Target State Estimator Design Using FIR Filter and Smoother

Jae-Hun Kim and Joon Lyou

Abstract: The measured rate of the tracking sensor becomes biased under some operational situation. For a highly maneuverable aircraft in 3D space, the target dynamics changes from time to time, and the Kalman filter using position measurement only can not be used effectively to reject the rate measurement bias error. To cope with this problem, we present a new algorithm which incorporate FIR-type filter and FIR-type fixed-lag smoother, and demonstrate that it has the optimal performance in terms of both estimation accuracy and response time through an application example to the anti-aircraft gun fire control system(AAGFCS).

Keywords: target state estimator, finite impulse response filter, fixed-lag smoother, rate measurement bias

I. Introduction

The Target State Estimator(TSE) takes a role of estimating the target state through combining the measured data from tracking sight and a prior knowledge of the target motion in a statistically optimal fashion. Commonly, the filter is designed and evaluated in time domain and such is the case of the well-known Kalman filter. This filter requires us to know the exact knowledges of the target motion which can be described by the dynamic state equation and measurement model so as to compute the state estimate accurately. However, real motion of the target is frequently deviated from the assumed model, and estimation error of the filter grows and diverges in some worst case.

In this situation, adaptive tracking filters may be used, and they can be conveniently categorized into three different groups. First, switching of filter model is introduced by monitoring the estimated error. When the model needs to be changed, the statistics of the model uncertainty may be adjusted depending on the magnitude of the residual[1] or the order of the state model may be augmented[2] or the forcing input of the model may be computed directly through manipulation of the residual sequence[3][4]. Second, the multiple model filters are presumed and each of the filter estimate is combined using some probabilistic method[5][6]. Third, the model is assumed to be valid for a finite time only. And the filter is designed for a finite time interval, and it uses the finite window of the past measurement data to adapt to the changed model[1],[7]-[10], in contrast to the Kalman filter which uses all the past measurements.

All aforementioned filters assume that the measurement data has no bias error throughout the whole estimation period. But when the behavior of measurement sensor deviates from the assumed model at some unknown time, the estimate of the filter would be inevitably biased while the measurement model mismatch, and remains biased some time afterwards depending on the response time of the filter. If we have multiple measurement sensors, we can detect performance degradation of the specific sensor using the functional redundancy and detection mechanism[17]. However if the filter model and measurement model change randomly at unknown time, it is very difficult to adapt to the changed state model while eliminating the measurement bias simultaneously.

In the 3D TSE problem using the tracking sight, the target dynamics changes from time to time and the measured rate of the tracking sensor becomes biased at some situation. It is assumed that the tracking sight can provide both position and rate measurement and keep good accuracy in position measurement even when the rate measurement becomes biased under the operational environment, thanks to the advanced image processing of the tracking system. It is necessary for the position filter, which uses position measurement only, to use larger interval of past position measurements to get the reliable estimate of velocity component. However it is also needed for the filter to have higher dynamic bandwidth and use smaller interval of position measurements in order to retain adaptive capability to the changed state model. Increasing bandwidth makes the rate estimate noisy, and reducing the bandwidth makes the rate estimate biased when it is influenced by the mismatched filter model. This makes the design more complicated.

In this paper, we propose two stage estimator not only to keep the model adaptive capability but also to reject the measurement bias. First, to estimate the state of the highly maneuvering aircraft, the FIR type filter based on the finite time measurement model, which uses position and rate measurement simultaneously, is used for a main target state estimator(MTSE). Note that it has such good characteristics as BIBO stability, parameter insensitivity to the model change[8] and especially the fast response[9][10] when the flying object frequently changes its moving behavior. Next, FIR type fixed-lag smoother, which uses the position measurement only, is used as an auxiliary TSE(ATSE) parallel to the MTSE so as to compute the rate estimate error of MTSE and correct the MTSE. The superiority of performance of the smoother to that of the filter has been well known[12]-[16] even though it has the time delay problem in real time applications. And the idea of the utilization of the fixed-lag smoother to eliminate the rate bias of the TSE is natural, but it has not been fully appreciated yet in TSE area to the authors' knowledge. Effectiveness of the proposed method is shown through an application to the 3D TSE in the AAGFCS.

II. The FIR filter/FIR smoother

When the target changes its motion dynamics, the past measurement data has a little information about the current motion.

Manuscript received: Oct. 31, 2001., Accepted: May. 27, 2002.

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The valid duration of the model might as well be limited to the recently finite time as far as we concern the single filter model, and it is the basic theory of the limited memory filters.[1],[7]-[10] We choose the FIR type filter with finite measurement window for this case. It guarantees absolute stability and it can be shown to be algebraically equivalent to the Kalman filter without initial information should the measurement interval grow infinitely[7]-[10].

Here we use the constant acceleration model as a representative filter model, and derive 3D FIR filters which have the identical structure decoupled each other in the rectangular coordinate, mostly in the sense of sub-optimality. The assumption of the constant acceleration model can not be justified in general, since the target can change its course arbitrarily in 3D space. The effect of changing course in 3D space makes each filter model coupled one another and vary from time to time. However so long as the duration of the filter model is limited to be appropriately small, the constant acceleration model can be naturally assumed resulting from the Newton's second law(motion law). The equation for this model and the parameters are as follows

$$x(k+1) = Ax(k) + Bw(k) \tag{1}$$

$$y(k) = Cx(k) + v(k) \tag{2}$$
$$E[w(k)w(m)^T] = Q\delta(k-m) \tag{3}$$
$$E[v(k)v(n)^T] = Q\delta(k-n) \tag{4}$$
$$E[w(k)] = 0 \tag{5}$$
$$E[v(k)] = 0 \tag{6}$$

where

$$A = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$B = \begin{bmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{bmatrix} \tag{8}$$

$$C = \begin{bmatrix} 1\,0\,0 \\ 0\,1\,0 \end{bmatrix} \; (pos. + rate\ meas.) \tag{9}$$

$$C = \begin{bmatrix} 1\,0\,0 \end{bmatrix} \; (pos.\ meas.\ only) \tag{10}$$

1. The FIR filter

The estimate of the FIR filter can be derived for a N-frame measurement time-window as

$$\hat{x}(k) = \sum_{i=0}^{N} H(i) \cdot y(k-i) \tag{11}$$

The Gain H(i) of the filter can be derived from the method reported in the references [7]-[10]. Here we describe the result of FIR filter and FIR smoother from [8] for later use. By successively using the equations of (12)-(18) for each i which lies in the range of $0 \le i \le N$, each gain of H(i) can be computed when H(i;n) reach H(i;N) starting n from -1.

$$H(i) = H(i;N) \tag{12}$$
$$H(i;N) = S^{-1}(N) \cdot L(i;N), \tag{13}$$
$$0 \le i \le N, p \le N < \infty$$
$$L(i;n+1) = A^{-T}[I - S(n)\bar{S}(n)]L(i;n), \tag{14}$$
$$N-i \le n \le N-1$$
$$L(i;N-i) = C^T \tag{15}$$
$$S(n+1) = A^{-T}S(n)A^{-1} + C^TC - A^{-T}S(n)\bar{S}(n)S(n)A^{-1}, \tag{16}$$
$$-1 \le n \le N-1$$
$$S(-1) = 0 \tag{17}$$

where p is the order of the state and

$$\bar{S}(n) = A^{-1}BQ^{\frac{1}{2}} \times [I + Q^{\frac{1}{2}}B^TA^{-T}S(n)A^{-1}BQ^{\frac{1}{2}}]^{-1}(Q^T)^{\frac{1}{2}} \times B^TA^{-T} \tag{18}$$

In above equations, the covariance of the filter can be expressed as

$$P_f = S(N)^{-1} \tag{19}$$

2. The FIR smoother

The estimate of the FIR smoother can be derived for a N-frame measurement time-window as

$$\hat{x}_b(k-N) = \sum_{i=0}^{N} G(i) \cdot y(k-i) \tag{20}$$

And by successively using the equations of (21)-(29) for each i which lies in the range of $0 \le i \le N$, each gain of G(i) can be computed when G(i;n) reach G(i;N) starting n from -1.

$$G(i) = G(i;N) \tag{21}$$
$$G(i;N) = Q(N;N)C', \tag{22}$$
$$0 \le i \le N, p \le N \le \infty$$
$$Q(N;N) = \Omega(N)S^{-1}(N)V_v(i;N) \tag{23}$$
$$V_v(i;n+1) = (I + C'C[S(n+1) - C'C]^{-1}) \times A'^{-1}V_v(i;n) - C'CAS^{-1}(n)L_l(i;n) \tag{24}$$
$$N-i \le n \le N-1$$
$$L_l(i;n+1) = A'^{-1}[I - S(n)\bar{S}(n)]L_l(i;n) \tag{25}$$
$$N-i \le n \le N-1$$
$$V_v(i;N-i) = I \tag{26}$$
$$L_l(i;N-i) = I \tag{27}$$
$$\Omega(n+1) = \Omega(n)[I - \bar{S}(n)S(n)]A^{-1} \tag{28}$$
$$0 \le n \le N-1$$
$$\Omega(0) = I \tag{29}$$

$S(n), \bar{S}(n)$: computed by (16)-(18)

The covariance of the smoother can be computed by

$$P_b = Q(N;N) \tag{30}$$

III. Two stage estimator

If rate measurement sensor becomes biased under some operational situation, the bias can be eliminated by the estimator which uses the position measurement only combining it with the detection mechanism of the rate estimate bias[17]. However when the target model changes from the presumed model, it is not easy for the conventional-type filter which uses position measurement only to track the variable model properly and to eliminate the rate bias at the same time. The position filter has more elimination capability of the measurement noise proportional to the size of the time window, but it needs to have smaller window to be able to adapt to the model change. When the window size is reduced to deal with the increased dynamic bandwidth, it shows ill side-effects of differentiation rather than smoothing of measurement noise, for it is less aided by the information of the model dynamic.

But if we use the uncausal estimator such as the fixed-lag smoother based on the finite time model, it can solve the above two problems at a time even though the estimate is performed not at the current time but at the past time. The smoother can reject the dynamic model bias at the delayed time point, for it can estimate the model uncertainty from the time point of estimation to the current time point helped by the measurement information[13]-[14]. Also it can achieve the good measurement noise rejection due to the increased information from past and future measurements simultaneously.

Nevertheless the fixed-lag smoother suffer time delay problem and it can not provide the current time estimate properly. So we take two different estimators. One is the position-and-velocity-using FIR filter (PVFIR), which is used as a main TSE(MTSE). The other one is the position-using FIR-type fixed-lag smoother, which is used as an auxiliary TSE(ATSE) to compute the rate estimate error of MTSE. In case that the rate bias is sufficiently slowly time varying compared with the fixed-lag time interval of the smoother, we can effectively compensate for the PVFIR filter by scaling the computed rate bias using the weight between 0 and 1.0 depending on the changing speed of the rate bias. The structure of two stage estimator is shown in figure 1.

1. Main Target State Estimator(MTSE)

The main target state estimator is designed to take the form of FIR filter which uses both position and velocity measurements in the interval of (i-N, i). The filter model is defined in (1)-(9), and the size of measurement window(N) and the variance of the system model uncertainty(Q) is chosen simultaneously to minimize the covariance of the filtered estimate referenced to the given real system once the variance of the measurement sensor(R) is known a priori. (filter tuning) To use unit variance matrix(I) instead of R, the measurement equation can be easily re-scaled for symmetric positive definite R matrix by using the decomposition of $R = \Lambda^T \Lambda$ [11].

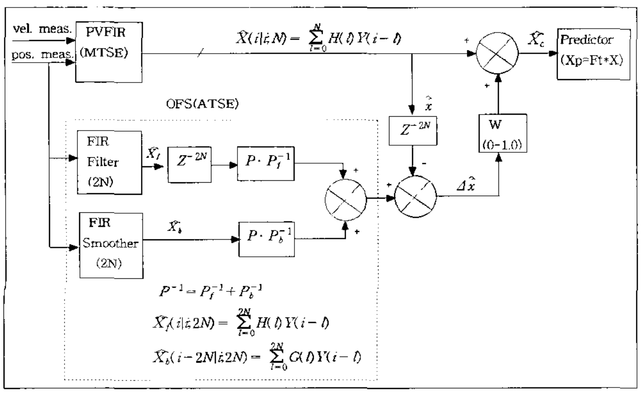

Fig. 1. Structure of Two Stage Estimator.

The information of the initial filter state is considered as unknown (zero) while computing the FIR filter gain [7]-[10]. The estimate of the position-and-velocity using FIR filter(PVFIR) is given as

$$\hat{x}(i;N) = \sum_{l=0}^{N} H(l) \cdot y(i-l) \tag{31}$$

The filter gain can be computed by (12)-(18),and the covariance can be given by (19).

2. Auxiliary Target State Estimator(ATSE)

The ATSE is utilized to monitor and eliminate the bias of the rate estimate in the MTSE. This estimator, which takes the form of FIR fixed-lag smoother, can be easily designed by incorporating the FIR filter and the FIR smoother which are given in section II using the information fusion approach[11][12].

2.1 Structure of the fixed-lag smoother

It is known that the interval of the fixed delay doesn't need to be taken for more than two or three times the settling time of the filter to get the optimal effect of the smoother[13]. We take the interval as twice(2N) of the PVFIR filter(N) for computational advantage with suboptimal sense in this paper. The smoother uses position measurement only and it also has the FIR type structure taking the measurement window from the time point of (i-4N) to (i) to compute the smoothed estimate at (i-2N), so it has equal data window(2N) for the past and future time interval, respectively, seeing from the smoothing point. It is a fixed-lag(2N) FIR smoother.

2.2 Smoother equations

The estimation of the fixed-lag smoother can be decomposed into 2 filtering process. That is, one is the time-forward filtering using (i-4N, i-2N) measurement to compute the filtered estimate at (i-2N) (forward filter). The other one is the time-reverse filtering using (i-2N, i) measurement to compute the smoothed estimate at (i-2N)(backward filter)[12]. Let the estimate and covariance of the forward filter and backward filter as $\hat{x}_f(i-2N)$ and $P_f(i-2N)$, $\hat{x}_b(i-2N)$ and $P_f(i-2N)$, respectively. The optimal smoother can be derived from the following equations which combines the past and future information by the statistically optimal method[11]-[12].

$$\begin{aligned} x^*(i-2N) &= P(i-2N) \\ &\quad \times [P_f^{-1}(i-2N) \cdot hatx_f(i-2N) \\ &\quad + P_b^{-1}(i-2N)] \cdot hatx_b(i-2N)] \quad (32) \\ P^{-1}(i-2N) &= P_f^{-1}(i-2N) + P_b^{-1}(i-2N) \quad (33) \end{aligned}$$

The forward filtering can be computed by the equations of (12) to (18) at the point of (i-2N) using the data of (i-4N, i-2N). And the backward filtering can be solved by the equations of (21) to (29) at the point of (i-2N) using the data of (i-2N, i). Each covariance of the forward filter and the backward filter can be computed using (19) and (30), respectively.

2.3 Comparison with the conventional fixed-lag smoother

Most of the conventional fixed-lag smoothers are designed using the all past measurement data of time interval (0, i), deriving from the Kalman filter equations[12]-[15]. Such fixed-lag smoother can also be represented by the equation of (32) combining the two kinds of information[11][12], replacing the forward filter by the Kalman filter which compute the filtered estimate at (i-2N) using (0, i-2N) measurement data, and keeping the backward filter the same as the aforementioned FIR smoother.

However when the system model changes frequently, the performance of the conventional smoother may be poorer than that of FIR-type fixed-lag smoother which is based on the finite-time model. This is demonstrated by an example during review of simulation in the next section.

IV. Simulation results

The real measured target path is used to assess the effectiveness of the TSE. The target is a small remotely-controlled aircraft for test purpose, and it is often under the effect of wind or short period of maneuver. During simulation the sampling time T is 0.02 (sec), and N is chosen to be 16 points for MTSE and 64 points for ATSE (32 points each for FIR filter and smoother, respectively) from compromise between the performance and computational ease. Q has been tuned for the Kalman filter and the same value is used for FIR estimator, and R is selected unity taking the appropriate dimension.

we used the error of the aim point as a performance criteria to evaluate the effectiveness of the TSE in this simulation. Figure 2 shows the flight path in 3D space, and Figure 3 shows the time of flight that is used for prediction of the aim point.

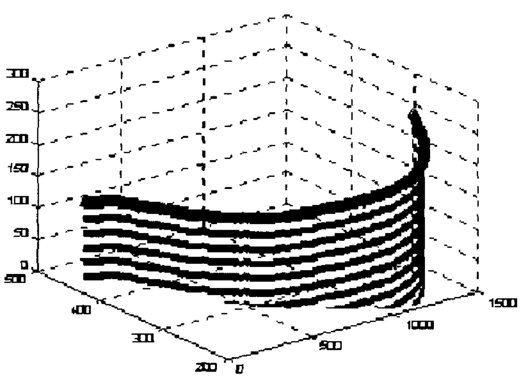

Fig. 2. 3D Target moving path.

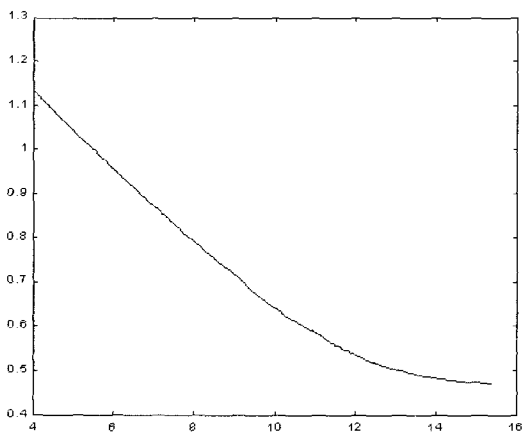

Fig. 3. Time of flight.

The errors of the fixed-lag smoothers between the conventional type and FIR type are shown in Figure 4(azimuth) and Figure 5(elevation). The proposed FIR fixed-lag smoother shows the superior performance to the conventional type which uses the all past measurements and can be regarded as the optimal fixed-lag smoother(OFS), since the latter one assumes the fixed system model(constant acceleration model) during the entire past period while the real motion conforms to the fixed model assumption only for a short duration. The difference of performance is contrasted in azimuth direction, for the target conducts the geometric turn mostly in such direction. In figure 6(azimuth) and figure 7(elevation), the aim point error of the proposed 2 stage estimator is compared with that of the other kind Kalman filters. The FIR filter (PVFIR) based on the accel-

Fig 4, Fig5:
1: FIR fixed-lag smoother
2: Conventional fixed-lag smoother

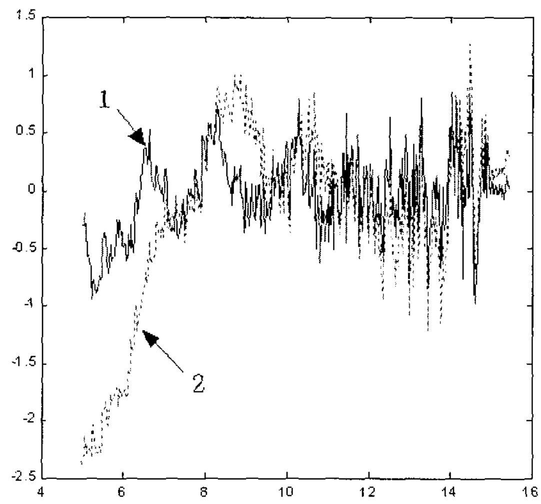

Fig. 4. Error of fixed-lag smoothers(azimuth).

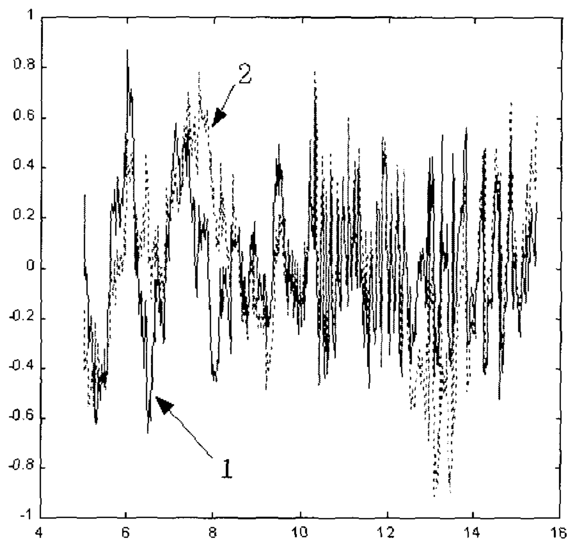

Fig. 5. Error of fixed-lag smoothers (elevation).

eration model which uses position and rate measurement shows the best performance compared with all Kalman filters such as PKF(position measurement)+CV(constant velocity model), PKF + CA(constant acceleration model), PVKF(position and velocity measurement)+CV, PVKF+CA in most of tracking time. However all filters including PVFIR show the aiming bias in the nearest crossing path due to mostly rate sensor bias. In this case the PVFIR+OFS(optimal fixed-lag smoother) can compensate the rate sensor bias well, and we can find the improved performance in Fig's 6 and 7. The computed rate bias is made by comparing the rate estimate between PVFIR and OFS, and it is scaled down to half (0.5) and added to the current rate estimate of the MTSE considering the time varying effect of the rate estimate bias.

Throughout simulations we can easily find that excellency of the FIR type estimator(filter and smoother) stems from its fast tracking capability compared with other kinds of Kalman estimator which use the all past measurement data, and this advantage has already been marked through previous literature [7]-[10].

Fig 6, Fig7:
1: FIR fixed-lag smoother; ATSE (delayed time)
2: Main FIR filter(PVFIR,CA); MTSE(current time)
3: Two stage estimator(PVFIR,CA); (current time)
4: Kalman filter(PVKF, CV)
5: Kalman filter(PVKF,CA)

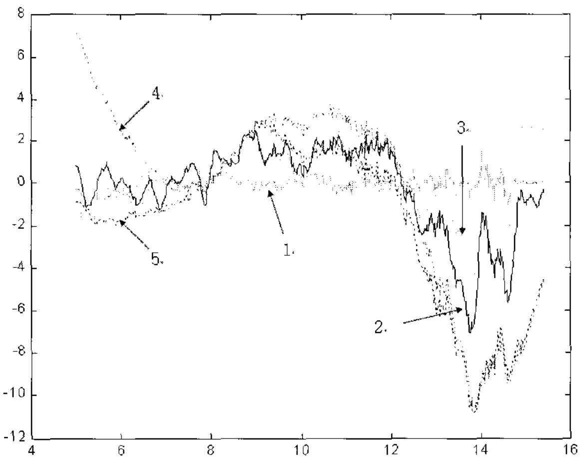

Fig. 6. Error of aim point (azimuth).

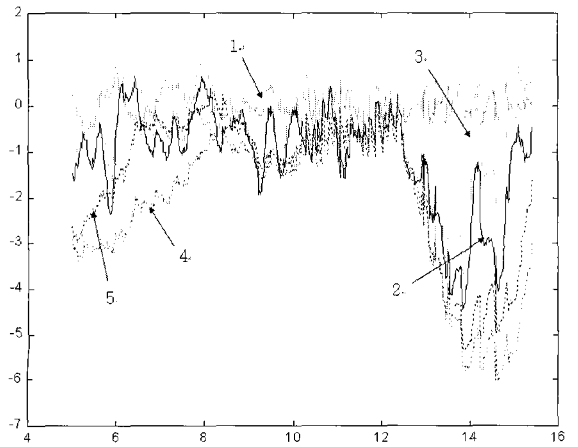

Fig. 7. Error of aim point (elevation).

V. Conclusion

In this paper we have presented a new scheme of target state estimator which can eliminate the sensor bias, especially rate measurement bias of the tracking sight when the system dynamic changes frequently. To keep wider dynamic capability as well as eradication capability of the rate sensor bias, two stage estimator has been constructed. First, to estimate the state of the highly maneuvering aircraft, the FIR type filter, which uses position and rate measurement simultaneously, is used for a main target state estimator (MTSE). Next, FIR type fixed-lag smoother, which uses the position measurement only, is used as an auxiliary TSE(ATSE) parallel to the MTSE so as to compute the rate estimate error of MTSE and correct the MTSE. The proposed estimator combining MTSE and ATSE, could be optimally used at all times regardless of whether the rate sensor became biased or not. Effectiveness of our method has been demonstrated via simulations to be applicable to anti-aircraft gun fire control system. But there still remains some concern about computation burden of the FIR estimator mainly due to its non-recursive structure for real-time applications, since it may not be small enough depending on the growth of the measurement window. Even though a recursive form of FIR estimator has also been proposed in [8], it is known to be numerically unstable and difficult to use. So an alternatively stable and recursive form of FIR estimator is much needed and it remains as the future study.

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