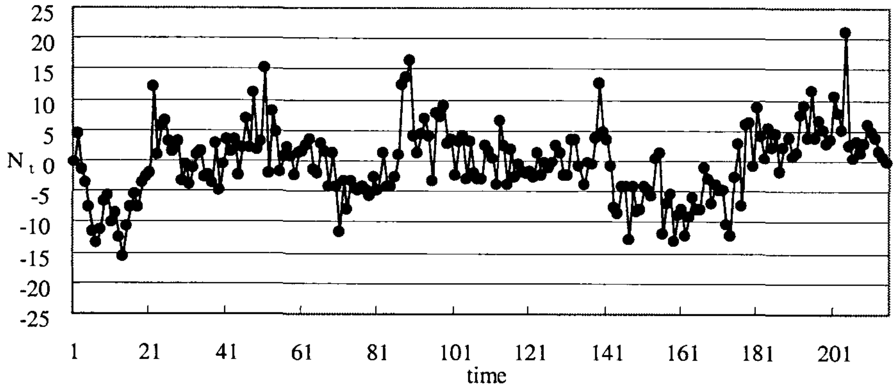

Figure 5. Plot of Noise Series

type of process : AR(3)
values of coefficient ϕ_1 =0.382
values of coefficient ϕ_2 =0.165
values of coefficient ϕ_3 =0.248

This meant that the noise series were identified as a third order autoregressive process shown as

$$N_t = \frac{1}{1 - 0.382B - 0.165B^2 - 0.248B^3} a_t \qquad (7)$$

For $L_2(B)/L_1(B) = 0.553$, $f=1$, and AR(3), we could find the following control equation from the tables, that is in the lower left cell (AR=3,MA=0) of Figure 2.

$$X_t = -\frac{\phi_1^2 + \phi_2}{\beta} Y_t - \frac{\phi_1\phi_2}{\beta} Y_{t-1} - \frac{\phi_1\phi_3}{\beta} Y_{t-2} + (\phi_1^2 + \phi_2) X_{t-2} + \phi_1\phi_2 X_{t-3} + \phi_1\phi_3 X_{t-4} \qquad (8)$$

By substituting the values of β and ϕ, the following control equation were obtained.

$$X_t = -0.562Y_t - 0.114Y_{t-1} - 0.171Y_{t-2} + 0.311X_{t-2} + 0.063X_{t-3} + 0.095X_{t-4} \quad (9)$$

So we used the control equation tables and obtained the control equation without derivation.

5. Concluding Remarks

We have proposed a list of control equations as a tool for improving dynamic process control. Its application along with statistical software packages will enable practitioners to easily utilize the MMSE control equations.

We have presented the obtained result to several engineers and operators. Their comments were "With this table, I can get the control equation by myself." Being able to apply time series analysis methods without understanding the detailed theories has large merit, especially in practical situations. We have to remember that we should not just apply it automatically and that we should comprehend the meanings and the usages of those statistical tools.

Acknowledgements

The authors wish to express appreciation to the editor and the referees for their comments and suggestions that improved the exposition of this paper.

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