Sliding Mode Control with Friction Observer for a Precise Mechanical System in the Presence of Nonlinear Dynamic Friction

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Abstract: A position tracking control schemes on the precise mechanical system in presence of nonlinear dynamic friction is proposed. A nonlinear dynamic friction is regarded as the bristle friction model to compensate effects of friction. The conventional sliding mode controller often has been used as a non-model-based friction controller, but it has a poor tracking performance in high-precision position tracking application since it completely cannot compensate the friction effect below a certain precision level. Thus to improve the precise position tracking performance, we propose the sliding mode control method combined with the friction-model-based observer having tunable structure of the transient response. Then this control scheme has a good transient response as well as the high precise tracking performance compared with the conventional sliding mode control without observer and the control system with similar type of observer. The experiments on the ball-screw drive table with the nonlinear dynamic friction show the feasibility of the proposed control scheme.

Keywords: Nonlinear Dynamic Friction, Bristle Friction Model, Sliding Mode Control, Friction Observer with Tunable Structure, Ball-screw Drive Table

I. Introduction

Friction is a phenomenon that almost appears in mechanical system contacted each other. It affects the tracking performance of the servo system such as machine tools and robots, etc. Specially, the tracking performance can be worse in the range of the low velocity when the precise motion is carried out. Thus, many researches have been executed to solve the effects of the friction problem. The friction effects at moderate velocity are somewhat predictable. In this velocity range, friction model called as a classical friction model is built by the combination of the Coulomb and viscous friction that considers the friction model as a static relation of friction force and velocity. A classical friction model, however, fails to capture the low velocity effects such as the Stribeck effect, stick-slip motion, pre-sliding motion, break-away force, etc, which play a significant role in high precise position tracking applications. Besides these friction effects, through many researches, it has been revealed that friction is also influenced by the factors such as interior temperature of friction surface, contacting time, magnitude of load, operating distance, etc. The dynamic friction depending on the internal dynamics of friction mechanism, which are very difficult to model, causes these low velocity friction effects.

From the researches on the more complete friction model, Canudas de Wit *et al.*[1] presented a new dynamic friction model which captures dynamic friction effects in low velocity as well as steady-state friction effects. They proposed a state variable bristle model, called the bristle model, to describe the friction between two contacting surfaces. Now, this bristle model has been often adopted as the friction model by many researchers for the friction compensation. The bristle model is explained that in low velocity, the bristles between two contacting surfaces maintain the elastic property within tiny relative displacement and when they deflect beyond the elastic range, the sliding motion appears. The bristle deflections, however, cannot be measured directly and so it must be estimated by the friction observer. In this paper, the dynamic friction of the ball-screw drive servo table is taken as the bristle model.

In general, the control methods for a compensation of nonlinear friction are divided into two sides of approaches: friction model-based method and non-friction model-based method (without friction observer). The latter approach is mainly used when the exact friction model cannot be constructed and no need precise tracking performance. In low precision level, this approach can be applied conveniently since the procedure of the friction parameter identification can be omitted and the structure of controller is also simple. The neural network control[2][3] and sliding mode control[4] (SMC) correspond to non-model based method. While the model-based method can be applied to the case that the identification for friction model is possible within a certain exact range of precision level, a more precise tracking performance can be obtained but cannot avoid a little complexity of control system and difficulty of the exact identification of friction parameters. PID/friction observer-based control method[1][5][6], Lyapunov-based method[6] and adaptive control method[8][9] are included in this friction model-based method.

In this paper, we propose the friction observer-based SMC method to improve the position tracking performance of ball-screw drive table that is frequently used in machine tool in the presence of dynamic nonlinear friction. The proposed SMC method is constituted by combing the reducing chattering structured[10] SMC of integral type[11][12] with friction model-based observer. Then, it will be shown that the proposed friction observer-based SMC method has a more precise position tracking performance compared with non-model based SMC method and similar friction observer-based control method via real time experiments

II. Dynamic friction modeling

In bristle friction model, the interface between two surfaces is modeled by contact between sets of elastic bristles. When a tangential torque is applied, the elastic bristle will deflect like spring that gives rises to the friction torque as shown in Fig. 1.

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If the torque is increased beyond a certain magnitude, some of the elastic bristles deflect so much and they will slip. The average deflection of the elastic bristle is defined by $z(t)$ and is model as follows[1]:

$$\dot{z}(t) = \dot{q}(t) - f(\dot{q})z(t) \qquad (1)$$

where

$$f(\dot{q}) = \frac{|\dot{q}|}{g(\dot{q})}. \qquad (2)$$

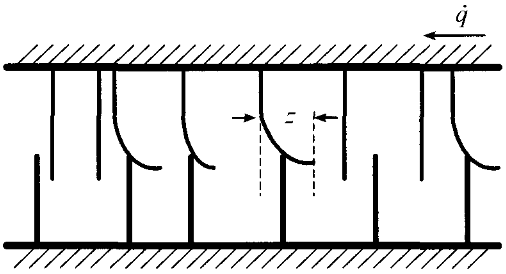

Fig. 1. The friction interfaces between two surfaces

In Eq. (2), $\dot{q}$ is a relative velocity between two contact surfaces. The function $g(\dot{q})$ is positive and depends on many factors such as the material properties, lubrication, and temperature. The friction torque excluding viscous friction torque is described by

$$T_f(t) = \sigma_0 z(t) + \sigma_1 \dot{z}(t) \qquad (3)$$

where σ_0 is a stiffness of the elastic bristle, σ_1 is a damping coefficient in elastic range. From Eq. (1) and Eq. (3), the friction term $T_f(t)$ can be also represented by

$$T_f(t) = \chi(\dot{q})z + \sigma_1 \dot{q} \qquad (4)$$

where the auxiliary function $\chi(\dot{q})$ is defined as follows:

$$\chi(\dot{q}) = \sigma_0 - \sigma_1 f(\dot{q}). \qquad (5)$$

In Eq. (2), the function $g(\dot{q})$ that includes the Stribeck effect is parameterized as

$$g(\dot{q}) = \frac{1}{\sigma_0}[T_c + (T_s - T_c)e^{-(\dot{q}/\dot{q}_s)^2}] \qquad (6)$$

where T_c is Coulomb friction level. T_s is stiction level, $\dot{q}_s$ is Stribeck velocity. For steady-state motion, the static friction torque containing viscous friction torque is given by

$$T_{fss}(\dot{q}) = [T_c + (T_s - T_c)e^{-(\dot{q}/\dot{q}_s)^2}]\mathrm{sgn}(\dot{q}) + C_t\dot{q} \qquad (7)$$

where C_t is viscous friction coefficient. The bristle friction model is characterized by six parameters, $\sigma_0, \sigma_1, T_c, C_t, T_s, \dot{q}_s$ and these parameters must be identified via experimental process.

In this paper, for the ball-screw drive table supported by LM(linear motion) guide, the experimental identification is executed to estimate the parameters of the bristle friction model. Sweeping slowly input voltage on the DC servo motor, the input current and velocity from the signal of encoder attached at each motor axis are measured. The measured input currents are converted to the friction torque using torque coefficients of DC servo motor. Through twenty-five experiments, the relation curves between the friction torque and velocity are obtained by the least square estimate. From these relation curves, the average values of four friction parameters $T_c, T_s, C_t, \dot{q}_s$ are estimated. Steady-state friction data yields no information about σ_0, which represents bristle stiffness. Then, in pre-sliding range, the linear relation between the input torque and corresponding small pre-displacement until the sliding motion starts can be approximately searched. From the slope of this linear relation, the value of σ_0 can be obtained. σ_1, which is related to damping during friction transients, is computed by pre-sliding motion equation with assuming a damping factor 0.5[6]. Fig. 2 shows the partly estimated result, and identified values of the bristle friction model are given in Table 1. J denotes the equivalent inertia of axis $(kgf \cdot cm \cdot \sec^2)$. In Fig. 3, it is shown that the results of the simulation and experiment for the PI control system to verify the identified values of friction parameters. For the command input of position $q = 0.5\sin t$ (rad), the output positions are converted as μm unit value. Fig. 3 shows that the estimated friction parameters are valid within a small range of estimation error.

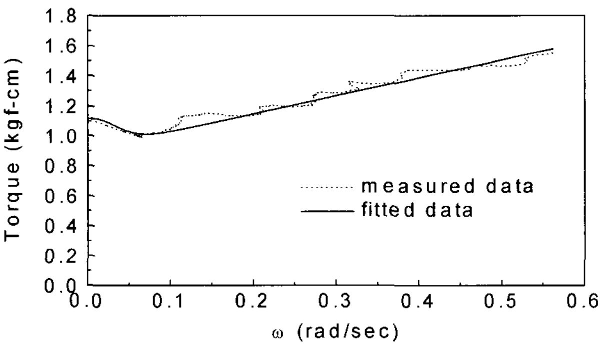

Fig. 2. The graph of measured data and estimated data

Table 1 Values of estimated friction and inertia parameter

T_c	T_s	C_t	$\dot{q}_s$	σ_0	σ_1	J
0.901	1.133	1.097	0.056	86.35	4.7	0.002509

III. Controller and observer design

1. Sliding mode controller design

The dynamic model for the ball-screw drive table in the presence of friction is

$$J\ddot{q} + C_t\dot{q} + T_f = u \qquad (8)$$

where $u(t)$ is the control input. Substituting Eq. (4) and Eq. (5) into Eq. (8), then the following expression can be obtained as

$$J\ddot{q} + C_{eq}\dot{q} + T_z = u \qquad (9)$$

where $C_{eq} = C_t + \sigma_1$ and $T_z = \chi(\dot{q})z$. In Eq. (9), T_z is the friction torque except viscous friction torque. We define the sliding surface of the variable structure controller of the integral type and the position tracking error $e(t)$ as

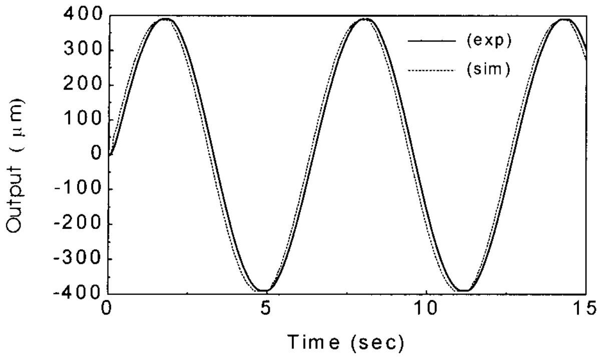

Fig. 3. Result of friction parameter estimation: experiment and simulation

follows[10].[11]:

$$s = \lambda_1 e + \dot{e} + \lambda_2 \int e dt \tag{10}$$

$$e = q_d - q \tag{11}$$

where λ_1 and λ_2 are the positive constant and q_d represents the desired position trajectory. Our objective is to choose the control force $u(t)$ such that the system state is driven to the sliding surface $s = 0$ regardless of friction. Choosing the Lyapunov function $V = \frac{1}{2}s^2$, the time-derivative of V is obtained by

$$\begin{aligned} \dot{V} &= s\dot{s} = s(\lambda_1 \dot{e} + \ddot{e} + \lambda_2 e) \\ &= s\left[\lambda_1 \dot{e} + \ddot{q}_d + \lambda_2 e + \frac{C_{eq}\dot{q} + T_z - u}{J}\right]. \end{aligned} \tag{12}$$

Let us choose the control force $u(t)$ of the form with unified smooth control law[9] instead of the traditional variable control law with switching or boundary layer term as follows:

$$\begin{aligned} u &= J(\lambda_1 \dot{e} + \ddot{q}_d + \lambda_2 e) + C_{eq}\dot{q} + \frac{\beta s}{\phi} \\ &= u_{eq} + u_f \end{aligned} \tag{13}$$

where $u_{eq} = J(\lambda_1 \dot{e} + \ddot{q}_d + \lambda_2 e) + C_{eq}\dot{q}$, $u_f = \frac{\beta s}{\phi}$, β is a positive constant whose magnitude depends on the system uncertainties and disturbances and ϕ is a positive constant relating to the thickness of the boundary layer. It was reported that the smooth control law has the advantages such as chattering free behavior, better robustness and ease of adaptability than the traditional variable control law with switching or boundary layer term[9]. Let us substitute Eq.(13) into Eq.(12), then

$$\dot{V} = -\frac{s}{J}\left(\frac{\beta s}{\phi} - T_z\right). \tag{14}$$

Now, β and ϕ must be chosen such that $|T_z| < \frac{\beta}{\phi}|s|$ in order to become $\dot{V} < 0$. Thus, the position tracking errors asymptotically go to zero even in the presence of friction. But in case that precision tracking performance is required, the condition of $|T_z| < \frac{\beta}{\phi}|s|$ leads to raise magnitude of control input excessively even though the smooth control law is introduced. In addition, a tracking performance can be worsening at a high precise tracking level if not compensating the friction effects.

Thus, in order to compensate the friction torque, let us add the estimated friction term $\hat{T}_z$ to the control input. Then the control input can be rewritten as

$$\begin{aligned} u &= J(\lambda_1 \dot{e} + \ddot{q}_d + \lambda_2 e) + C_{eq}\dot{q} + \frac{\beta s}{\phi} + \hat{T}_z \\ &= u_{eq} + u_f + \hat{T}_z \end{aligned} \tag{15}$$

where $\hat{T}_z = \chi(\dot{q})\hat{z}$ and $\hat{z}$ is the estimated state variable of the bristle deflection state z. Thus, Eq. (14) is also rewritten as

$$\dot{V} = -\frac{s}{J}\left(\frac{\beta s}{\phi} + \hat{T}_z - T_z\right). \tag{16}$$

From Eq. (16), if the friction torque is estimated such that $\hat{T}_z \to T_z$, Eq. (16) is expressed as

$$\dot{V} = -\frac{\beta}{J\phi}s^2 \tag{17}$$

and then the tracking error $e(t)$ will go to zero asymptotically fast.

2. Exponentially stable observer with tunable transient response

Since the state $z(t)$ cannot be measured directly as stated in Section 1, the observer that dynamically estimates unmeasurable state $z(t)$ is required to use it with the controller. Now, we suggest the exponentially stable observer with tunable transient response in order to estimate the friction. Let us substitute Eq. (15) into Eq. (9) and rearrange it in terms of tracking error as follows:

$$J[(\ddot{q}_d - \ddot{q}) + \lambda_1 \dot{e} + \lambda_2 e] = T_z - \hat{T}_z - \frac{\beta s}{\phi} \tag{18}$$

Let us define the observation error for unmeasurable friction state as follows:

$$\tilde{z} = z - \hat{z} \tag{19}$$

Then

$$T_z - \hat{T}_z = \chi(\dot{q})\tilde{z} \tag{20}$$

so the following closed-loop error system is obtained by

$$\dot{s} = \frac{1}{J}\left(\chi(\dot{q})\tilde{z} - \frac{\beta s}{\phi}\right). \tag{21}$$

Define the Lyapunov function as the following nonnegative function as follows:

$$V_s = \frac{1}{2}Js^2 \tag{22}$$

Taking the time-derivative of Eq. (22) and substituting it into Eq. (21), the following expression can be obtained by

$$\dot{V}_s = s\left[\chi(\dot{q})\tilde{z} - \frac{\beta s}{\phi}\right]. \tag{23}$$

An observer, which exponentially estimates the state $z(t)$ and can tune a transient response, is given by

$$\hat{z} = w + \frac{J}{\sigma_1}s + (k_1 e + k_2 \dot{e}) \tag{24}$$

where w is an auxiliary variable and satisfy the following equation:

$$\begin{aligned}\dot{w} = \frac{1}{\sigma_1}[&-\sigma_0 w + (-C_{eq} + \sigma_1)\dot{q} - J\frac{\sigma_0}{\sigma_1}s + u + \chi(\dot{q})s \\ &- \sigma_0(k_1 e + k_2\dot{e}) - J(\ddot{q}_d + \lambda_1 e + \lambda_2 \dot{e})] - (k_1\dot{e} + k_2\ddot{e})\end{aligned} \tag{25}$$

The basic concept of this observer is mainly originated from the observer proposed by Nicosia *et. al.*[13] In order to prove exponential stability, let us redefine the following nonnegative Lyapunov function:

$$V_{s2} = V_s + \frac{1}{2}\sigma_1 \tilde{z}^2 \tag{26}$$

In order to formulate the error dynamics, we first take the time-derivative of Eq. (26) and substitute Eq. (23) into it, the resulting expression is obtained as

$$\dot{V}_{s2} = s\left[\chi(\dot{q})\tilde{z} - \frac{\beta s}{\phi}\right] - \sigma_1 \tilde{z}\dot{\tilde{z}}. \tag{27}$$

Next, substituting Eq. (25) into the result of taking the time-derivative of Eq. (24) and in next substituting it into Eq. (27) with the time-derivative of Eq. (19), then

$$\begin{aligned}\dot{V}_{s2} &= s\left[\chi(\dot{q})\tilde{z} - \frac{\beta s}{\phi}\right] - \sigma_1 \tilde{z}\left[\frac{\chi(\dot{q})s}{\sigma_1} + \frac{\sigma_0}{\sigma_1}\tilde{z}\right] \\ &= -\frac{\beta s^2}{\phi} - \sigma_0 \tilde{z}^2.\end{aligned} \tag{28}$$

In Eq. (26), V_{s2} is positive-definite and in Eq. (28), $\dot{V}_{s2}$ is negative-definite since β, ϕ and σ_0 are positive. Then, $\tilde{z}(t)$ go to zero exponentially fast and in Eq. (16), T_z approaches fast to $\hat{T}_z$. Therefore, since the tracking error $e(t)$ goes to zero exponentially, the effect of friction torque is almost disappeared and the observer-based SMC system maintains a good tracking performance under small scale of control input when comparing it with non-observer based SMC system. Fig. 4 indicates the block diagram of the proposed observer-based SMC system.

Remark 1: The observer in Eq. (24) and (25) is similar to the exponentially stable observer suggested by Vedagarbha, *et al.* [5], who proposed the following exponentially stable observer:

$$\hat{z} = p - \frac{J}{\sigma_1}\dot{q} \tag{29}$$

where p is an auxiliary variable and satisfy the following equation.

$$\dot{p} = \frac{1}{\sigma_1}\left[-\sigma_0 p + (-C_{eq} + \sigma_1 + J\frac{\sigma_0}{\sigma_1})\dot{q} + u + \chi(\dot{q})r\right] \tag{30}$$

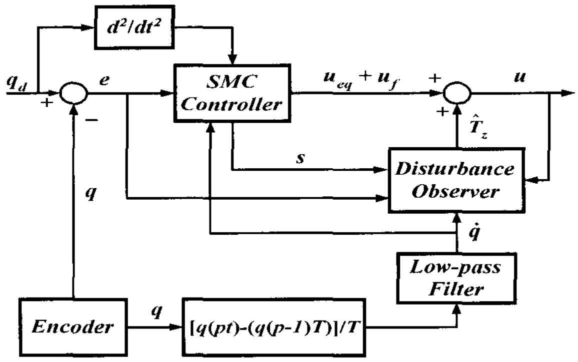

Fig. 4. Block diagram of the proposed observer-based SMC system

But since the transient response of the observer suggested by Vedagarbha, *et al.* is limited by the value of the dynamic friction parameter (i.e., σ_0, σ_1), the transient response cannot be adjusted independently. Thus they presented an observer that can improve the transient response [5]. An observer that estimates the state $z(t)$ with tunable transient response is given by

$$\hat{z} = p - \kappa_0 J \int_0^{\dot{q}} C(\dot{q})d\dot{q} \tag{31}$$

where $p(t)$ is the auxiliary variable and is updated according to

$$\begin{aligned}\dot{p} = &-\kappa_0 C(\dot{q})[C_{eq}\dot{q} + \chi(\dot{q})(p - \kappa_0 J\textstyle\int_0^{\dot{q}} C(\dot{q})d\dot{q}) - u] \\ &- f(\dot{q})(p - \kappa_0 J\textstyle\int_0^{\dot{q}} C(\dot{q})d\dot{q}) + \chi(\dot{q})r + \dot{q}\end{aligned}. \tag{32}$$

In Eq. (30) and (32), κ_0 is a tuning parameter and the filtered tracking error $r(t)$ is defined as follows[5]:

$$r = \dot{e} + \mathrm{L}^{-1}\left\{\frac{1}{s}K_F(s)e(s)\right\} \tag{33}$$

where L^{-1} denotes the inverse Laplace transform operation, s denotes the Laplace transform variable, and $K_F(s)$ denotes a linear filter which is selected to ensure that the transfer function given by

$$\frac{e(s)}{r(s)} = \frac{s}{s^2 + K_F(s)} \tag{34}$$

is strictly proper and exponentially stable. For example, a proportional integral derivative (PID) feedback law can be designed by defining $K_F(s)$ can be as follows:

$$K_F(s) = \alpha s + \beta \tag{35}$$

where α and β are a positive, scalar, constant control gains.

Remark 2: The observer of Eq. (31) and (32) suggested by Vedagarbha, *et al.*, has the limitation that $C(\dot{q})$ must be constructed to ensure the condition that $C(\dot{q})\chi(\dot{q}) \geq 0$ for guaranteeing exponential convergence of $\tilde{z}(t)$. Also the form of $C(\dot{q})$ is not unique since many different functions can be constructed to satisfy the requisite conditions and it is unknown how a particular choice of $C(\dot{q})$ from many different functions to satisfy the requisite conditions effects on the observer performance although Vedagarbha, *et al.* suggest that $C(\dot{q})$ can be constructed graphically only.

Remark 3: Our proposed observer has the tunable structure contrast to the observer of Eq. (29) and Eq. (30), which the transient response the observer is fixed by the value of system parameter. Also no limitation exists such as $C(\dot{q})\chi(\dot{q}) \geq 0$ for guaranteeing exponential convergence of $\tilde{z}(t)$ in the observer of Eq. (31) and Eq. (32). Simply tuning the gains k_1 and k_2 such as PD controller type, the transient response can be handled easily. As a result, our observer has the structure to combine two observer suggested by Vedagarbha, *et al.* without any limitation for convergence of the estimation error $\tilde{z}(t)$.

IV. Experimental results

1. System description

Fig. 5 shows the photograph of the ball-screw drive table system. The motion resolution of the ball-screw is 5mm pitch/rev. The LM guide supports the linear motion. In one side of axis, the DC servo motor is attached with the reduction gear of 20:1 of the gear ratio. The rotary incremental type of encoder reads the angular position with the resolution of *1000* pulse/rev.

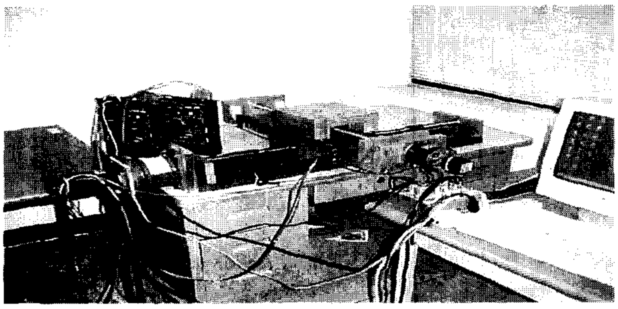

Fig. 5. Photograph of the ball-screw drive table system

In the computer, the control system is implemented by Simulink of Mathworks company and the TMS320C30 DSP system. The control signal is to send to the motor amplifier through the interface system to drive the servo motor. The torque constant of the motor is $2.27\,kg_f cm/A$ and the motor amplifier gain is obtained as $2.7273A/V$ by the experiment. Since the angular velocity cannot be measured directly, we obtain the angular velocity $\dot{q}$ applying the backward difference algorithm to the position signal from encoder as follows with resulting signal being filtered by the low pass filter:

$$\dot{q}_i = \frac{q_i(pT) - q_i[(p-1)T]}{T}, \quad p = 1, \cdots, n \qquad (36)$$

where p, T and nT denote respectively the sampling instance, sampling interval, and total experimental time.

2. Experiment on the SMC system without observer

In this section, we execute the position tracking experiment on the SMC system without observer. For the variable input command, $q_d = 0.62445 \sin(1.26t) \cdot 0.625445 \cdot \sin(0.21t)$ (rad) (maximum amplitude of $300\mu m$) and the design parameter $\lambda_1 = 30$, $\lambda_2 = 60$, $\phi = 2$ and $\beta = 10$, the command input and output are shown in Fig. 6 and tracking error is shown in

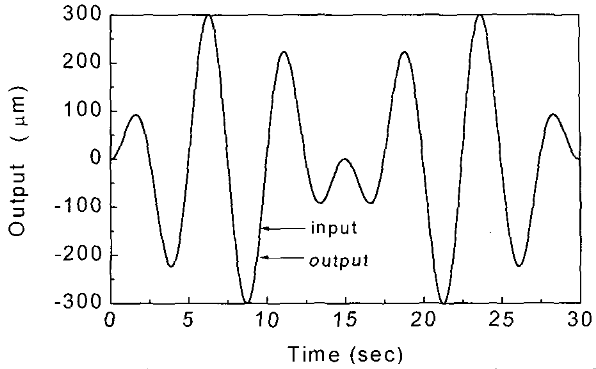

Fig. 6. The desired command input and output of the SMC system without the observer

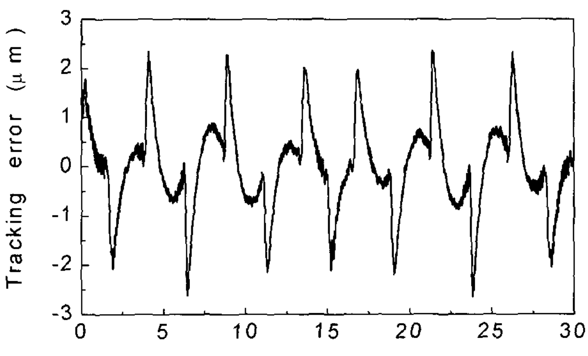

Fig. 7. Tracking error of the SMC system without the observer: $\lambda_1 = 30$, $\lambda_2 = 60$, $\phi = 2$ and $\beta = 10$

Fig. 7. The control input torque is shown in Fig. 8. In Fig. 6 and Fig 7, it is seen that the tracking performance seems to be well since the size of the tracking error with respect to the size of the magnitude command is relatively small. This reason is that the magnitude of the control input torque is much larger than the friction torque and thus friction effects on the tracking performance can be neglected. However, since the magnitude of control input torque is decreased if more precise position tracking is required, the relative tracking error can be increased. In order to show this, the same experiment is executed for the small variable input command $q_d = 0.062445 \cdot \sin(1.26t) \cdot 0.0625445 \sin(0.21t)$ (rad) (maximum amplitude of $3\mu m$) with the same values of design parameters. In this case, as shown in Fig. 9, the relative size of the tracking error with respect to size of the magnitude command is larger than the large command input case. To decrease the size of tracking error, we choose the design parameters as $\lambda_1 = 40$, $\lambda_2 = 80$, $\phi = 2$, $\beta = 36$ to enhance the robustness under the condition $|T_z| < \frac{\beta}{\phi}|s|$. In the results of Fig. 10 and 11, the size of tracking error is decreased than the previous SMC system, but the precise position tracking performance cannot be obtained yet and the magnitude of control input is increased greatly. Since the excessive control input may lead to wear or tear of actuator, this choice of control parameters is not appropriate in real application side.

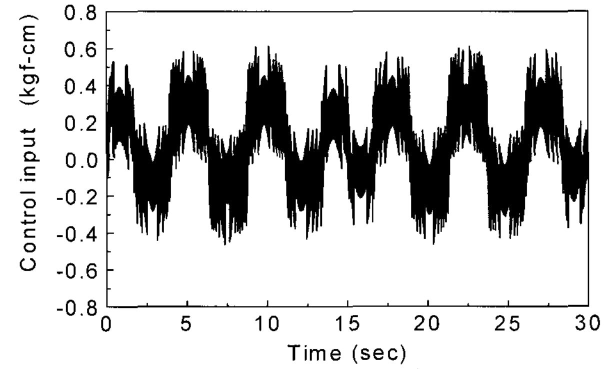

Fig. 8. Control input of the SMC system without the observer: $\lambda_1 = 30$, $\lambda_2 = 60$, $\phi = 2$ and $\beta = 10$

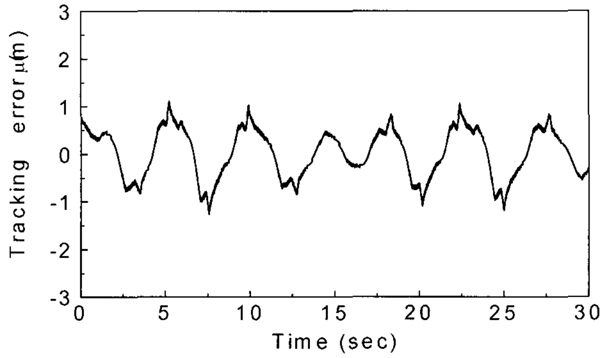

Fig. 9. Tracking error of the SMC system without the observer for a small input command: $\lambda_1 = 30$, $\lambda_2 = 60$, $\phi = 2$ and $\beta = 10$

3. Experiment on the SMC system with observer

For the SMC system with the proposed tunable observer, we execute the position tracking experiment for the ball-screw drive system. First, the small variable input command $q_d = 0.062445 \sin(1.26t) \cdot 0.0625445 \sin(0.21t)$ (rad) (maximum amplitude of $3 \mu m$) like Section 4.2 is selected with the design parameters $\lambda_1 = 30$, $\lambda_2 = 60$, $\phi = 2$, $\beta = 10$ and $k_1 = 0.3$ and $k_2 = 10^{-5}$ as tuning parameters. It is shown that the tracking error and control input are shown in Fig. 12 and Fig. 13. The tracking performance is improved under the proper size of control input than that of the SMC system without the observer. While the tracking error of the SMC system without observer in case of the increased values of design parameters is stable within about $\pm 0.17 \mu m$, the tracking error of the SMC system with the proposed observer is stable within about $\pm 0.05 \mu m$. The size of control input is much decreased as half level of that of the SMC system without observer in case of the increased values of design parameters. For the above input command, the plant friction and estimated friction are given in Fig. 14, where the results are obtained by computer simulation since the friction state $z(t)$ cannot be measured directly.

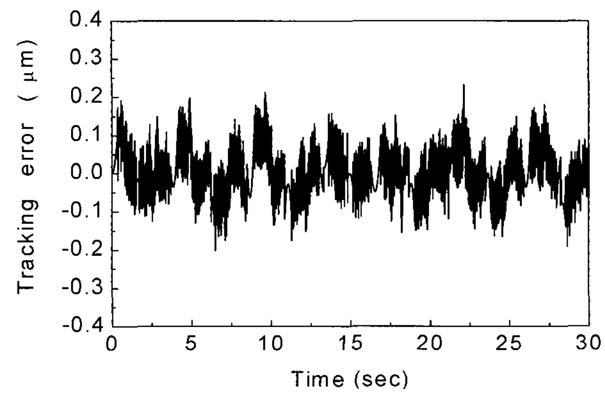

Fig. 10 Tracking error of the SMC system without observer for a small variable input command: $\lambda_1 = 40$, $\lambda_2 = 80$, $\phi = 2$ and $\beta = 36$

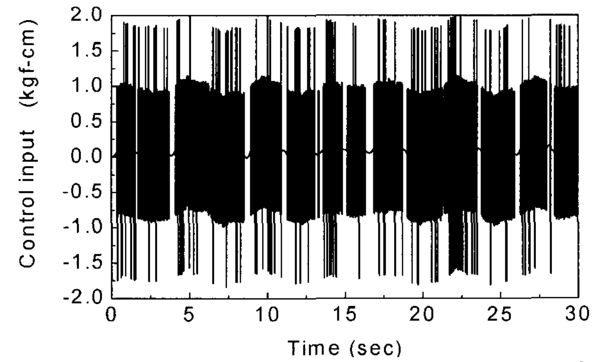

Fig. 11. Control input of the SMC system without observer for a small variable input command: $\lambda_1 = 40$, $\lambda_2 = 80$, $\phi = 2$ and $\beta = 36$

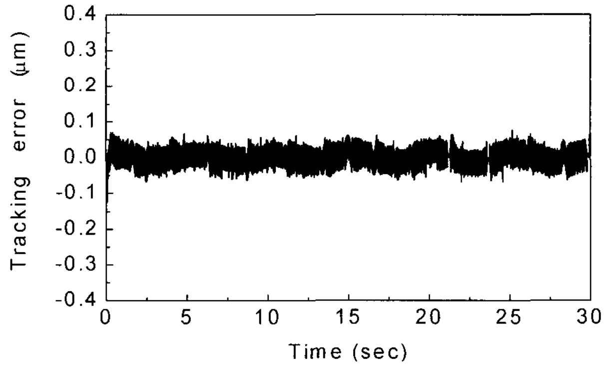

Fig. 12. Tracking error of the proposed SMC system with the tunable observer for a small variable input command: $\lambda_1 = 30$, $\lambda_2 = 60$, $\phi = 2$, $\beta = 10$, $k_1 = 0.3$ and $k_2 = 10^{-5}$

Next, let us compare the performance of our proposed observer with observer proposed by Vedagarbha, *et al.*. First, for two observers of Eq. (29) and Eq. (31) with controller of Eq. (35), the small variable input command $q_d = 0.062445 \cdot \sin(1.26t) \cdot 0.0625445 \sin(0.21t)$ (rad) (maximum amplitude of $3 \mu m$) is selected with the design parameters $\alpha = 30$, $\beta = 60$ and $\kappa_0 = 0.05$ to make similar experiment conditions of our proposed control system. For the fixed parameter observer system, the tracking error and control input are shown in Fig. 15 and Fig. 16. In Fig. 15 and Fig. 16, it is appeared that the tracking error and control input are a little high than those of our proposed observer system. For another tunable parameter observer system, the tracking error and control input are shown in Fig. 17 and Fig. 18, where the tracking and

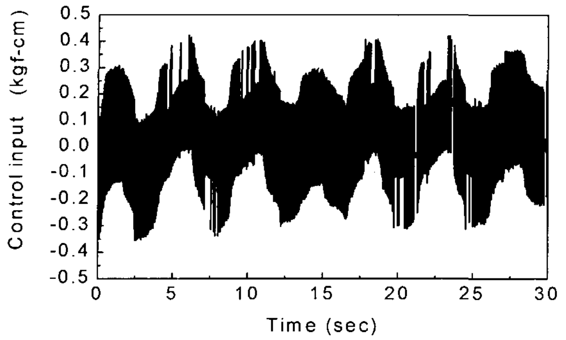

Fig. 13. Control input of the proposed SMC system with the tunable observer for a small variable input command: $\lambda_1 = 30$, $\lambda_2 = 60$, $\phi = 2$, $\beta = 10$, $k_1 = 0.3$ and $k_2 = 10^{-5}$

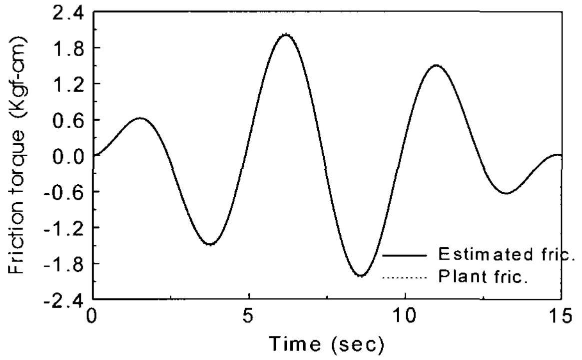

Fig. 14. The plant friction and estimated friction (simulation)

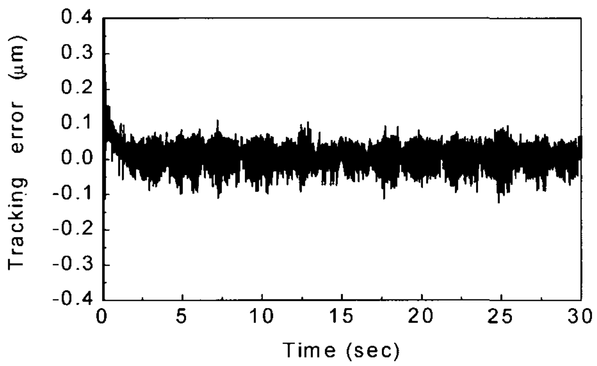

Fig. 15. Tracking error of the fixed parameter observer system for a small variable input command: $\alpha = 30$, $\beta = 60$

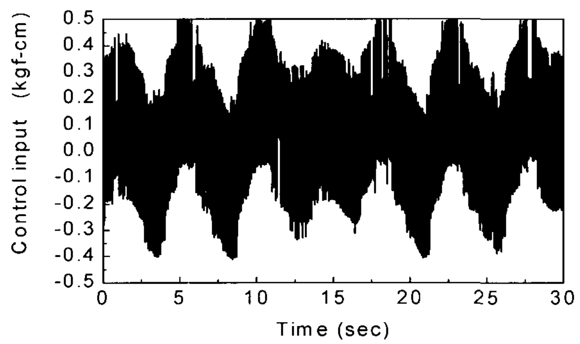

Fig. 16. Control input of the fixed parameter observer system for a small variable input command: $\alpha = 30$, $\beta = 60$

control input are similar to those of our proposed observer system. Thus, it can be seen that a steady state response and control input of our proposed observer are similar to those of observer proposed by Vedagarbha, *et al.*. However, a transient response will differ as the structure of observer according to tuning method of transient response. Next, this will be discussed.

For two observers proposed by Vedagarbha, *et al.*, the command input $q_d = 0.0062832\ \cos(1.25t)$ (rad) (maximum amplitude of $5\mu m$) is selected to examine transient response when $\alpha = 30$ and $\beta = 60$ are fixed. The transient outputs of the observer system of fixed parameter in Eq. (29) are shown in Fig. 19. But the overshoot is very high and this overshoot cannot be adjusted by the observer itself without changing control parameter α and β. Then for the observer of Eq. (31), which can adjust the transient response, the same experiment is executed with respect to variations of tuning parameter κ_0. In Fig. 20, the transient response can be adjusted according to the value of κ_0, but the overshoot is high as before. For our proposed observer system, the same input commandas the previous case is selected to examine transient response for fixed value of $\lambda_1 = 30$ and $\lambda_2 = 60$. In Fig. 21, it is shown that transient response can be adjusted by tuning proportional gain k_1, while k_2 is fixed. In Fig. 21, the overshoot is not occurred, but the rising time is a little longer than

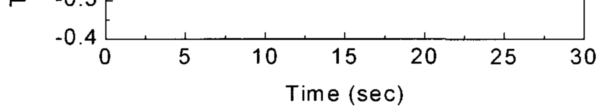

Fig. 17. Tracking error of the tunable parameter observer system for a small variable input command: $\alpha = 30$, $\beta = 60$ and $\kappa_0 = 0.05$

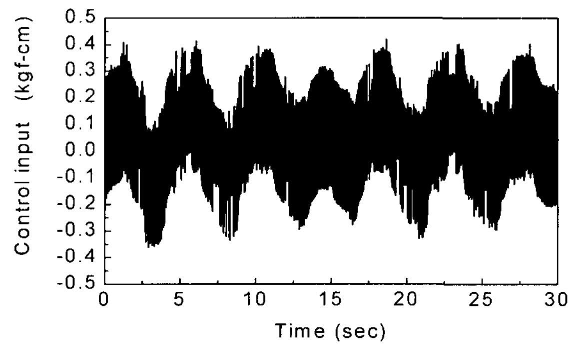

Fig. 18. Control input of the tunable parameter observer system for a small variable input command: $\alpha = 30$, $\beta = 60$ and $\kappa_0 = 0.05$

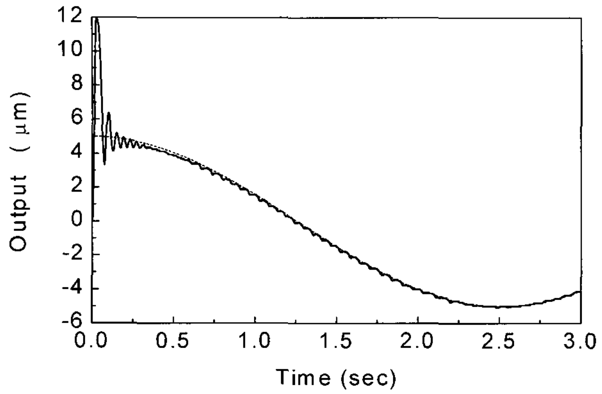

Fig. 19. Transient responses of fixed parameter observer system: $\alpha = 30$ and $\beta = 60$

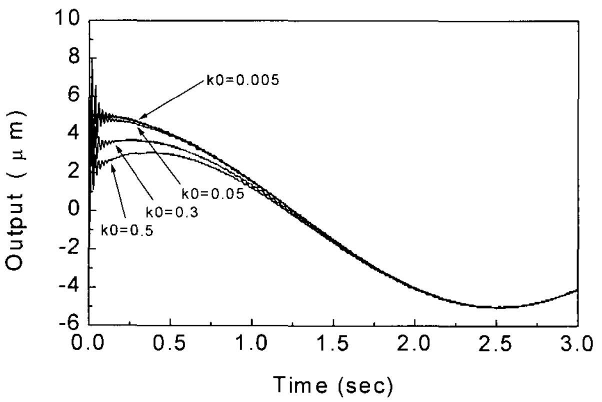

Fig. 20. Transient responses of tunable parameter observer-system as variations of $\kappa_0 : \alpha = 30$ and $\beta = 60$

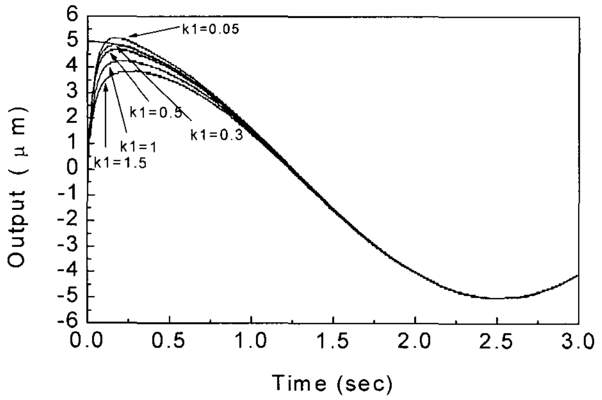

Fig. 21. Transient responses of the proposed SMC system with the tunable observer as variations of $k_1 : \lambda_1 = 30$, $\lambda_2 = 60$, $\phi = 2$, $\beta = 10$

two observers proposed by Vedagarbha, *et al.* But this time can be shorting by increasing value of λ_1 as shown in Fig. 22 without increasing control input excessively higher. This fact can be shown in Fig. 23 and Fig. 24. In Fig. 23 and Fig. 24, the magnitude of the transient response of control input for our proposed observer system with $k_1 = 0.3$, $\lambda_1 = 90$ is smaller than the tunable observer proposed by Vedagarbha, *et al.* with $\kappa_0 = 0.05$, $\alpha = 30$. Thus, adjusting transient response can be more flexibly done in our proposed method under mild control input than the tunable observer proposed by Vedagarbha, *et al.*.

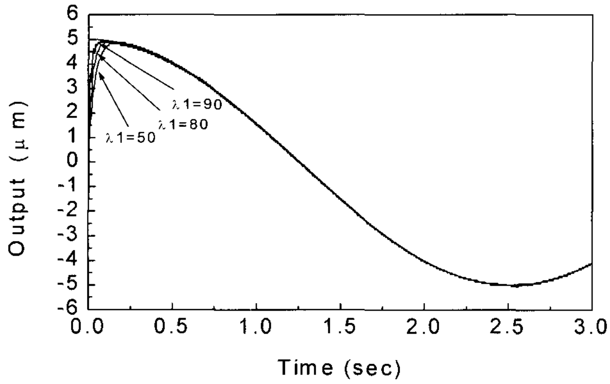

Fig. 22. Transient responses of the proposed SMC system with the tunable observer as variations of $\lambda_1 : k_1 = 0.3$

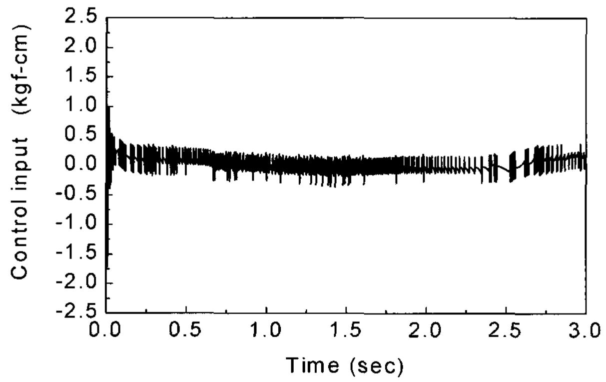

Fig. 23. Control input of the proposed SMC system with the tunable observer for a cosine input command: $k_1 = 0.3$, $\lambda_1 = 90$

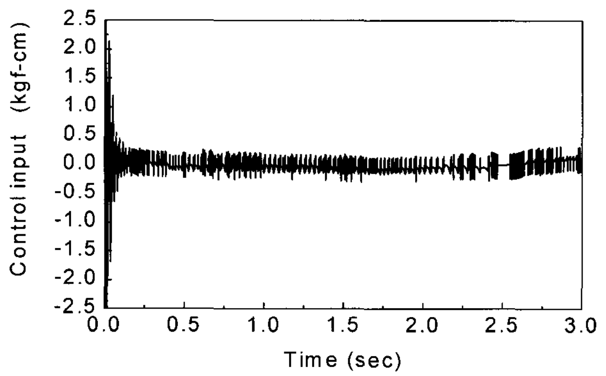

Fig. 24. Control input of the observer system proposed Vedagarbha, *et al.* for a cosine input command: $\kappa_0 = 0.05$, $\alpha = 30$

V. Conclusion

For a precise servo system in the presence of nonlinear dynamic friction, a tunable observer-based sliding mode control scheme is proposed. Taking nonlinear dynamic friction as a bristle friction model, the parameters of bristle friction model are estimated by experiment identifications. When a precise level of position tracking is required, a conventional SMC system without friction observer cannot guarantee a good position tracking performance despite of increasing control parameter related to robustness for uncertainty and it may cause the control input excessively. Thus, we propose the SMC system combined with friction model-based observer with simple tunable structure such as PD controller type to improve the transient response as well as position tracking performance under a mild control input. To show the effectiveness of our proposed control system, the experiment on the ball-screw drive table is executed via DSP system. The experimental results show that the precise position tracking performance is accomplished and the transient response of output can be improved by tuning parameters of the observer when comparing with a similar observer proposed Vedagarbha, *et al.*.

Therefore, our proposed control scheme can be applied to control a precise servo system in the presence of the dynamic friction with the capability of adjusting transient response as well as satisfying precise tracking position performance. As a future research, a design of observer that can be estimated the friction state and velocity simultaneously will be needed since the velocity information obtained by the backward difference algorithm is weakly exact than directly obtained velocity information.

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