Robust Fuzzy Feedback Linearization Control Based on Takagi-Sugeno Fuzzy Models

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Abstract: In this paper, well-known Takagi-Sugeno fuzzy model is used as the nonlinear plant model and uncertainty is assumed to be included in the model structure with known bounds. Based on the fuzzy models, a numerical robust stability analysis for the fuzzy feedback linearization regulator is presented using Linear Matrix Inequalities (LMI) Theory. For these structured uncertainty, the closed system can be cast into Lur'e system by simple transformation. From the LMI stability condition for Lur'e system, we can derive the robust stability condition for the fuzzy feedback linearization regulator based on Takagi-Sugeno fuzzy model. The effectiveness of the proposed analysis is illustrated by a simple example.

Keywords: fuzzy control, feedback linearization, linear matrix inequalities, robust stability, takagi-sugeno model

I. Introductions

A feedback linearization control has been widely used in nonlinear control theory [1]-[2]. On the other hand, since nonlinearity can be efficiently modeled and canceled by fuzzy logic system, fuzzy feedback linearization has attracted the attention of many control researchers [3]-[7]. Fuzzy feedback linearization is a feedback linearization method which uses a fuzzy model as a nonlinear system model. A fuzzy model has excellent capability in nonlinear system description and is particularly suitable for the complex and uncertain system[9]. In [3], the fuzzy feedback linearization concept was introduced using Takagi-Sugeno fuzzy model. However, robustness issue which is significant in practical applications was not considered in this work. In some previous researches, adaptive techniques were applied and adaptive fuzzy feedback linearization methods were suggested to guarantee robustness [4]-[7].

While adaptive fuzzy feedback linearization guarantees Lyapunov stability in the presence of uncertainty, it has some practical limitations because of its complex structures. From a practical point of view, robust approach is more suitable for fuzzy feedback linearization to overcome uncertainty. In [8], the L_2 robust stability analysis technique of the fuzzy feedback linearization regulator via multivariable circle criterion has been proposed.

However, it based on graphical stability analysis method there exist some difficulties to apply it to control problems directly.

In this paper, inspired by the work in [8], we have studied a numerical stability analysis method for the robust fuzzy feedback linearization regulator using Takagi-Sugeno fuzzy model. To analyze the robust stability, we assume that uncertainty is included in the model structure with known bounds. For these structured uncertainty, the robust stability of the closed system is analyzed by applying Linear Matrix Inequalities (LMI) theory.

LMI theory is the new and fast growing field and an valuable alternative to the analytical method [10], [11]. A variety of problems arising in system and control theory can be reduced to a few standard convex or quasiconvex optimization problems involving LMI. Since these resulting optimization problems can be easily solved by numerical computation, LMI techniques are very efficient and practical tools for the complex control problems. Specifically, a class of fuzzy control problems which is difficult to solve analytically, LMI techniques can afford the practical solutions. In the recent papers [12]-[16], the applicability of LMI techniques were showed excellently to fuzzy control systems.

To apply LMI techniques to our stability analysis problems, the closed system should be transformed into the standard form which has available LMI solution. By the simple transformation, our closed system can be cast into the well-known Lur'e system [17]. With the derived sufficient stability condition of the Lur'e system via LMI theory, we can extract the sufficient condition for the robust stable fuzzy feedback linearization regulator.

II. The feedback linearization based on T-S fuzzy models

Consider the regulation problem of the following n-th order nonlinear SISO system

$$x^{(n)} = f(\boldsymbol{x}) + g(\boldsymbol{x})u \tag{1}$$

where f and g are unknown or uncertain, but bounded continuous nonlinear functions. Let $\boldsymbol{x}=[x, \dot{x}, \cdots, x^{(n-1)}]^T \in R^n$ be the state vector of the system which is assumed to be available.

In this paper, well-known Takagi-Sugeno fuzzy model is used to identify the unknown nonlinear system (1). Takagi-Sugeno fuzzy model is available in IF-THEN form (2) or Input-Output form (3).

plant rule i:

$$\begin{aligned}&IF\ x\ is\ M_{i1}\ \text{and}\ \dot{x}\ is\ M_{i2}\ \text{and}\ \cdots\ \text{and}\ x^{(n-1)}\ is\ M_{in}\\ &\quad THEN\ x^{(n)}=(\boldsymbol{a}_i+\Delta\boldsymbol{a}_i(t))^T\cdot\boldsymbol{x}+(b_i+\Delta b_i(t))u,\\ &\quad i=1,2,\cdots,r\end{aligned} \tag{2}$$

where $\boldsymbol{x}=[x, \dot{x}, \cdots, x^{(n-1)}]^T$,

$$\boldsymbol{a}_i,\ \Delta\boldsymbol{a}_i(t)\in R^{1\times n},\ b_i,\ \Delta b_i(t)\in R$$

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In (2), M_{ij} is the fuzzy set and r is the number of rules. Also, $\Delta a_i(t)$ and $\Delta b_i(t)$ denotes the norm-bounded time-varying modeling uncertainty.

$$x^{(n)} = \frac{\sum_{i=1}^{r} w_i(x)\{(a_i + \Delta a_i(t))^T \cdot x + (b_i + \Delta b_i(t))u\}}{\sum_{i=1}^{r} w_i(x)}$$

$$= \sum_{i=1}^{r} h_i(x)\{(a_i + \Delta a_i(t))^T \cdot x + (b_i + \Delta b_i(t))u\} \quad (3)$$

where

$$w_i(x) = \prod_{j=1}^{n} M_{ij}(x^{(j-1)}), \quad h_i(x) = \frac{w_i(x)}{\sum_{i=1}^{r} w_i(x)}$$

$M_{ij}(x^{(j-1)})$ is the grade of membership of $x^{(j-1)}$ in M_{ij}. It is assumed in this paper that $w_i(x) \geq 0$, $i = 1,2,\cdots,r$, $\sum_{i=1}^{r} w_i(x) > 0$

Therefore,

$$h_i(x) \geq 0, \quad i=1,2,\cdots,r, \quad \sum_{i=1}^{r} h_i(x) = 1$$

For (3) to be controllable, $\sum_{i=1}^{r} h_i(x)\, b_i \neq 0$ for x in certain controllability region $U_c \subset R^n$ is required. If this controllability requirement is satisfied, the following fuzzy feedback linearization regulator (4) can cancel the nonlinearity of (3) and achieve perfect linearization (5).

$$u = \frac{\hat{a}^T \cdot x - \sum_{i=1}^{r} h_i(x)\; a_i^T \cdot x}{\sum_{i=1}^{r} h_i(x) b_i} = \frac{\sum_{i=1}^{r} h_i(x)(\hat{a}^T - a_i^T) \cdot x}{\sum_{i=1}^{r} h_i(x) b_i} \quad (4)$$

where we use the same a_i, b_i and $h_i(x)$ with the fuzzy model (3) for all i and $\hat{a} \in R^{1 \times n}$ is the linear state feedback gain vector. The perfectly linearized system can be written as (5).

$$x^{(n)} = \hat{a}^T \cdot x \quad (5)$$

However, due to the inevitable uncertainty, perfect linearization can not be achieved in practical application. By substituting (4) into (3), the imperfectly linearized system can be written as (6). From the bounds of $\Delta a_i(t)$ and $\Delta b_i(t)$, the bound of $a_N(t)$ can be derived as in Appendix A. Thus, the closed system (6) can be treated as the linear system with the sector bounded nonlinearities. In the next section, the numerical robust stability analysis via LMI for the closed system (6) will be presented.

$$x^{(n)} = \hat{a}^T \cdot x + \sum_{i=1}^{r} h_i(x)\; \Delta a_i(t)^T \cdot x + \frac{\sum_{i=1}^{r} h_i(x)\Delta b_i(t)}{\sum_{i=1}^{r} h_i(x)\, b_i}\{\sum_{i=1}^{r} h_i(x)(\hat{a} - a_i) \cdot x\}$$

$$= \hat{a}^T \cdot x + a_N(t)^T \cdot x \quad (6)$$

where

$$a_N(t) = \sum_{i=1}^{r} h_i(x)\; \Delta a_i(t)^T \cdot x + \frac{\sum_{i=1}^{r} h_i(x)\Delta b_i(t)}{\sum_{i=1}^{r} h_i(x)\, b_i}\{\sum_{i=1}^{r} h_i(x)\,(\hat{a} - a_i)^T \cdot x\} \quad (7)$$

III. LMI-based robust stability analysis

Consider the following Lur'e system (8)

$$\dot{x} = Ax + Bp, \qquad p_i(t) = \phi_i(x_i(t)), \quad i = 1, \cdots, n_p \quad (8)$$

where $p(t) \in R^{n_p}$, and the functions ϕ_i satisfy the [0, 1] sector conditions

$$0 \leq \sigma\phi_i(\sigma) \leq \sigma^2 \qquad \text{for } all \;\; \sigma \in R \quad (9)$$

or equivalently,

$$\phi_i(\sigma)(\phi_i(\sigma) - \sigma) \leq 0 \qquad \text{for } all \;\; \sigma \in R$$

The linear system with the sector bounded nonlinearities can be cast into Lur'e system. Therefore, the closed system (6) can be cast into Lur'e system. In Theorem 1, Lyapunov stability condition for Lur'e system is derived using LMI Theory. In the proof of Theorem 1, S-procedure in LMI techniques [10] is used.

Theorem 1: Lur'e system (8) is stable in the sense of Lyapunov if there exist $P > 0$, $\Lambda = \mathbf{diag}(\lambda_1, \cdots, \lambda_{n_p}) \geq 0$ and $T = \mathbf{diag}(\tau_1, \cdots, \tau_{n_p}) \geq 0$ which satisfy LMI (10).

$$\begin{bmatrix} A^TP + PA & PB + A^T\Lambda + T \\ B^TP + \Lambda A + T & \Lambda B + B^T\Lambda - 2T \end{bmatrix} < 0 \quad (10)$$

Proof: Let us choose a Lyapunov function

$$V(x) = x^TPx + 2\sum_{i=1}^{n_p} \lambda_i \int_0^{x_i} \phi_i(\sigma)\, d\sigma \quad (11)$$

Thus the data describing the Lyapunov function are the matrix P and the scalars λ_i, $i = 1, \cdots, n_p$. For $V(x)$ to be positive for nonzero x, we require $P > 0$ and $\Lambda = \mathbf{diag}(\lambda_1, \cdots, \lambda_{n_p}) \geq 0$.

The time derivative of $V(x)$ is

$$\frac{dV(x)}{dt} = 2(x^TP + \sum_{i=1}^{n_p} \lambda_i p_i I_i)(Ax + Bp) \quad (12)$$

where I_i denotes the i-th row of $n\times n$ identity matrix. Lyapunov stability condition $\frac{dV(x)}{dt} < 0$ holds for all nonzero x if and only if

$$(\ x^TP+\sum_{i=1}^{n_p}\lambda_i p_i I_i\ )\ (\ Ax\ +\ Bp\ ) < 0 \tag{13}$$

for all nonzero x.

The S-procedure in LMI techniques then yields the following LMI condition

$$\begin{bmatrix} A^TP+PA & PB+A^T\Lambda+T \\ B^TP+\Lambda A+T & \Lambda B+B^T\Lambda-2T \end{bmatrix} < 0$$

where

$$T = diag\,(\tau_1,\ \cdots,\ \tau_{n_p}) \geq 0,$$
$$\Lambda = diag\,(\lambda_1,\ \cdots,\ \lambda_{n_p}) \geq 0.$$

Therefore, Lur'e system (8) is stable in the sense of Lyapunov if there exist $P>0, \Lambda\geq 0$ and $T\geq 0$ which satisfy LMI (10).

Remark: When we set $\Lambda = 0$, we obtain the LMI

$$\begin{bmatrix} A^TP+PA & PB+T \\ B^TP+T & -2T \end{bmatrix} < 0$$

which can be interpreted as a condition for the existence of a quadratic Lyapunov function $V(x) = x^TPx$ for Lur'e system.

To apply Theorem 1, the closed system (6) should be transformed into Lur'e system (8).

First, we divide (6) into the linear and the nonlinear part as (14)

$$x^{(n)} - \hat{a}^T\cdot x = a_N(t)^T\cdot x \tag{14}$$

Then, the differential equation (14) can be represented by the state-space equation of (15).

$$\dot{x} = \bar{A}x + \bar{B}\,\bar{p} \tag{15}$$
$$\bar{p}_i(t) = \bar{\phi}_i(\ x_i(t)\ ),\qquad i = 1,\ \cdots,\ n$$

where,

$$\bar{A}=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \\ & & \vdots & & \\ \hat{a}_1 & \hat{a}_2 & \hat{a}_3 & \cdots & \hat{a}_n \end{bmatrix},\qquad \bar{B}=\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ & & \vdots & & \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$$

and

$$\bar{\phi}_i(\ x_i(t)\ ) = a_{Ni}(t)x_i(t),\qquad i = 1,\ \cdots,\ n$$

The sector condition of $\bar{\phi}_i$ is

$$\alpha_i\sigma^2 \leq \sigma\bar{\phi}_i(\ \sigma\ ) \leq \beta_i\sigma^2,\qquad i = 1,\ \cdots,\ n \tag{16}$$

where $\alpha_i = \min_t\,(a_{Ni}(t))$ and

$$\beta_i = \max_t\,(a_{Ni}(t)).$$

The sector bounds α_i and β_i can be obtained by the method in Appendix.

By substituting the equations (17) into (15) and (16), the general $[\alpha_i,\ \beta_i]$ sector condition for $\bar{\phi}_i$ (16) can be cast into the [0, 1] sector condition (9) for ϕ_i. This substitution procedure is called 'loop transformation'.

$$\bar{p}_i = \bar{\phi}_i(x_i(t)) = (\beta_i-\alpha_i)\,p_i + \alpha_i x_i \tag{17}$$
$$p_i(t) = \phi_i(\ x_i(t)\ ),\qquad i = 1,\ \cdots,\ n$$

or in the matrix form.

$$\bar{p} = Mp + Nq$$

where $M = diag\,(\alpha_1,\ \cdots,\ \alpha_n)$ and $N = diag\,(\beta_1-\alpha_1,\ \cdots,\ \beta_n-\alpha_n)$

And the resulting Lur'e system of loop transformation can be expressed as (18)

$$\dot{x} = Ax + Bp \tag{18}$$
$$p_i(t) = \phi_i(\ x_i(t)\ ),\qquad i = 1,\ \cdots,\ n_p$$

where $A = \bar{A} + \bar{B}N$ and $B = \bar{B}M$

$$\phi_i(\ x_i(t)\ ) = \frac{a_{Ni}(t)-\alpha_i}{\beta_i-\alpha_i}\,x_i(t),\qquad i = 1,\ \cdots,\ n$$

Applying Theorem 1 to the transformed Lur'e system (18), we can derive the robust stability condition for the closed system (6), as in Theorem 2.

Theorem 2: The closed system (6) is robust stable in the sense of Lyapunov if the corresponding Lur'e system (18) of the closed system (6) satisfies Theorem 1.

Proof: The closed system (6) can be transformed into the corresponding Lur'e system (18) by the above-mentioned loop transformation. Therefore, the stability of Lur'e system (18) implies the robust stability of the closed system (6). Thus, if the corresponding Lur'e system (18) of the closed system (6) satisfies Theorem 1, then we can conclude that the closed system (6) is robust stable in the sense of Lyapunov.

IV. Examples

Consider the problem of balancing and swing-up of an inverted pendulum on a cart shown in Fig. 1. The equations of motion for the pendulum are

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x) + g(x)\,u + d(t)$$
$$= \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} + d(t)$$

where $x = [\ x_1\ \ x_2\ ]^T$ and x_1 denotes the angle (in radians) of the pendulum from the vertical, and x_2 is the angular velocity. $g = 9.8\,m/s^2$ is the gravity constant,

m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, u is the control force applied to the cart (in Newtons). $d(t)$ is the external disturbance and $a = \frac{1}{m+M}$. We choose $m = 2.0\,kg$, $M = 8.0\,kg$ and $2l = 1.0\,m$ in the simulation.

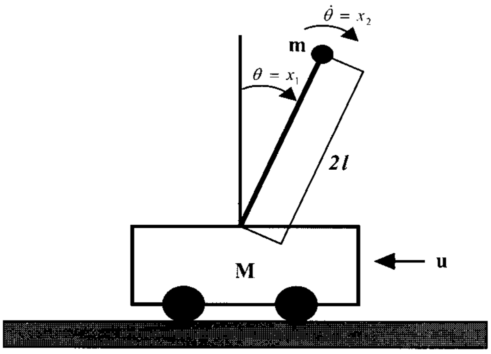

Fig. 1. The inverted pendulum system.

The dynamic equations can be approximated by the following two fuzzy rules and the membership functions used in this fuzzy model are shown in Fig. 2.

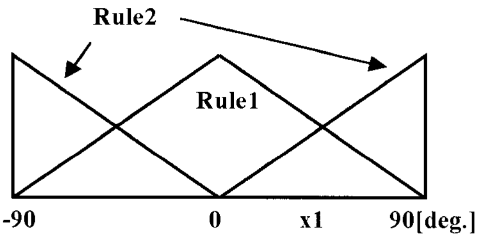

Fig. 2. Membership functions.

Rule 1 : *IF* x *is about* 0

THEN $\dot{x} = (\boldsymbol{a}_1 + \boldsymbol{\Delta a}_1(t))^T \cdot \boldsymbol{x} + (b_1 + \Delta b_1(t))u + d$

Rule 2 : *IF* x *is about* $\pm\frac{\pi}{2}$ $(|x| < \frac{\pi}{2})$

THEN $\dot{x} = (\boldsymbol{a}_2 + \boldsymbol{\Delta a}_2(t))^T \cdot \boldsymbol{x} + (b_2 + \Delta b_2(t))u + d$

(19)

(19) can be inferred as

$$\dot{x} = \frac{\sum_{i=1}^{2} w_i(\boldsymbol{x})\{(\boldsymbol{a}_i + \boldsymbol{\Delta a}_i(t))^T \cdot \boldsymbol{x} + (b_i + \Delta b_i(t))u\}}{\sum_{i=1}^{2} w_i(\boldsymbol{x})} + d$$

$$= \sum_{i=1}^{2} h_i(\boldsymbol{x})\{(\boldsymbol{a}_i + \boldsymbol{\Delta a}_i(t))^T \cdot \boldsymbol{x} + (b_i + \Delta b_i(t))u\} + d \tag{20}$$

where, $w_i(\boldsymbol{x}) = \prod_{j=1}^{2} M_{ij}(x^{(j-1)})$, $h_i(\boldsymbol{x}) = \frac{w_i(\boldsymbol{x})}{\sum_{i=1}^{2} w_i(\boldsymbol{x})}$ and,

$$\boldsymbol{a}_1 = \left[\frac{g}{4l/3 - aml} \quad 0\right] = [17.29 \quad 0],$$

$$\boldsymbol{a}_2 = \left[\frac{2g}{\pi(4l/3 - aml\beta^2)} \quad 0\right] = [9.35 \quad 0],$$

$$b_1 = -\frac{a}{4l/3 - aml} = -0.1765,$$

$$b_2 = -\frac{a\beta}{4l/3 - aml\beta^2} = -0.0052$$

we assume that $\boldsymbol{\Delta a}_1(t)$, $\boldsymbol{\Delta a}_2(t)$, $\Delta b_1(t)$, $\Delta b_2(t)$ are unknown but bounded as follows.

$$-1 \le \Delta a_{11}(t) \le 1,\ -0.5 \le \Delta a_{12}(t) \le 0.5,$$
$$-1 \le \Delta a_{21}(t) \le 1,\ -0.5 \le \Delta a_{22}(t) \le 0.5,$$
$$-0.001 \le \Delta b_1(t) \le 0.001,\ -0.001 \le \Delta b_2(t) \le 0.001$$

In this example, the following fuzzy feedback linearization regulator (21) is used to stabilize the system (19) or (20)

$$u = \frac{\hat{\boldsymbol{a}}^T \cdot \boldsymbol{x} - \sum_{i=1}^{2} h_i(\boldsymbol{x})\ \boldsymbol{a}_i^T \cdot \boldsymbol{x}}{\sum_{i=1}^{2} h_i(\boldsymbol{x}) b_i} = \frac{\sum_{i=1}^{2} h_i(\boldsymbol{x})(\hat{\boldsymbol{a}}^T - \boldsymbol{a}_i^T) \cdot \boldsymbol{x}}{\sum_{i=1}^{2} h_i(\boldsymbol{x}) b_i} \tag{21}$$

where $\hat{\boldsymbol{a}}$ = [-50 ,-30]

The resulting closed system is

$$\dot{x} = \hat{\boldsymbol{a}}^T \cdot \boldsymbol{x} + \sum_{i=1}^{2} h_i(\boldsymbol{x})\ \boldsymbol{\Delta a}_i(t)^T \cdot \boldsymbol{x} + \frac{\sum_{i=1}^{2} h_i(\boldsymbol{x})\Delta b_i(t)}{\sum_{i=1}^{2} h_i(\boldsymbol{x})\, b_i}\{\sum_{i=1}^{2} h_i(\boldsymbol{x})(\hat{\boldsymbol{a}} - \boldsymbol{a}_i) \cdot \boldsymbol{x}\}$$
$$= \hat{\boldsymbol{a}}^T \cdot \boldsymbol{x} + \boldsymbol{a}_N(t)^T \cdot \boldsymbol{x} \tag{22}$$

where $\boldsymbol{a}_N(t) = \sum_{i=1}^{2} h_i(\boldsymbol{x})\ \boldsymbol{\Delta a}_i(t)^T \cdot \boldsymbol{x} + \frac{\sum_{i=1}^{2} h_i(\boldsymbol{x})\Delta b_i(t)}{\sum_{i=1}^{2} h_i(\boldsymbol{x})\, b_i}\{\sum_{i=1}^{2} h_i(\boldsymbol{x})\ (\hat{\boldsymbol{a}} - \boldsymbol{a}_i)^T \cdot \boldsymbol{x}\}$

The maximum and minimum sector bounds of $a_{Nj}(t)$ for j=1, 2 can be found from (25) and (26) in Appendix A.

$$\max_t a_{N1}(t) = 4.12,\ \min_t a_{N1}(t) = -4.12$$
$$\max_t a_{N2}(t) = 1.03,\ \min_t a_{N2}(t) = -1.03$$

Then, the closed system (22) can be cast into the following Lur'e system (23) by the loop transformation.

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{p} \tag{23}$$
$$p_i(t) = \phi_i(x_i(t)),\qquad i = 1, \cdots, n_p$$

where

$$\boldsymbol{A} = \overline{\boldsymbol{A}} + \overline{\boldsymbol{B}}\boldsymbol{N}$$
$$= \begin{bmatrix} 0 & 1 \\ -50 & -30 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 8.24 & 0 \\ 0 & 2.06 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -41.76 & -27.94 \end{bmatrix}$$

$$B = \overline{B}\,M = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -4.12 & 0 \\ 0 & -1.03 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ -4.12 & -1.03 \end{bmatrix}$$

$$\phi_i = \frac{1}{\max_t a_{Ni}(t) - \min_t a_{Ni}(t)} \cdot \left(a_{Ni}(t)\, x_i - \min_t a_{Ni}(t)\, x_i \right)$$

For this transformed Lur'e system (23), we check if the sufficient condition of Theorem 1 is satisfied. To simplify our analysis, we consider the quadratic stability condition, i.e. $\Lambda = 0$. Then, we use the interior-point method [10] in LMI techniques to obtain P and T which satisfy LMI (10). It can be easily verified that the following P and T in (24) satisfy LMI (10), which can be obtained by some computer-aided optimization tools [18].

$$P = \begin{bmatrix} 37.8 & 1.1 \\ 1.1 & 0.9 \end{bmatrix} \text{ and } \quad T = \begin{bmatrix} 11.2 & 0 \\ 0 & 8.5 \end{bmatrix} \tag{24}$$

Therefore, we can conclude that the sufficient condition of Theorem 1 is satisfied and the closed system (22) is robust stable. To verify the Lyapunov stability, computer simulation is performed with the initial condition $x_0 = [1 \;\; 0]$. Fig. 3 and Fig. 4 illustrate the simulation results of the state variables.

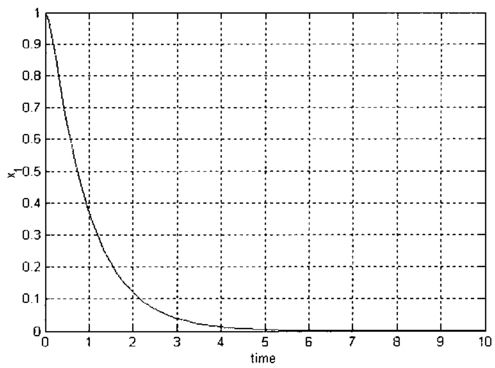

Fig. 3. Simulation result of state x_1.

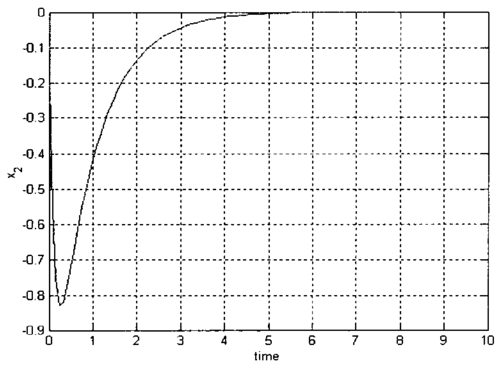

Fig. 4. Simulation result of state x_2.

V. Conclusion

In this paper, we have presented the LMI-based robust stability condition which can be solved numerically for the fuzzy feedback linearization regulator via Takagi-Sugeno fuzzy model. Feedback linearization is a very useful control scheme in nonlinear control theory. But, various analytic constraints and uncertainty make it difficult to design and implement the robust stable feedback linearization controller. To overcome these difficulties, we implement the fuzzy feedback linearization regulator based on Takagi-Sugeno fuzzy model and propose the LMI-based robust stability condition. Through a simple example, we illustrate the effectiveness of the proposed method.

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Appendix A

In the followings, we need the basic assumption that

$$\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) = 1 \quad \text{and} \quad \max_{\boldsymbol{x}} \; h_i(\boldsymbol{x}(t)) = 1.$$

The maximum and minimum sector bounds of $a_{Nj}(t)$ can be computed from (25) and (26)

$$= \max_t \left\{ \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta a_{ij}(t) \right\} + \max_t \left\{ \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) b_i} \cdot \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) e_{ij} \right\} \tag{25}$$

$$\min_t \,(a_{Nj}(t)) = \min_t \left\{ \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta a_{ij}(t) \right\} + \min_t \left\{ \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) b_i} \cdot \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) e_{ij} \right\} \tag{26}$$

where $e_{ij} = \widehat{a}_j - a_{ij}$

The second terms of (25) and (26) can be computed using the following property.

$$\min_i \,(\Delta a_{ij}(t)) \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta a_{ij}(t) \le \max_i \,(\Delta a_{ij}(t)) \tag{27}$$

The third terms of (25) and (26) can be computed from (28) and (29)

$$\max_t \left\{ \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) b_i} \cdot \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) e_{ij} \right\}$$
$$= \max\left\{ \frac{\overline{\Delta b^{nn}}}{\underline{b^p}} \cdot \overline{e_j^{nn}},\ \frac{\underline{\Delta b^n}}{\underline{b^p}} \cdot \underline{e_j^n},\ \frac{\underline{\Delta b^n}}{\overline{b^n}} \cdot \overline{e_j^{nn}},\ \frac{\overline{\Delta b^{nn}}}{\overline{b^n}} \cdot \underline{e_j^n},\ \frac{\underline{\Delta b^{nn}}}{\overline{b^p}} \cdot \overline{e_j^n},\ \frac{\overline{\Delta b^n}}{\overline{b^p}} \cdot \underline{e_j^{nn}},\ \frac{\overline{\Delta b^n}}{\underline{b^n}} \cdot \overline{e_j^n},\ \frac{\underline{\Delta b^{nn}}}{\underline{b^n}} \cdot \underline{e_j^{nn}} \right\} \tag{28}$$

$$\min_t \left\{ \frac{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) \Delta b_i(t)}{\sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) b_i} \cdot \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) e_{ij} \right\}$$
$$= \min\left\{ \frac{\underline{\Delta b^{nn}}}{\overline{b^p}} \cdot \underline{e_j^{nn}},\ \frac{\overline{\Delta b^n}}{\overline{b^p}} \cdot \overline{e_j^n},\ \frac{\overline{\Delta b^n}}{\underline{b^n}} \cdot \underline{e_j^{nn}},\ \frac{\underline{\Delta b^{nn}}}{\underline{b^n}} \cdot \overline{e_j^n},\ \frac{\overline{\Delta b^{nn}}}{\underline{b^p}} \cdot \underline{e_j^n},\ \frac{\underline{\Delta b^n}}{\underline{b^p}} \cdot \overline{e_j^{nn}},\ \frac{\underline{\Delta b^n}}{\overline{b^n}} \cdot \underline{e_j^n},\ \frac{\overline{\Delta b^{nn}}}{\overline{b^n}} \cdot \overline{e_j^{nn}} \right\} \tag{29}$$

where, $b_i^p = \{\ b_i \mid b_i > 0\ \}$, $b_i^n = \{\ b_i \mid b_i < 0\ \}$

$e_{ij}^{nn} = \{\ e_{ij} \mid e_{ij} \ge 0\ \}$, $e_{ij}^{n} = \{\ e_{ij} \mid e_{ij} < 0\ \}$

$$\underline{b^p} \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) b_i^p \le \overline{b^p}, \quad \underline{b^n} \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) b_i^n \le \overline{b^n}$$

$$\underline{e_j^{nn}} \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) e_{ij}^{nn} \le \overline{e_j^{nn}},$$

$$\underline{e_j^{n}} \le \sum_{i=1}^{r} h_i(\boldsymbol{x}(t)) e_{ij}^{n} \le \overline{e_j^{n}}$$

$$\overline{\Delta b^{nn}} = \max_{i,t} \{ (\Delta b_i(t)) \mid \Delta b_i(t) \ge 0 \},$$

$$\overline{\Delta b^{n}} = \max_{i,t} \{ (\Delta b_i(t)) \mid \Delta b_i(t) < 0 \}$$

$$\underline{\Delta b^{nn}} = \min_{i,t} \{ (\Delta b_i(t)) \mid \Delta b_i(t) \ge 0 \},$$

$$\underline{\Delta b^{nn}} = \min_{i,t} \{ (\Delta b_i(t)) \mid \Delta b_i(t) < 0 \}$$

Appendix B

· S-procedure of LMI theory [10]

Let $F_0, \cdots, F_p$ be quadratic functions of the variable $\xi \in R^n$ such that

$$F_i(\xi) \equiv \xi^T \boldsymbol{T}_i \xi + 2\boldsymbol{u}_i^T \xi + v_i, \quad i = 0, \cdots, p, \quad \boldsymbol{T}_i = \boldsymbol{T}_i^T. \tag{30}$$

We consider the following condition on $F_0, \cdots, F_p$: $F_0(\xi) \ge 0$ for all ξ such that

$$F_i(\xi) \ge 0, \qquad i = 0, \cdots, p$$

Obviously, if there exists $\tau_1 \ge 0, \cdots, \tau_p \ge 0$ such that

for all ξ, $F_0(\xi) - \sum_{i=1}^{p} \tau_i F_i(\xi) \ge 0$,

then (30) holds or equivalently (32) holds..

$$\begin{bmatrix} \boldsymbol{T}_0 & \boldsymbol{u}_0 \\ \boldsymbol{u}_0^T & v_0 \end{bmatrix} - \sum_{i=1}^{p} \tau_i \begin{bmatrix} \boldsymbol{T}_i & \boldsymbol{u}_i \\ \boldsymbol{u}_i^T & v_i \end{bmatrix} \ge 0 \tag{31}$$

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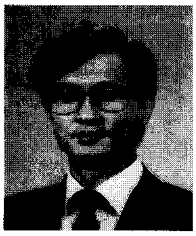

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