Discrete-Time Robust Guaranteed Cost Filtering for Convex Bounded Uncertain Systems With Time Delay

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Abstract: In this paper, the guaranteed cost filtering design method for linear time delay systems with convex bounded uncertainties in discrete-time case is presented. The uncertain parameters are assumed to be unknown but belonging to known convex compact set of polytotype less conservative than norm bounded parameter uncertainty. The main purpose is to design a stable filter which minimizes the guaranteed cost. The sufficient condition for the existence of filter, the guaranteed cost filter design method, and the upper bound of the guaranteed cost are proposed. Since the proposed sufficient conditions are LMI(linear matrix inequality) forms in terms of all finding variables, all solutions can be obtained simultaneously by means of powerful convex programming tools with global convergence assured. Finally, a numerical example is given to check the validity of the proposed method.

Keywords: Guaranteed cost filter, time delay, linear matrix inequality, convex bounded uncertainty

I. Introduction

Since the Kalman filtering theory has been introduced, much effort has been devoted to the development of filtering design algorithms. Also, the extensive use of optimization criteria like the H_2 and/or H_∞ norm has consolidated the importance of estimation and filtering in linear system theory during the last decades. In the guaranteed cost filtering approach[1], the design methods were developed for guaranteeing the upper bound of guaranteed cost function. However, there are a few results considering guaranteed cost filtering. Recently, the H_∞ filtering approach has been developed from the loose assumption of boundedness of the noise variance. In this case, the H_∞ performance index to be minimized being the worst case H_∞ norm from the process noise to the estimation error[2-4]. Petersen and McFarlane[1] presented the results on the design of robust state feedback controllers and steady-state robust state estimator for a class of uncertain linear systems with norm bounded uncertainty. Wang *et al.*[5] considered the robust H_2/H_∞ state estimation for discrete-time systems with error variance constraints. Xie and Soh[6] studied the problem of Kalman filter design for uncertain systems using Riccati equation approach. However, they just considered parameter uncertain systems without time delay using Riccati equation technique. Geromel *et al.*[2] dealt with H_2 and H_∞ robust filtering for discrete time systems and convex bounded uncertain systems by LMI techniques. Also, Palhares *et al.*[7] considered the problem of designing a full order stable linear filter that minimized the worst-case peak value of the filtering error output signal with respect to all bounded energy inputs, in such a way that the filtering error system remained quadratically stable.

On the other hand, the delayed state is very often causes for instability and poor performance of systems[8] and references therein. Recently, Souza *et al.*[9] considered the problem of robust H_∞ filtering for continuous-time uncertain linear systems with multiple time-varying delays in the state variables by LMI technique. Wang *et al.*[10] investigated the robust filter design problem for a class of nonlinear time delay stochastic systems by ARI(algebraic Riccati inequality) approach. This is somewhat conservative because some variables should be pre-determined to find a robust filter. Kim[11] proposed the continuous-time guaranteed cost filtering design method to guarantee the minimization of upper bound in guaranteed cost function for time-varying delay systems with parameter uncertainties by LMI approach. However, there are no results on the problem of discrete-time robust guaranteed cost filter design for discrete-time uncertain systems with time delay.

In this paper, we present the robust guaranteed cost filter design algorithm for time delay systems with convex bounded uncertainties in discrete-time case. The optimization problem to get the filter and upper bound of guaranteed cost function is given. Also, It is shown that the system without time delay can be solvable using the proposed design method. A numerical example is demonstrated to show the validity.

The notations are fairy standard. The notations are fairy standard. I and 0 stands for the identity and the zero matrices with proper dimensions, respectively. The symbol $*$ represents the submatrices that lie below the main diagonal and $tr(\cdot)$ denotes the trace of the matrix $(\cdot)$. $X > 0$ (or $X < 0$) means positive (or negative) definite symmetric matrix. And, $Diag$ means block diagonal matrix.

II. Problem statements

Consider a linear discrete-time system with time delay

$$\begin{aligned} x(k+1) &= Ax(k) + A_d x(k-d) \\ y(k) &= Cx(k) \\ x(k) &= \phi_1(k), \quad -d \le k \le 0 \end{aligned} \tag{1}$$

where $x(k) \in \mathbf{R}^n$ is the state vector, $y(k) \in \mathbf{R}^r$ is the measurement output vector, and $\phi_1(k)$ is an initial value function. Time delay d is a positive integer. For simplicity, we just consider linear time delay systems. In the rear part, we explain the filter design method for time delay systems with convex

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bounded uncertainties. And, we assume that the system (1) is asymptotically stable and detectable. This assumption guarantees that the boundedness of the filtering error holds, since the asymptotic stability of the filtering error dynamics depends on the states and error state vectors of the filtering error dynamics. The aim is to design a stable guaranteed cost filter described by

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + Ky(k) \tag{2}$$

where, $\hat{A}$ and K are design parameters. If we take the error state vector as follows:

$$e(k) = x(k) - \hat{x}(k), \tag{3}$$

then the error dynamics is obtained

$$\begin{aligned} e(k+1) &= \hat{A}e(k) + (A - KC - \hat{A})x(k) + A_d x(k-d) \\ z(k) &= Le(k) \end{aligned} \tag{4}$$

by defining the error state output as $z(k) = Le(k)$. Define the following augmented state vector

$$x_f(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \tag{5}$$

such that the filtering error dynamics is given by

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + A_{df} x_f(k-d) \\ z(k) &= C_f x_f(k) \\ x_f(k) &= \phi_f(k) = \begin{bmatrix} \phi_1(k) \\ \phi_2(k) \end{bmatrix}, \quad -d \le k \le 0 \end{aligned} \tag{6}$$

where some notations are denoted by

$$\begin{aligned} A_f &= \begin{bmatrix} A & 0 \\ A - KC - \hat{A} & \hat{A} \end{bmatrix}, \\ A_{df} &= \begin{bmatrix} A_d & 0 \\ A_d & 0 \end{bmatrix}, \\ C_f &= \begin{bmatrix} 0 & L \end{bmatrix}. \end{aligned} \tag{7}$$

Associated with guaranteed cost filter (2), we introduce the following filtering design objective:

Determine stabilizing filter parameters $\hat{A}$ and K that achieve minimization of guaranteed cost in filtering error dynamics. (8)

Also, we introduce guaranteed cost function as follows:

$$J = \sum_{k=0}^{\infty} z(k)^T z(k). \tag{9}$$

III. Robust guaranteed cost filtering

In this section, we present guaranteed cost filter design methods of discrete-time polytopic uncertain systems with time delay. The objective of guaranteed cost filtering is to determine filter variables $\hat{A}$ and K that achieve asymptotic stability and minimization of guaranteed cost function in filtering error dynamics. First, the sufficient condition for the existence of guaranteed cost filter and guaranteed cost filter design method for the system without uncertainty are established. And then, the result is extended to the discrete-time linear polytopic type convex bounded uncertain system with time delay.

Theorem 1: If there exist positive definite matrices(or scalar) P_1, P_2, S_1, S_3, α, Q, and matrices S_2, M_1, M_2 satisfying the following optimization problem:

$$minimize\ \{\alpha + tr(Q)\}\ subject\ to$$

$$i)\ \begin{bmatrix} -P_1 & 0 & P_1 A & 0 & P_1 A_d & 0 \\ * & -P_2 & P_2 A - M_2 C - M_1 & M_1 & P_2 A_d & 0 \\ * & * & -P_1 + S_1 & S_2 & 0 & 0 \\ * & * & * & -P_2 + S_3 + L^T L & 0 & 0 \\ * & * & * & * & -S_1 & -S_2 \\ * & * & * & * & * & -S_3 \end{bmatrix} < 0, \tag{10}$$

$$ii)\ -\alpha + \phi_1(0)^T P_1 \phi_1(0) + \phi_2(0)^T P_2 \phi_2(0) < 0,$$

$$iii)\ -Q + N_1^T S_1 N_1 + N_2^T S_2 N_1 + N_1^T S_2 N_2 + N_2^T S_3 N_2 < 0,$$

then (2) is an optimal discrete-time guaranteed cost filter and $J^* = \alpha + tr(Q)$ is an upper bound of discrete-time guaranteed cost. Here, some notations are defined as

$$\begin{aligned} M_1 &= P_2 \hat{A}, \\ M_2 &= P_2 K, \\ \textstyle\sum_{i=-d}^{-1} \phi_f(i)\phi_f(i)^T &= NN^T \\ &= \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \begin{bmatrix} N_1^T & N_2^T \end{bmatrix}. \end{aligned} \tag{11}$$

Proof: Define a Lyapunov functional as follows:

$$V(x_f(k)) = x_f(k)^T P x_f(k) + \sum_{i=k-d}^{k-1} x_f(i)^T S x_f(i). \tag{12}$$

The difference of the (12) is given

$$\begin{aligned} \Delta V &= V(x_f(k+1)) - V(x_f(k)) \\ &= x_f(k+1)^T P x_f(k+1) - x_f(k)^T P x_f(k) \\ &\quad + x_f(k)^T S x_f(k) - x_f(k-d)^T S x_f(k-d). \end{aligned} \tag{13}$$

From the difference (13) and the guaranteed cost function (9), the following to satisfy the asymptotic stability and minimization of the guaranteed cost in filtering error dynamics implies the linear matrix inequality (i) of (10).

$$\Delta V < -z(k)^T z(k) < 0. \tag{14}$$

Therefore, we have

$$\begin{bmatrix} x_f(k) \\ x_f(k-d) \end{bmatrix}^T \times \begin{bmatrix} A_f^T P A_f - P + S + C_f^T C_f & A_f^T P A_{df} \\ * & -S + A_{df}^T P A_{df} \end{bmatrix} \times \begin{bmatrix} x_f(k) \\ x_f(k-d) \end{bmatrix} < 0, \tag{15}$$

which ensures asymptotic stability of filtering error dynamics. And if we set

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \quad S = \begin{bmatrix} S_1 & S_2 \\ * & S_3 \end{bmatrix}, \tag{16}$$

then the following inequality in (15)

$$\begin{bmatrix} A_f^T P A_f - P + S + C_f^T C_f & A_f^T P A_{df} \\ * & -S + A_{df}^T P A_{df} \end{bmatrix} < 0 \tag{17}$$

is changed to

$$\begin{bmatrix} -P^{-1} & A_f & A_{df} \\ * & -P + S + C_f^T C_f & \\ * & * & -S \end{bmatrix} < 0 \tag{18}$$

using Schur complements[13]. By pre and post multiplication with $Diag[P \; I \; I]$, the inequality (18) is equivalent to

$$\begin{bmatrix} -P & PA_f & PA_{df} \\ * & -P + S + C_f^T C_f & 0 \\ * & * & -S \end{bmatrix} < 0. \tag{19}$$

By substituting the (7) and (16) into (19), the LMI (19) is equal to

$$\begin{bmatrix} -P_1 & 0 & P_1 A & 0 & P_1 A_d & 0 \\ * & -P_2 & P_2 A - P_2 K C - P_2 \hat{A} & P_2 \hat{A} & P_2 A_d & 0 \\ * & * & -P_1 + S_1 & S_2 & 0 & 0 \\ * & * & * & -P_2 + S_3 + L^T L & 0 & 0 \\ * & * & * & * & -S_1 & -S_2 \\ * & * & * & * & * & -S_3 \end{bmatrix} < 0. \tag{20}$$

Using some changes of variables, $M_1 = P_2 \hat{A}$, $M_2 = P_2 K$, (20) is transformed into (i) of (10). Furthermore, by the summation of both sides in the inequality (14) from 0 to $T_f - 1$, we obtain

$$\begin{aligned} &-\textstyle\sum_{k=0}^{T_f-1} z(k)^T z(k) \\ &> x_f(T_f)^T P x_f(T_f) - x_f(0)^T P x_f(0) \\ &\quad + \textstyle\sum_{i=T_f-d}^{T_f-1} x_f(i)^T S x_f(i) \\ &\quad - \textstyle\sum_{i=-d}^{-1} x_f(i)^T S x_f(i). \end{aligned} \tag{21}$$

As the closed loop system is asymptotically stable, when $T_f \to \infty$(or $T_f - 1 \to \infty$) and using initial condition, some terms go to zero. Hence we get

$$\sum_{k=0}^{\infty} z(k)^T z(k) \le \phi_f(0)^T P \phi(0) + \sum_{i=-d}^{-1} \phi_f(i)^T S \phi_f(i). \tag{22}$$

This is an upper bound of guaranteed cost. The first term of right hand side in (22) is changed to $-\alpha + \phi_f(0)^T P \phi_f(0) < 0$. This is equivalent to (ii) in (10). The second term of right hand side in (22) has the following relations:

$$\begin{aligned} &\textstyle\sum_{i=-d}^{-1} \phi_f(i)^T S \phi_f(i) = \sum_{i=-d}^{-1} tr(\phi_f(i)^T S \phi_f(i)) \\ &= tr(NN^T S) = tr(N^T S N) < tr(Q), \end{aligned} \tag{23}$$

where Q is a positive definite matrix to determine the upper bound of $tr(N^T S N)$. Therefore, $-Q + N^T S N < 0$ is equal to (iii) in (10). □

Hence, we can get the optimal discrete-time guaranteed cost filter satisfying the filtering design objective. Also, all solutions including filter variables($\hat{A} = P_2^{-1} M_1$, $K = P_2^{-1} M_2$) and the upper bound of guaranteed cost($J^* = \alpha + tr(Q)$) can be calculated simultaneously because the proposed sufficient conditions are LMIs regarding all finding variables. The guaranteed cost controller design algorithm can be directly applied to linear systems without time delay by simple manipulations in the following Corollary 1.

Corollary 1: Consider a linear system without time delay in (1), i.e., $A_d = 0$. If the following optimization problem

$$minimize \;\; \alpha \;\; subject \; to$$

$$i) \begin{bmatrix} -P_1 & 0 & P_1 A & 0 \\ * & -P_2 & P_2 A - M_2 C - M_1 & M_1 \\ * & * & -P_1 & 0 \\ * & * & * & -P_2 + L^T L \end{bmatrix} < 0,$$

$$ii) \; -\alpha + \phi_1(0)^T P_1 \phi_1(0) + \phi_2(0)^T P_2 \phi_2(0) < 0, \tag{24}$$

has a solution, positive definite matrices(or scalar) P_1, P_2, α, and matrices M_1, M_2, then (2) is an optimal guaranteed cost filter and $J^* = \alpha$ is an upper bound of discrete-time guaranteed cost. Here, $M_1 = P_2 \hat{A}$ and $M_2 = P_2 K$.

Now, we treat the problem of time delay systems with convex bounded uncertainties. The robust guaranteed cost filter of convex bounded uncertain system with time delay can be designed using the proposed result and slight modifications of Theorem 1.

Consider the following linear convex bounded uncertain system with time delay.

$$\begin{aligned} x(k+1) &= \mathcal{A} x(k) + \mathcal{A}_d x(k-d) \\ y(k) &= \mathcal{C} x(k) \\ x(k) &= \phi_1(k), \quad -d \le k \le 0 \end{aligned} \tag{25}$$

Here, system matrices are assumed to be unknown but belonging to a known convex compact set of polytope type, i.e.,

$$(\mathcal{A}, \mathcal{A}_d, \mathcal{C}) \in \Psi \tag{26}$$

where

$$\begin{aligned} \Psi := \{(\mathcal{A}, \mathcal{A}_d, \mathcal{C}) | (\mathcal{A}, \mathcal{A}_d, \mathcal{C}) = \textstyle\sum_{i=1}^{l} \tau_i (\mathcal{A}_i, \mathcal{A}_{di}, \mathcal{C}_i), \\ \textstyle\sum_{i=1}^{l} \tau_i = 1\} \end{aligned} \tag{27}$$

and $\Lambda(\cdot)$ denotes the set of i, $i = 1, 2, ..., l$ vertices of the above convex polytope. The kind of convex bounded parameter uncertainty has been widely used[12] and references therein. Note that in many practical cases, very often, only a few entries of the matrices of systems dynamics contain uncertain parameters.

Theorem 2: Consider a uncertain system with time delay (23). If there exist positive definite matrices(or scalar) P_1, P_2, S_1,

S_3, α, Q, and matrices S_2, M_1, M_2 satisfying the following optimization problem

$$minimize\ \{\alpha + tr(Q)\}\ subject\ to$$

$$i)\ \begin{bmatrix} -P_1 & 0 & P_1\mathcal{A}_i & 0 & P_1\mathcal{A}_{di} & 0 \\ * & -P_2 & P_2\mathcal{A}_i - M_2\mathcal{C}_i - M_1 & M_1 & P_2\mathcal{A}_{di} & 0 \\ * & * & -P_1 + S_1 & S_2 & 0 & 0 \\ * & * & * & -P_2 + S_3 + L^TL & 0 & 0 \\ * & * & * & * & -S_1 & -S_2 \\ * & * & * & * & * & -S_3 \end{bmatrix} < 0,$$

$$ii)\ -\alpha + \phi_1(0)^TP_1\phi_1(0) + \phi_2(0)^TP_2\phi_2(0) < 0,$$

$$iii)\ -Q + N_1^TS_1N_1 + N_2^TS_2N_1 + N_1^TS_2N_2 + N_2^TS_3N_2 < 0 \tag{28}$$

for all $(\mathcal{A}_i, \mathcal{A}_{di}, \mathcal{C}_i) \in \Lambda(\Psi)$, then (2) is an optimal discrete-time guaranteed cost filter and $J^* = \alpha + tr(Q)$ is an upper bound of discrete-time guaranteed cost. Some notations are defined as same as (11). Here, filter variables can be determined from the obtained solutions, $\hat{A} = P_2^{-1}M_1$ and $K = P_2^{-1}M_2$.

Proof: The proof follows in a straightforward way from the proof of Theorem 1. □

Similarly to the Corollary 1, Theorem 2 can be applied to the systems without time delay.

Corollary 2: Consider a linear system without time delay in (25), i.e., $\mathcal{A}_d = 0$. If the following optimization problem

$$minimize\ \alpha\ subject\ to$$

$$i)\ \begin{bmatrix} -P_1 & 0 & P_1\mathcal{A}_i & 0 \\ * & -P_2 & P_2\mathcal{A}_i - M_2\mathcal{C}_i - M_1 & M_1 \\ * & * & -P_1 & 0 \\ * & * & * & -P_2 + L^TL \end{bmatrix} < 0, \tag{29}$$

$$ii)\ -\alpha + \phi_1(0)^TP_1\phi_1(0) + \phi_2(0)^TP_2\phi_2(0) < 0,$$

has a solution, positive definite matrices(or scalar) P_1, P_2, α, and matrices M_1, M_2 for all $(\mathcal{A}_i, \mathcal{C}_i) \in \Lambda(\Psi)$, then (2) is an optimal guaranteed cost filter and $J^* = \alpha$ is an upper bound of discrete-time guaranteed cost. Here, $M_1 = P_2\hat{A}$ and $M_2 = P_2K$.

IV. Numerical example

In order to check the validity of the proposed robust guaranteed cost filter design algorithm, an example is given. Consider an uncertain linear discrete-time system with time delay

$$\begin{aligned} x(k+1) &= \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.5+\Delta_1 \end{bmatrix} x(k) + \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.3 \end{bmatrix} x(k-d) \\ y(k) &= \begin{bmatrix} 1+\Delta_2 & 0 \end{bmatrix} x(k) \\ z(k) &= \begin{bmatrix} 1 & 1 \end{bmatrix} e(k) \\ \phi_f(k) &= \begin{bmatrix} 0 & 0 & 0.1 & 1 \end{bmatrix}^T \end{aligned} \tag{30}$$

Here, we treat $-0.1 \le \Delta_1 \le 0.1$ and $0 \le \Delta_2 \le 1$ yielding an uncertain systems of $l = 4$ vertices as follows:

$$\begin{aligned} &\mathcal{A}_1 = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.6 \end{bmatrix}, \mathcal{A}_{d1} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.3 \end{bmatrix}, \\ &\mathcal{C}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ &\mathcal{A}_2 = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.6 \end{bmatrix}, \mathcal{A}_{d2} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.3 \end{bmatrix}, \\ &\mathcal{C}_2 = \begin{bmatrix} 2 & 0 \end{bmatrix}, \\ &\mathcal{A}_3 = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.4 \end{bmatrix}, \mathcal{A}_{d3} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.3 \end{bmatrix}, \\ &\mathcal{C}_3 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ &\mathcal{A}_4 = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.4 \end{bmatrix}, \mathcal{A}_{d4} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.3 \end{bmatrix}, \\ &\mathcal{C}_4 = \begin{bmatrix} 2 & 0 \end{bmatrix}. \end{aligned} \tag{31}$$

First, the solutions satisfying Theorem 2 are obtained simultaneously using the command of '*mincx*' in LMI TOOLBOX[14] as follows:

$$\begin{aligned} &P_1 = \begin{bmatrix} 2.4845 & -0.0502 \\ -0.0502 & 4.7845 \end{bmatrix}, \\ &P_2 = \begin{bmatrix} 1.6574 & 0.9335 \\ 0.9335 & 1.0093 \end{bmatrix}, \\ &S_1 = \begin{bmatrix} 0.1542 & -0.3932 \\ -0.3932 & 1.3862 \end{bmatrix}, \\ &S_2 = \begin{bmatrix} -0.0086 & -0.0007 \\ 0.0190 & -0.0027 \end{bmatrix}, \\ &S_3 = \begin{bmatrix} 0.1406 & -0.0117 \\ -0.0117 & 0.0023 \end{bmatrix}, \\ &M_1 = \begin{bmatrix} 0.0165 & -0.0026 \\ 0.1206 & -0.0148 \end{bmatrix}, \\ &M_2 = \begin{bmatrix} -0.0486 \\ 0.4034 \end{bmatrix}, \\ &\alpha = 1.2129, \\ &Q = \begin{bmatrix} 0.0006 & 0 & 0 & 0 \\ 0 & 0.5575 & -0.0012 & 0 \\ 0 & -0.0012 & 0.0011 & 0 \\ 0 & 0 & 0 & 0.0006 \end{bmatrix}. \end{aligned} \tag{32}$$

Hence, $\hat{A}$ and K are determined from the changes of variables, $M_1 = P_2\hat{A}$ and $M_2 = P_2K$. Therefore, the robust guaranteed cost filter and the upper bound of guaranteed cost are

$$\begin{aligned} \hat{x}(k+1) &= \begin{bmatrix} -0.1196 & 0.0141 \\ 0.2301 & -0.0277 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} -0.5311 \\ 0.8909 \end{bmatrix} y(k), \\ J^* &= 1.7726. \end{aligned} \tag{33}$$

Moreover, the obtained filter guarantees not only the asymptotic stability of filtering error dynamics but also minimization of guaranteed cost. If we set time delay $d = 2$, the trajectories of error state vectors and error state output are shown in Fig. 1 $\sim$ Fig. 3. Therefore, the guaranteed cost filter guarantees the asymptotic stability of filtering error dynamics in Fig. 1 and Fig. 2 because the error states converge to zero as time goes to infinity. Also, the filter ensures the minimization of guaranteed cost function. From Fig. 3, J can be calculated as $J = 1.5974 (< J^*)$. Note that the gap between the upper bound of guaranteed cost(J^*) and the calculated value(J) from the simulation result is changed according to the value of initial conditions.

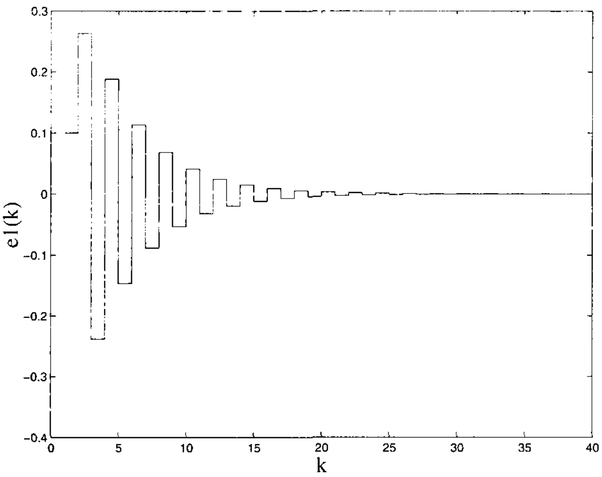

Fig. 1. The trajectory of $e_1(k)$.

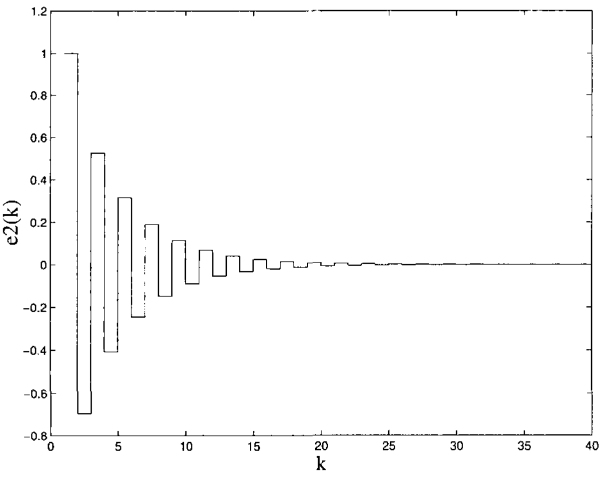

Fig. 2. The trajectory of $e_2(k)$.

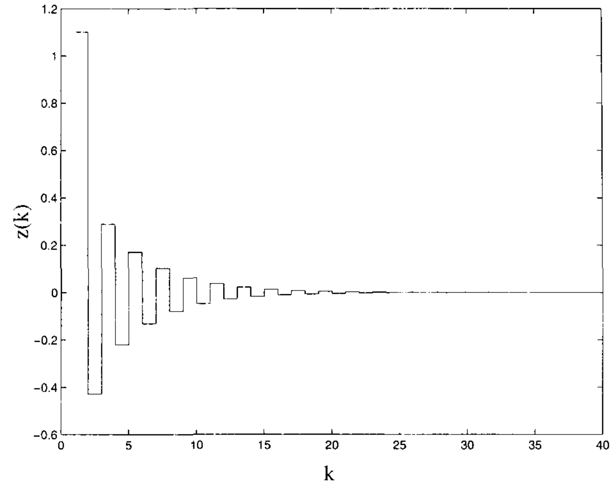

Fig. 3. The trajectory of $z(k)$.

V. Conclusions

In this paper, we proposed the guaranteed cost filtering design algorithms for time delay systems with convex bounded uncertainties in discrete-time case. The sufficient conditions for the existence of filter and guaranteed cost filter design methods were presented using LMI approach. The proposed stable filter guaranteed minimization of the upper bound in guaranteed cost. Also, it has been shown that the guaranteed cost filter design method for the systems without time delay could be solvable using the proposed method and slight modifications. The validity of the proposed design algorithm was checked by an example.

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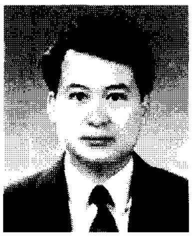

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