Tracking Control of Mechanical Systems with Partially Known Friction Model

Hyun Suk Yang, Martin C. Berg, and Bum Il Hong

Abstract: Two adaptive nonlinear friction compensation schemes are proposed for second-order nonlinear mechanical systems with a partially known nonlinear dynamic friction model to achieve asymptotic position and velocity tracking. The first scheme has auxiliary filtered states so that a simple open-loop observer can be used. The second one has a dual-observer structure to estimate two different nonlinear aspects of the friction state. Conditions for the parameter estimates to converge to the true parameter values are presented. Simulation results are utilized to show control performance and to demonstrate the convergence of the parameter estimates to their true values.

Keywords: adaptive control, mechanical systems with friction, bristle model

I. Introduction

Friction, which exists in virtually every mechanical dynamic system, is difficult to model accurately and frequently negatively impacts the performance of servomechanisms. Especially in applications with high precision position or velocity tracking objectives, the results are unsatisfactory in many cases. For controller design in such situations, it is desirable to have a good friction model available.

The classical friction models described by static maps between friction force and velocity capture static features such as Coulomb friction and viscous friction. They can not explain behaviours like hysteresis, pre-sliding displacement, varying break-away force, and Stribeck effects. The spring-like behavior during stiction is captured in the Dahl model (see [1]). The Dahl model was modified to capture Stribeck effects and hysteresis in [2] and [3], respectively. The bristle model introduced in [4] captures the slip-stick phenomenon well. Unfortunately, it is numerically difficult to implement. Another mathematical representation of the bristle model that captures most of the experimentally observed friction phenomena was proposed in [5].

While tribology researchers strive to better understand friction phenomenon, control system researchers are interested in high precision position and low-velocity tracking control of mechanical systems subject to friction. In [6], the authors proposed an adaptive controller for low-velocity robot position control replacing the conventional exponential function that represents the Stribeck effect with a linear parameterizable function. In [7], a nonlinear low-velocity friction problem with a simple PD controller was solved by reducing the number of parameters of the friction model using dimensional analysis. In [8], the authors proposed a model-based adaptive friction compensator for a DC motor servomechanism that consisted of a two-step off-line method to estimate the nonlinear static and dynamic parameters associated with the friction model and two adaptive globally stable mechanisms to deal with structured normal forces and temperature variations. A new Lyapunov-based continuous dynamic controller for a more general class of nonlinear systems with friction was proposed in [9]. In [10], an adaptive nonlinear friction compensation scheme based on a dual-observer structure was proposed to handle parametric uncertainties of the bristle friction model. In [11], an observer-based controller for exact model knowledge position tracking for a second-order mechanical system with the bristle model was proposed and asymptotical tracking was studied while compensating for seleted parametric uncertainty.

Even though the adaptive schemes in [10] and [11] gave good tracking results, they failed to show parameter convergence result. It is important to have parameter convergence result because firstly, it gives a good friction model, which can be used for other compensator design, and secondly, it may be useful to deal with full parametric uncertainties. To have parameter convergence result a condition such as the persistence of excitation condition is necessary but it is impossible to obtain such a condition with the schemes in [10] and [11]. The simulation results show that the estimated parameter values do not converge to the true values.

The dynamics of a mechanical system subject to friction are formulated in section 2 and two adaptive friction compensation schemes are proposed in section 3 and 4 to achieve asymptotic position and velocity tracking, which is obtained from the ones in [10] and [11] with slight modification. This modification is required to develop the persistence of excitation condition for the uncertain parameter convergence. The persistence of excitation condition and proof for parameter convergence are given in section 3 and 4, also. In section 5, simulation results are presented to support the theoretical concepts and this paper is concluded in section 6.

II. Problem formulation

The dynamics of many mechanical systems subject to friction are represented by

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(see [11])

$$M\ddot{q} + B_v\dot{q} + T_L(q,\dot{q}) + \xi(\dot{q})z = u \quad (1)$$

where M denotes the constant mechanical inertia of the system, B_v denotes the constant viscous friction coefficient, $T_L(q,\dot{q})$ denotes a scalar nonlinear load function of the position q and velocity $\dot{q}$, z denotes an unmeasurable internal friction state representing the average deflection of thousands of bristles, and u is the control input. The dynamics of the friction state z are represented by

$$\dot{z} = \dot{q} - f(\dot{q})z \quad (2)$$

where $f(\dot{q})$ is a non-negative function defined to be $|\dot{q}|/g(\dot{q})$ where the function $g(\dot{q})$ is used to describe various friction effects ([5]). For example, $g(\dot{q}) = \beta_0 + \beta_1 exp\left(-(\dot{q}/\beta_2)^2\right)$ has been used to represent the Stribeck effect. With $g(\dot{q}) = \beta_0$ this friction model reduces to the Dahl model [5].

The friction force due to the bristle deflection is

$$F = \theta_0 z + \theta_1\frac{dz}{dt} = \theta_1\dot{q} + (\theta_0 - f(\dot{q})\theta_1)z \quad (3)$$

where θ_0 and θ_1 are the bristle stiffness and viscous damping constants, respectively. Equation (1) then represents the dynamics of a mechanical system subject to friction with

$$\xi(\dot{q}) = \theta_0 - f(\dot{q})\theta_1 \quad (4)$$

In this paper, we assume that M, B_v, $T_L(q,\dot{q})$, and $f(\dot{q})$ are known and will present a control input $u(t)$, the dynamics of a friction state estimate $\hat{z}$, and update rules for $\hat{\theta}_0$ and $\hat{\theta}_1$, which are the estimates of θ_0 and θ_1, respectively, so that given a reference signal $q_d(t)$, the position and velocity tracking errors and the estimation error of the internal friction state approach zero asymptotically. We modify adaptive control laws in [10] and [11] so that they are different in the sense that asymptotic position and velocity trackings are achieved. We also present conditions for parameter convergence which was not done in either [10] or [11].

III. An adaptive scheme with filtered states

1. Adaptive control laws

In this section, an adaptive control algorithm with filtered states for a system represented by (1) is proposed. Let q_d and $\dot{q}_d$ be the desired position and velocity trajectories, respectively. Define the error vector $\mathbf{e(t)} \in R^{2\times 1}$ as

$$\mathbf{e(t)} = \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} = \begin{pmatrix} q(t) - q_d(t) \\ \dot{q}(t) - \dot{q}_d(t) \end{pmatrix}$$

Then, from (1), we obtain the following state equations

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= M^{-1}[u - B_v\dot{q} - T_L(q,\dot{q}) - \xi(\dot{q})z] - \ddot{q}_d \end{aligned} \quad (5)$$

Equation (5) can be written in the more compact form

$$\dot{\mathbf{e}} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\mathbf{e} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} M^{-1}v = A\mathbf{e} + BM^{-1}v \quad (6)$$

where the input v is given by

$$v = u - B_v\dot{q} - T_L(q,\dot{q}) - \xi(\dot{q})z - M\ddot{q}_d \quad (7)$$

Since (A, B) is a controllable pair, we can choose a matrix $F \in R^{1\times 2}$ so that $A_m = A - BF$ is Hurwitz. Then equation (6) becomes

$$\dot{\mathbf{e}} = A_m\mathbf{e} + BM^{-1}(v + MF\mathbf{e}) \quad (8)$$

Since A_m is Hurwitz there exist symmetric positive definite matrices P and Q such that $PA_m + A_m^T P = -Q$. Now, we are ready to state the result.

Theorem 1: Consider a system with friction represented by (1)-(4). We assume that q_d and $\dot{q}_d$ are bounded and smooth. Let the control $u(t)$ be given by

$$\begin{aligned} u = B_v\dot{q} + T_L(q,\dot{q}) + M\ddot{q}_d - MF\mathbf{e} + (\hat{\theta}_0 - f(\dot{q})\hat{\theta}_1)\hat{z} \\ + \hat{\theta}_0\zeta_0 - f(\dot{q})\hat{\theta}_1\zeta_1 \end{aligned} \quad (9)$$

where the update rules for the parameter estimates $\hat{\theta}_0$ and $\hat{\theta}_1$, the friction state observer $\hat{z}$, and the auxiliary filtered state dynamics $\zeta_0(t)$ and $\zeta_1(t)$ are given by

$$\begin{aligned} \dot{\hat{\theta}}_0 &= -\gamma_0(\hat{z} + \zeta_0)M^{-1}B^T P\mathbf{e} \\ \dot{\hat{\theta}}_1 &= \gamma_1 f(\dot{q})(\hat{z} + \zeta_1)M^{-1}B^T P\mathbf{e} \\ \dot{\hat{z}} &= \dot{q} - f(\dot{q})\hat{z} \\ \dot{\zeta}_0 &= -f(\dot{q})\zeta_0 - M^{-1}B^T P\mathbf{e} \\ \dot{\zeta}_1 &= -f(\dot{q})\zeta_1 + f(\dot{q})M^{-1}B^T P\mathbf{e} \end{aligned} \quad (10)$$

where γ_0 and γ_1 are positive design constants. Then globally asymptotic tracking of the position and velocity trajectories are achieved.

Proof: Choose the candidate Lyapunov function V for the system as

$$V = \mathbf{e}^T P\mathbf{e} + \frac{1}{2}\tilde{z}^2 + \frac{1}{\gamma_0}\tilde{\theta}_0^2 + \frac{1}{\gamma_1}\tilde{\theta}_1^2 + \theta_0(\tilde{z} - \zeta_0)^2 + \theta_1(\tilde{z} - \zeta_1)^2 \quad (11)$$

where $\tilde{\theta}_0 = \theta_0 - \hat{\theta}_0$, $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$, and $\tilde{z} = z - \hat{z}$. It can be easily shown that since $f(\dot{q})$, θ_0, and θ_1 are non-negative, the derivative of V is given by

$$\begin{aligned} \dot{V} = -\mathbf{e}^T Q\mathbf{e} - f(\dot{q})\tilde{z}^2 - \theta_0 f(\dot{q})(\tilde{z} - \zeta_0)^2 \\ -f(\dot{q})\theta_1(\tilde{z} - \zeta_1)^2 \le -\mathbf{e}^T Q\mathbf{e} < 0 \end{aligned} \quad (12)$$

Then we have that $V(t) \in L_\infty$, which implies that $\mathbf{e(t)}$, $\tilde{\theta}_0(t)$, $\tilde{\theta}_1(t)$, $\tilde{z}(t)$, $\zeta_0(t)$, and $\zeta_1(t)$ are in L_∞. As a result, $\hat{\theta}_0$ and $\hat{\theta}_1$ are bounded. Since $\dot{q}_d$ is bounded, so is $\dot{q}$, which results in the boundedness of the friction state z and its estimate $\hat{z}$. The boundedness of the control u is then apparent.

From (8) and the boundedness of all internal signals in the system, the derivative of the error $\mathbf{e}$ is bounded. Also, from the fact that $\dot{V} \le -\mathbf{e}^T Q\mathbf{e}$, it is apparent that $\mathbf{e}(t) \in L_2$. Then we have that $\mathbf{e}(t) \to 0$ as $t \to \infty$, which completes the proof. ■

Remark: The adaptive gains γ_0 and γ_1 in (10) can be any positive nondecreasing functions of time. Then since the time derivatives of $1/\gamma_0$ and $1/\gamma_1$ are non-positive the derivative of the Lyapunov function in (11) is non-positive.

2. Convergence of the estimated internal friction state

Convergence of the estimate of the internal friction state, $\hat{z}$, to z is considered in this section. We know that the friction state error $\tilde{z}$ satisfies $\dot{\tilde{z}} = -f(\dot{q})\tilde{z}$. Note that non-negativeness of $f(\dot{q})$ does not guarantee that $\tilde{z}$ converges to zero asymptotically. The following assumption resolves this problem.

Assumption 1: Suppose that $q_d(t)$ is the desired reference signal. We assume that there exists $\hat{\delta} > 0$ such that for any $\delta > \hat{\delta}$, there exists $\alpha > 0$ such that, for all $t \geq 0$,

$$\int_t^{t+\delta} f(\dot{q_d})d\tau \geq \alpha \tag{13}$$

is satisfied.

Now, we are ready to state a theorem.

Theorem 2: Consider the system represented by (1)-(3) with the control $u(t)$ and update laws given as in Theorem 1. Suppose that Assumption 1 is satisfied and that q_d and $\dot{q}_d$ are bounded smooth functions. Then the estimate of the friction state, $\hat{z}$, converges to the actual state z asymptotically. Furthermore, the state z converges asymptotically to a state z_d whose state dynamics are represented by

$$\dot{z_d} = \dot{q_d} - f(\dot{q_d})z_d \tag{14}$$

with an arbitrary initial state.

Proof: First, we prove asymptotic convergence of $\hat{z}$ to z. We know that

$$\dot{\tilde{z}} = -f(\dot{q})\tilde{z} = -f(\dot{q_d})\tilde{z} + (f(\dot{q_d}) - f(\dot{q}))\tilde{z} \tag{15}$$

Let $V(t) = \frac{1}{2}\tilde{z}^2(t)$. Then its derivative is

$$\dot{V} = -f(\dot{q_d})\tilde{z}^2 + (f(\dot{q_d}) - f(\dot{q}))\tilde{z}^2 \tag{16}$$

Pick any $\delta > \hat{\delta}$ and $\alpha > 0$ that satisfies (13). Since the function f is continuous and the fact that $\mathbf{e(t)} \to 0$ as $t \to \infty$ from Theorem 1, for any $\epsilon > 0$ there exists a $T_\epsilon > 0$ such that $|f(\dot{q_d}) - f(\dot{q})| < \epsilon$ for all $t \geq T_\epsilon$. Let $\epsilon = \alpha/2\delta$. Then for any $t \geq T_\epsilon$, by integrating (16) from t to $t+\delta$, we obtain that

$$\begin{aligned} & V(t+\delta) - V(t) \\ = & -\int_t^{t+\delta} f(\dot{q_d})\tilde{z}^2(\tau)d\tau + \int_t^{t+\delta}(f(\dot{q_d}) - f(\dot{q}))\tilde{z}^2(\tau)d\tau \\ \leq & -\int_t^{t+\delta} f(\dot{q_d})2V(t+\delta)d\tau + \tfrac{1}{2}\alpha 2V(t) \\ \leq & -2\alpha V(t+\delta) + \alpha V(t) \end{aligned} \tag{17}$$

Here, we used the monotone nonincreasing property of V. Then, (17) can be written as $V(t+\delta) \leq \gamma V(t)$ where

$$\gamma = (1+\alpha)/(1+2\alpha) < 1 \tag{18}$$

This implies that for any positive integer N

$$V(t+N\delta) \leq \gamma^N V(t) \leq \gamma^N V(T_\epsilon) \tag{19}$$

is satisfied for any $t \in [T_\epsilon,\ T_\epsilon + \delta)$, which implies that $V(t) \to 0$ as $t \to \infty$. Therefore, we have that $\hat{z}(t) \to z(t)$ as $t \to \infty$.

Let us prove that $z(t) \to z_d(t)$ as $t \to \infty$. Let $e_z = z - z_d$. Then we have that

$$\begin{aligned} \dot{e}_z &= \dot{q} - f(\dot{q})z - \dot{q_d} + f(\dot{q_d})z_d \\ &= -f(\dot{q_d})e_z + (\dot{q} - \dot{q_d}) + (f(\dot{q_d}) - f(\dot{q}))z \end{aligned} \tag{20}$$

Let $\epsilon = (\dot{q} - \dot{q_d}) + [f(\dot{q_d}) - f(\dot{q})]z$. Because f is continuous, $\mathbf{e}$ approaches zero asymptotically, $\dot{q}_d$ is bounded, and the state z is bounded, $\epsilon(t)$ approaches zero asymptotically. Then by Lemma 1 below, we have that $e_z(t) \to 0$ as $t \to \infty$, which completes the proof. ■

It is well-known that a persistence of excitation condition is needed for convergence of the parameter estimates to their actual values. Unfortunately, if the friction state z and/or its estimate $\hat{z}$ are needed in the condition, we have an unverifiable condition since the friction state z is unmeasurable and its estimate is not known a priori. In the next section, the persistence of excitation condition will be constructed with z_d, instead of z or $\hat{z}$ which is known a priori.

The following lemma is needed in the proof of Theorem 2 and later claims.

Lemma 1: Suppose that Assumption 1 is satisfied. Let a signal, $w(t)$, satisfy the state equation

$$\dot{w}(t) = -f(\dot{q_d})w(t) + \epsilon(t) \tag{21}$$

where $\epsilon(t)$ approaches zero asymptotically. Then for any initial condition $w(0)$, $w(t)$ approaches zero asymptotically.

Proof: Suppose that $\phi(t, t_0)$ is the state transition function for (21), so that

$$\dot{\phi}(t, t_0) = -f(\dot{q_d})\phi(t, t_0) \tag{22}$$

is satisfied for any t and t_0 with $\phi(t,t) = 1$ for all t. Let $V(t) = \frac{1}{2}\phi(t, t_0)^2$. Then $\dot{V} = -f(\dot{q_d})\phi(t, t_0)^2$. Given that $f(\dot{q_d}) \geq 0$ for all t, it is easy to see that $|\phi(t, t_0)|$ is nonincreasing for all $t \geq t_0$. By integrating $\dot{V}$ from t to $t+\delta$, we obtain that

$$V(t+\delta) - V(t) = -\int_t^{t+\delta} f(\dot{q_d})\phi(\tau, t_0)^2 d\tau \tag{23}$$

Since $V(t)$ is nonincreasing we can say that

$$V(t+\delta) - V(t) \leq -\int_t^{t+\delta} f(\dot{q_d})d\tau 2V(t+\delta) \leq -2\alpha V(t+\delta) \tag{24}$$

where α is chosen by Assumption 1. Then we obtain the relationship $V(t+\delta) \leq \gamma^2 V(t)$ for all $t \geq t_0$ where $\gamma^2 = 1/(1+2\alpha) < 1$. This implies that

$$|\phi(t+\delta, t_0)| \leq \gamma|\phi(t, t_0)| \leq \gamma|\phi(t_0, t_0)| = \gamma \tag{25}$$

Now, we can write that

$$w(t) = \phi(t, t_0)w(t_0) + \int_{t_0}^{t} \phi(t, \tau)\epsilon(\tau)d\tau \tag{26}$$

Since $\epsilon(t) \to 0$ as $t \to \infty$, there exists $T \geq 0$ such that $\int_t^{t+\delta} |\epsilon(\tau)|d\tau \leq \gamma$ for all $t \geq T$. Then for any $t \in [T, T+\delta)$ and an arbitrary positive integer N, we have that

$$\begin{aligned} & |w(t+N\delta)| \\ \leq & \ |\phi(t+N\delta, t+(N-1)\delta)||w(t+(N-1)\delta)| \\ & + \int_{t+(N-1)\delta}^{t+N\delta} |\phi(t+N\delta, \tau)||\epsilon(\tau)|d\tau \\ \leq & \ \gamma|w(t+(N-1)\delta)| + \int_{t+(N-1)\delta}^{t+N\delta} |\epsilon(\tau)|d\tau \\ \leq & \ \gamma[|\phi(t+(N-1)\delta, t+(N-2)\delta)||w(t+(N-2)\delta)| \\ & + \int_{t+(N-2)\delta}^{t+(N-1)\delta} |\phi(t+(N-1)\delta, \tau)||\epsilon(\tau)|d\tau] \\ & + \int_{t+(N-1)\delta}^{t+N\delta} |\epsilon(\tau)|d\tau \end{aligned}$$

$$\leq \gamma^2|w(t+(N-2)\delta)| + \gamma\int_{t+(N-2)\delta}^{t+(N-1)\delta}|\epsilon(\tau)|d\tau + \int_{t+(N-1)\delta}^{t+N\delta}|\epsilon(\tau)|d\tau$$

Letting $\epsilon_i = \int_{t+i\delta}^{t+(i+1)\delta}|\epsilon(\tau)|d\tau$, $i = 0, \cdots, (N-1)$, and continuing the process, we obtain that

$$|w(t+N\delta)| \leq \gamma^N|w(t)| + \sum_{i=0}^{N-1}\gamma^{N-1-i}\epsilon_i \tag{27}$$

Since $\epsilon_i \leq \gamma$ for all i and ϵ_i converges to zero, there exists a positive integer k such that $\gamma^{k+1} < \epsilon_i \leq \gamma^k$ for $i = 0, \cdots, (N-1)$. Furthermore, there exist $N_1, \cdots, N_k$ such that $0 \leq N_1 \leq N_2 \leq \cdots \leq N_k \leq N$ and $\gamma^{j+1} < \epsilon_i \leq \gamma^j$ for $N_{j-1} < i \leq N_j$, $j = 1, 2, \cdots, (k+1)$ with $N_0 = 0$ and $N_{k+1} = N$. Note that as $N \to \infty$, so does k and that $(N - N_j) \geq (k - j)$ for $j = 1, \cdots, k$. Therefore, we have the following:

$$\begin{aligned}\sum_{i=0}^{N-1}\gamma^{N-1-i}\epsilon_i &= \sum_{j=1}^{k+1}\sum_{i=N_{j-1}}^{N_j-1}\gamma^{N-1-i}\epsilon_i \\ &\leq \sum_{j=1}^{k+1}\sum_{i=N_{j-1}}^{N_j-1}\gamma^{N-1-i+j} \\ &\leq \sum_{j=1}^{k+1}\gamma^{N-N_j+j}\frac{1}{1-\gamma} \\ &\leq \frac{1}{1-\gamma}\sum_{j=1}^{k+1}\gamma^k = \frac{k+1}{1-\gamma}\gamma^k\end{aligned}$$

Since $k\gamma^k \to 0$ as $k \to \infty$, we have that $\sum_{i=0}^{N-1}\gamma^{N-1-i}\epsilon_i \to 0$ as $N \to \infty$. This implies that $|w(t)| \to 0$ as $t \to \infty$ and this completes the proof. ■

Remark: Since $\mathbf{e(t)} \to 0$ asymptotically, Lemma 1 implies that $\zeta_0(t)$ and $\zeta_1(t)$ in (10) approach zero asymptotically when Assumption 1 is satisfied.

3. Convergence of the estimates of the parameters

In this section, we show that the parameter estimate $\hat{\theta} = \left(\hat{\theta}_0\ \hat{\theta}_1\right)^T$ approaches the true unknown parameter $\theta = (\theta_0\ \theta_1)^T$ under an additional persistence of excitation condition. Let $\tilde{\theta} = \theta - \hat{\theta} = \left(\theta_0 - \hat{\theta}_0\ \ \theta_1 - \hat{\theta}_1\right)^T$. Rewrite the error equation (8) and the parameter update laws (10) as follows.

$$\begin{aligned}\dot{\mathbf{e}} &= A_m\mathbf{e} + BM^{-1}(v + MF\mathbf{e}) \\ &= A_m\mathbf{e} + BM^{-1}(\hat{\theta}_0(\hat{z}+\zeta_0) \\ &\quad - f(\dot{q})\hat{\theta}_1(\hat{z}+\zeta_1) - \theta_0 z + f(\dot{q})\theta_1 z) \\ &= A_m\mathbf{e} + BM^{-1}\left(\tilde{\theta}^T\hat{\mathbf{w}} - \theta^T\hat{\mathbf{w}} - \theta_0 z + f(\dot{q})\theta_1 z\right) \qquad (28) \\ &= A_m\mathbf{e} + BM^{-1}\left(\tilde{\theta}^T\hat{\mathbf{w}} - \theta^T\tilde{\mathbf{w}} + 2\theta^T(\zeta_0\ \ - f(\dot{q})\zeta_1)^T\right) \\ \dot{\tilde{\theta}} &= -\dot{\hat{\theta}} = \begin{pmatrix}(\hat{z}+\zeta_0)M^{-1}B^TP\mathbf{e} \\ -f(\dot{q})(\hat{z}+\zeta_1)M^{-1}B^TP\mathbf{e}\end{pmatrix} \\ &= -M^{-1}B^TP\mathbf{e}\hat{\mathbf{w}} \qquad (29)\end{aligned}$$

where $\hat{\mathbf{w}} = (-(\hat{z}+\zeta_0)\ \ f(\dot{q})(\hat{z}+\zeta_1))^T$ and $\tilde{\mathbf{w}} = (-(\tilde{z}+\zeta_0)\ \ f(\dot{q})(\tilde{z}+\zeta_1))^T$. Let $\epsilon = -\theta^T\tilde{\mathbf{w}} + 2\theta^T(\zeta_0\ \ - f(\dot{q})\zeta_1)^T$. Then because $\tilde{z}$, $\zeta_0(t)$, and $\zeta_1(t)$ approach zero asymptotically, $\epsilon(t)$ approaches zero asymptotically. Let $\mathbf{x} = \left(\mathbf{e}^T\ \tilde{\theta}^T\right)^T$. Then (28) and (29) can be written as

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{pmatrix}A_m & M^{-1}B\hat{\mathbf{w}}^T(t) \\ -M^{-1}\hat{\mathbf{w}}(t)B^TP & 0\end{pmatrix}\mathbf{x} + \begin{pmatrix}M^{-1}B \\ 0\end{pmatrix}\epsilon \\ &= A_x(t)\mathbf{x} + B_x\epsilon \qquad (30)\end{aligned}$$

Let $Q = L^TL + \gamma I$ for any $L \in R^{2\times2}$ and $\gamma > 0$. Obviously Q is a symmetric positive definite matrix. Since A_m is Hurwitz, there exists a symmetric positive definite matrix P such that $A_m^TP + PA_m = -Q$. Let $\mathbf{x_m}(t) = (\mathbf{e_m}(t)^T\ \ \tilde{\theta_m}(t)^T)^T$ and consider a system represented by $\dot{\mathbf{x}}_\mathbf{m} = A_x(t)\mathbf{x_m}$. Let a Lyapunov function be given by

$$V(t) = \mathbf{e_m}^T(t)P\mathbf{e_m}(t) + \tilde{\theta_m}^T(t)\tilde{\theta_m}(t) \tag{31}$$

Then its derivative $\dot{V}$ will be

$$\dot{V}(t) = -\mathbf{x_m}^T(t)C^TC\mathbf{x_m}(t),\ C = \begin{pmatrix}L & 0 \\ \sqrt{\gamma}I & 0\end{pmatrix} \tag{32}$$

From Sections 4.5 and 5.6 of [12], if the signal $\hat{\mathbf{w}}(t)$ satisfies the persistence of excitation condition, which is that there exist α_1, α_2, and $\delta > 0$ such that

$$\alpha_1 I \leq \int_t^{t+\delta}\hat{\mathbf{w}}(\tau)\hat{\mathbf{w}}^T(\tau)d\tau \leq \alpha_2 I \tag{33}$$

is satisfied for all t, the system is exponentially stable. In other words, there exist k and $\alpha > 0$ such that for any bounded initial state $x_m(t_0)$

$$||\mathbf{x_m}(t)|| \leq ||\mathbf{x_m}(t_0)||ke^{-\alpha(t-t_0)} \tag{34}$$

This implies that the state transition matrix $\Phi(t, t_0)$ for (30) satisfies that $||\Phi(t,t_0)|| \leq ke^{-\alpha(t-t_0)}$. Since $\epsilon(t)$ in (30) approaches zero asymptotically, we can prove that the state $\mathbf{x}(t)$ approaches zero asymptotically using arguments similar to those in Lemma 1. This implies that the parameter estimate $\hat{\theta}$ converges to the true value θ asymptotically.

The persistence of excitation condition on $\hat{\mathbf{w}}(t)$ is not a verifiable condition because the signal $\hat{\mathbf{w}}(t)$ utilizes the signals $\hat{z}(t)$, $\zeta_0(t)$, and $\zeta_1(t)$ which are generated in the adaptive system. Therefore, there is no way to check this persistence condition. However, notice that $\hat{z}(t)$ converges to $z_d(t)$ asymptotically by Theorem 2 and that $\zeta_0(t)$ and $\zeta_1(t)$ approach zero asymptotically, which implies that the signal $\hat{\mathbf{w}}(t)$ converges to $\mathbf{r(t)} = (-z_d(t)\ \ f(\dot{q}_d)z_d(t))^T$, which is known a priori. Then, provided $\hat{\mathbf{w}}(t)$ is persistently exciting, there exists $t_0 \geq 0$ such that

$$\frac{1}{2}\alpha_1 I \leq \int_t^{t+\delta}\mathbf{r}(\tau)\mathbf{r}(\tau)^Td\tau \leq 2\alpha_2 I \tag{35}$$

is satisfied for all $t \geq t_0$, which implies that $\mathbf{r}(t)$ is also persistently exciting. This is true vice versa, i.e., if $\mathbf{r}(t)$ is persistently exciting, so is $\hat{\mathbf{w}}(t)$.

Now, we are ready to state the result on the convergence of the parameter estimates. We need the following assumption known as the persistence of excitation condition.

Assumption 2: The reference signal $q_d(t)$ is such that q_d and $\dot{q}_d$ are bounded smooth differentiable functions and there exist α_1, α_2, and $\delta > 0$ such that, for all t,

$$\begin{aligned}\alpha_1 I &\leq \int_t^{t+\delta} \begin{pmatrix} -z_d(\tau) \\ f(\dot{q}_d)z_d(\tau)\end{pmatrix}\begin{pmatrix} -z_d(\tau) \\ f(\dot{q}_d)z_d(\tau)\end{pmatrix}^T d\tau \\ &= \int_t^{t+\delta} \mathbf{r}(\tau)\mathbf{r}(\tau)^T d\tau \leq \alpha_2 I \end{aligned} \quad (36)$$

where $z_d(t)$ is a solution of the system represented by $\dot{z}_d = \dot{q}_d - f(\dot{q}_d)z_d$ with an arbitrary initial state $z_d(0)$. □

Note that the upper bound in (36) is satisfied whenever $z_d(t)$ and $f(\dot{q}_d)$ are bounded. This is the case because q_d and $\dot{q}_d$ are bounded.

Theorem 3: Suppose that Assumption 1 and Assumption 2 are satisfied. If the control $u(t)$ and the update laws in Theorem 1 are applied to the system (1), the parameter error $\tilde{\theta} = \theta - \hat{\theta}$ approaches zero asymptotically.

Proof: The persistence of excitation condition (36) implies that $\hat{\mathbf{w}}(t)$ is persistently exciting. Then, by the arguments given above, the state $\mathbf{x}(t)$ approaches zero asymptotically, which completes the proof. ■

IV. An adaptive scheme with a dual-observer

In this section, we propose an adaptive scheme with a dual-observer structure similar to the one in [10]. Note that when the term $\xi(\dot{q})z = (\theta_0 - \theta_1 f(\dot{q}))z$ in (5) is estimated by $(\hat{\theta}_0 - \hat{\theta}_1 f(\dot{q}))\hat{z}$, the estimation error which can be written as $\tilde{\theta}_0 z + \hat{\theta}_0\tilde{z} - \tilde{\theta}_1 f(\dot{q})z - \hat{\theta}_1 f(\dot{q})\tilde{z}$ contains two terms of the friction state estimation error $\tilde{z}$: one with and the other without the term $f(\dot{q})$. In the previous section, two filtered states, ζ_0 and ζ_1, are used to deal with those effects. In this section, instead, two observers are used in parallel to estimate two different effects of the unmeasurable friction state. Again, we deal with both the position and velocity tracking errors whose dynamics are represented by (8). We choose the control input $u(t)$ as

$$u = B_v\dot{q} + T_L(q,\dot{q}) + M\ddot{q}_d - MF\mathbf{e} + \hat{\theta}_0\hat{z}_0 - \hat{\theta}_1 f(\dot{q})\hat{z}_1 \quad (37)$$

where $\hat{z}_0$ and $\hat{z}_1$ are two different estimates of the internal friction state z. The estimate updating rules and the observers are given by

$$\begin{aligned}\dot{\hat{\theta}}_0 &= -\gamma_0 M^{-1}BP\mathbf{e}\hat{z}_0 \\ \dot{\hat{\theta}}_1 &= \gamma_1 f(\dot{q})M^{-1}BP\mathbf{e}\hat{z}_1 \\ \dot{\hat{z}}_0 &= \dot{q} - f(\dot{q})\hat{z}_0 - M^{-1}BP\mathbf{e} \\ \dot{\hat{z}}_1 &= \dot{q} - f(\dot{q})\hat{z}_1 - f(\dot{q})M^{-1}BP\mathbf{e}\end{aligned} \quad (38)$$

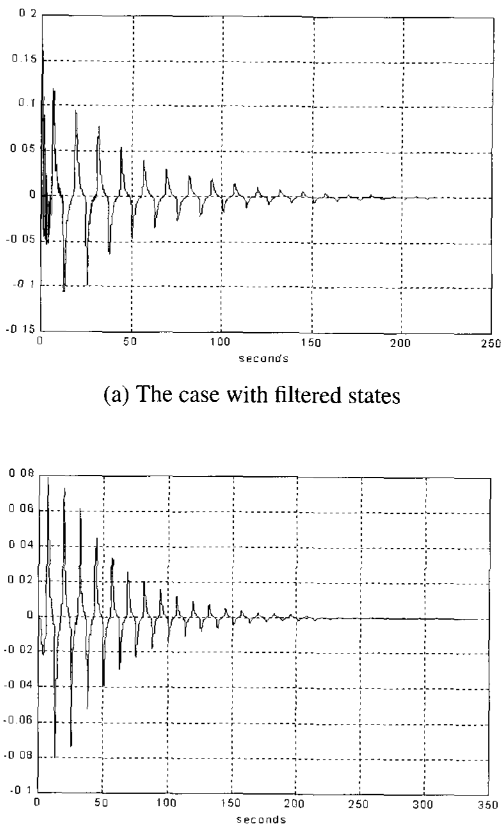
where the update gains γ_0 and γ_1 are any positive constants or positive nondecreasing functions of time as in Section 3. Now, let $\tilde{z}_0 = z - \hat{z}_0$ and $\tilde{z}_1 = z - \hat{z}_1$. Let the candidate Lyapunov function be

$$V = \mathbf{e}^T P\mathbf{e} + \frac{1}{\gamma_0}\tilde{\theta}_0^2 + \frac{1}{\gamma_1}\tilde{\theta}_1^2 + \tilde{z}_0^2 + \tilde{z}_1^2 \quad (39)$$

Then, its derivative satisfies that

$$\dot{V} = -\mathbf{e}^T Q\mathbf{e} - 2f(\dot{q})(\tilde{z}_0^2 + \tilde{z}_1^2) \leq -\mathbf{e}^T Q\mathbf{e} < 0 \quad (40)$$

since $f(\dot{q})$ is non-negative for all time.



(a) The case with filtered states

(b) The case with a dual-observer

Fig. 1. Position Tracking Errors

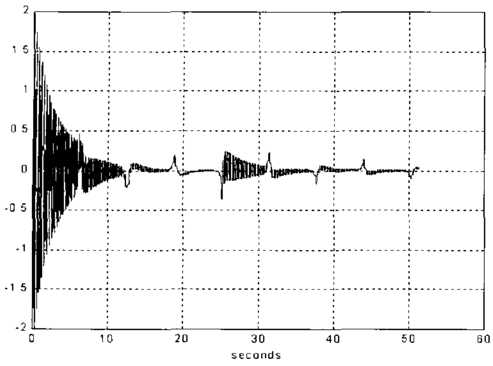
Theorem 4: Consider a mechanical system with friction represented by (1)-(4). Assume that the reference signal q_d and its derivative $\dot{q}_d$ are bounded smooth functions. Let the adaptive nonlinear controller be given by (37) with the parameter update laws and the friction observers given in (38). Then globally asymptotic tracking of the desired position and velocity trajectories is achieved.

Proof: The global uniform boundedness of all errors are guaranteed by the definition of the Lyapunov function in (39) and the non-positivity of its derivative. Then, by arguments similar to those in Theorem 1, the asymptotic tracking properties of the position and velocity trajectories are achieved. □

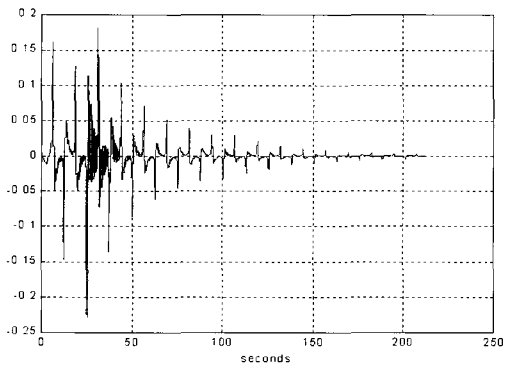
The convergences of the estimates $\hat{z}_0$ and $\hat{z}_1$ of the friction state to the actual state z are achieved by Lemma 1 if Assumption 1 is satisfied since the error $\mathbf{e}(t)$ approaches zero asymptotically. Furthermore, it can be easily shown that the state z approaches a reference state z_d whose dynamics are represented by $\dot{z}_d = \dot{q}_d - f(\dot{q}_d)z_d$ with an arbitrary initial condition. Then, again by the arguments in Section 3.3, it can be easily shown that if the persistence of excitation condition (36) is satisfied, the parameter estimates $\hat{\theta}_0$ and $\hat{\theta}_1$ converge to the actual values θ_0 and θ_1, respectively.

V. Simulation results

In this section we apply our adaptive controllers to the same mathematical model of a motor/mechanical friction system as in [11]. The parameter values are $M = 0.125kg \cdot m^2$, $B_v = 1.42Nm \cdot s/rad$, $T_L(q,\dot{q}) = 0$, $\theta_0 = 12Nm/rad$, $\theta_1 =$

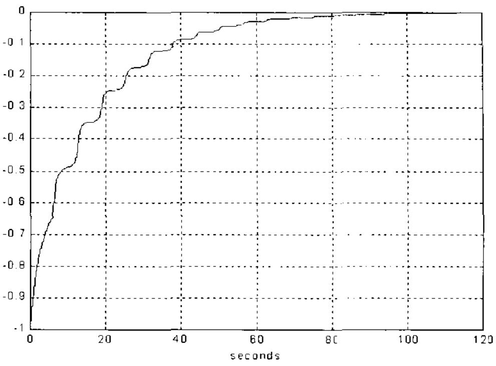


(a) The case with filtered states

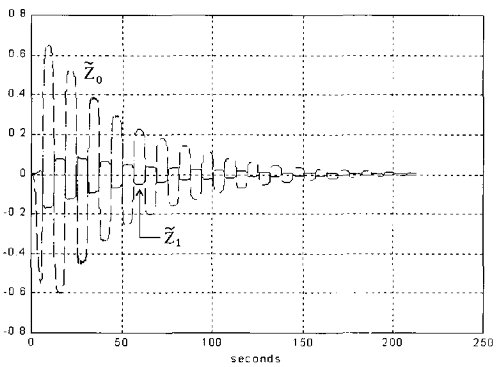


(b) The case with a dual-observer.

Fig. 2. Velocity tracking errors.



(a) The case with filtered states



(b) The case with a dual-observer

Fig. 3. Estimation errors of the internal friction state.

$0.1Nm \cdot s/rad$, $\beta_0 = 3.24$, $\beta_1 = 5.21$, and $\beta_2 = 3.00$. The desired position trajectory is $q_d(t) = tan^{-1}(4sin(0.5t))(1 - exp(-0.01t^3))$ rads. The adaptive gains γ_0 and γ_1 are increased from 3 to 40 and from 0.01 to 1000 as time increases, respectively. The changes in the gains are not optimal in any sense but were chosen arbitrarily. Simulation was done using MATLAB SIMULINK with ode23 for solving the nonlinear state equations. The initial estimates were $\hat{\theta}_0(0) = 6$ and $\hat{\theta}_1(0) = 0.05$ which are the half of the actual values. All other initial values were set to zero.

The position and velocity errors are shown in Fig.1 and Fig.2, respectively. For the position tracking error, the two controllers produce similar results. For the velocity tracking error, the controller with a dual-observer has smaller error than the one with filtered states. This is because the control input of the error dynamics represented by (5) contains the term $\hat{\theta}_0\zeta_0$, which is large for some period of time.

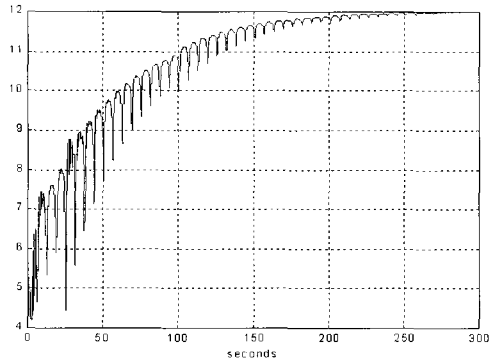
The estimation errors of the unmeasurable internal friction state are shown in Fig.3. For the adaptive controller with filtered states, the initial error was set to be -1 (if the initial error was set to zero, the estimate error would be zero for all time). For the one with a dual-observer, the initial errors were set to zero. It is easy to see that the errors approach zero asymptotically.

Fig.4 and Fig.5 show the estimation for the parameters θ_0 and θ_1, respectively. The convergence rates of the two controllers are virtually the same because their update rules for θ_0 and θ_1 are the same. The estimate $\hat{\theta}_0$ converges to the actual value θ_0 faster than the estimate $\hat{\theta}_1$ converges to θ_1 because the term containing $\hat{\theta}_0$ in the error dynamics (5) is dominant compared with the term containing $\hat{\theta}_1$.

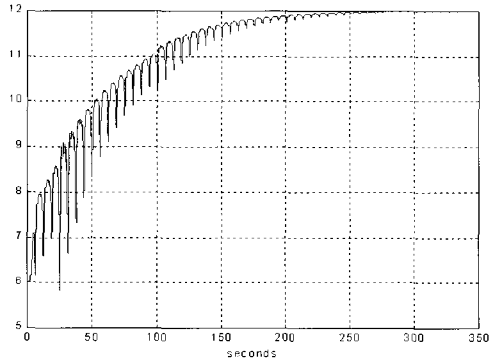
VI. Conclusion

In this paper, two adaptive control algorithms were proposed to achieve position and velocity tracking for a second-order nonlinear mechanical system using the nonlinear dynamic friction model proposed in [5]. The adaptive schemes are obtained from the schemes in [10] and [11] with slight modification. They are different in that the position and the velocity trajectory tracking errors are treated together. Also, conditions for the parameter estimates to converge to their actual values are also given, which was neither mentioned in [10] nor in [11]. Simulation results are presented to illustrate the performance of the controllers and to demonstrate that all of the parameter estimates converge to their actual values.

Further research will be done to deal with the nonlinear parameter uncertainties caused by β_0, β_1, and β_2. Due to their severe nonlinearities asymptotic tracking properties can not be obtained with simple linear adaptive control laws. We will need sophisticated nonlinear adaptive control law to deal with these uncertainties. Also, developing new schemes for faster asymptotic tracking and parameter convergence are remained as future researches.

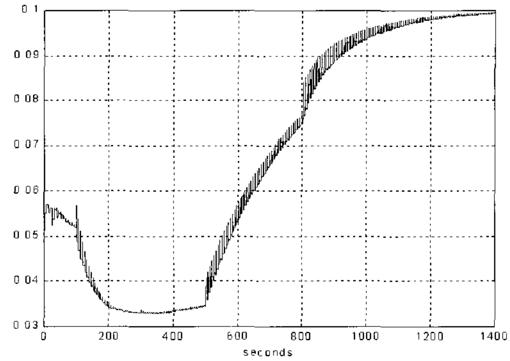


(a) The case with filtered states

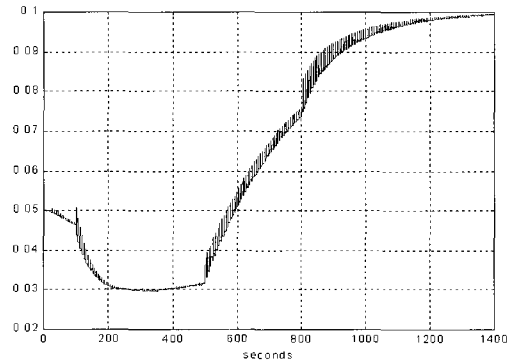


(b) The case with a dual-observer

Fig. 4. Estimation of the parameter θ_0.



(a) The case with filtered states



(b) The case with a dual-observer

Fig. 5. Estimation of the parameter θ_1.

References

[1] P. Dahl, "A solid friction model", Aerospace Corp. El Segundo CA Tech. Rep TOR-0158(3107-18)-1, 1968

[2] P. A. Bliman, "Mathematical study of the Dahl's friction model", *European Journal of Mechanics*, **11**, pp. 835-848, 1992.

[3] P. A. Bliman, and M. Sorine, "Friction modelling by hysteresis operators, application to Dahl, stricktion, and Stribeck effects". *Proceedings of Conference on Models and Hysteresis*, Ternto Italy 1991.

[4] D. A. Haessin Jr., and B. Friedland, "On the modeling and simulation of friction", *ASME Journal of Dynam. Systems, Measurements, and Control*, **113**, pp. 354-362, 1991.

[5] C. Canudas De Wit, H. Olsson, K. J. Astrom, and P. Lischinsky, "A new model for control of systems with friction", *IEEE Transactions on Automatic Control*, **40**, pp. 419-425, 1995.

[6] C. Canudas De Wit, C. P. Noel, and B. Brogliato, "Adaptive friction compensation in robot manipulations: Low velocities", *International Journal of Robotics Research*, **10**, pp. 189-199, 1991.

[7] B. Amstrong-Helouvry, "Strick-slip and control in low-speed motion", *IEEE Transactions on Automatic Control*, **38**, pp. 1483-1496, 1993.

[8] C. Canudas De Wit and P. Lischinsky, "Adaptive friction compensation with partially known dynamic friction model", *International Journal of Adaptive Control and Signal Processing*, **11**, pp. 65-80, 1997.

[9] R. M. Hirscorn and G. Miller, "Control of nonlinear systems with friction", *IEEE Transactions on Control System Technology*, **7**, pp. 588-595, 1999.

[10] Y. Tan and I. Kanellakopoulos, "Adaptive nonlinear friction compensation with parametric uncertainties", Proceedings of American Control Conference, pp. 2511-2515, 1999.

[11] P. Vedagarbha, D. M. Dawson, and M. Feemster, "Tracking control of mechanical systems in the presence of nonlinear dynamic friction effects", *IEEE Transactions on Control Systems Technology*, **7**, pp. 446-456, 1999.

[12] H. Khalil, *Nonlinear Systems*, Macmillan Co, 1992

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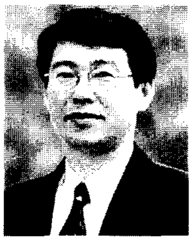

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