Robust Predictive Control of Uncertain Nonlinear System With Constrained Input

Won Kee Son, Jin Young Choi, and Oh Kyu Kwon

Abstract: In this paper, a linear matrix inequality(LMI)-based robust control method, which combines model predictive control(MPC) with the feedback linearization(FL), is presented for constrained nonlinear systems with parameter uncertainty. The design procedures consist of the following 3 steps: Polytopic description of nonlinear system with a parameter uncertainty via FL, Mapping of actual input constraint by FL into constraint on new input of linearized system, Optimization of the constrained MPC problem based on LMI. To verify the performance and usefulness of the control method proposed in this paper, some simulations with application to a flexible single link manipulator are performed.

Keywords: robust predictive control, feedback linearization, parameter uncertainties, flexible single link manipulator, input constraint

I. Introduction

Recently, the differential geometric control methods have led to a new class of control technique for nonlinear systems which are affine in the control inputs[1]-[7]. Also, this method has proved to be a very effective for transforming a nonlinear system into linear one without using standard Jacobian linearization. Basically, feedback linearization(FL) scheme uses nonlinear feedback to cancel out any nonlinearity in the input-output relation of system. In addition, once a nonlinear system is linearized by feedback, then all of linear control methods can be directly applied to the new linearized system. However, one of major disadvantages of FL is a lack of uncertainty handling ability. Uncertainties can lead to performance deterioration as well as instability of system, if not properly accounted for in the control design procedure. Therefore, to overcome this problem, it is important to design a robust controller taking the uncertainty into consideration for the efficient control system design and performance improvement. In this paper for the bounded nonlinear state-dependent uncertain terms, a system description that the nonlinear system with parameter uncertainties is transformed into polytopic systems by nonlinear feedback, is presented.

Also, most of physical systems have limitations to the amplitude that can be made to the manipulated variables. Therefore, a control algorithm should be designed to have the ability to account for such limitations. To fulfill overall operational requirements, it is necessary to explicitly consider all constraints in the formulation of the control cost function. In this paper, to preserve the linearity established by FL, a constraint mapping method[8], [9] is presented to deal with nonlinear state-dependent constraint transformed by FL and, an LMI-based method[10] is also proposed to handle these input constraints in the controller design procedure. To take account of such constraints, the IHRMPC (Infinite Horizon Robust Model Predictive Control)[11] is adopted as a control technique. From these results, the MPC problem of minimizing an upper bound on robust performance objective function over the infinite horizon which is subject to constraint on the input of linearized system, is reduced to a convex LMI-based optimization problem. The configuration of overall closed loop system for control law with feedback linearization is shown in Fig. 1.

The layout of the paper is as follows: In section II, the description procedure that a uncertain nonlinear system is transformed into polytopic systems is given. In section III, the LMI-based model predictive control law for the constrained uncertain nonlinear system is presented. Also, the constraint mapping method transforming original input constraint into feedback linearized input constraint is given. To exemplify the performance of the control scheme some simulations with application to flexible single link manipulator are performed in section IV. Finally, conclusions are summarized in section V.

II. Polytopic systems via FL

Consider a single-input single-output(SISO) nonlinear system with parameter uncertainty as follows.

$$\begin{aligned} \dot{x} &= f(x,\theta) + g(x,\theta)u, \\ &= f_0(x) + g_0(x)u + \Delta f(x,\theta) + \Delta g(x,\theta)u, \\ y &= h(x), \end{aligned} \quad (1)$$

where $x \in R^n$ is the states, $u \in R$ the input, $y \in R$ the output, θ is a uncertain parameter vector in a compact set, and $f_0(\cdot), g_0(\cdot)$ represent the nominal parts of uncertain nonlinear system, and $\Delta f(\cdot,\theta), \Delta g(\cdot,\theta)$ represent the uncertain terms. For all θ, it is assumed that $f_0(\cdot), g_0(\cdot), \Delta f(\cdot,\theta)$ and $\Delta g(\cdot,\theta)$ are smooth vectors and $h(x)$ is a smooth function. The following lemma 2 and 3 show that the uncertain nonlinear system(1) can be linearized by the nominal feedback law and co-ordinate transformation based on nominal parameters($\theta = \theta_0$), and its configuration is shown in Fig. 2.

Lemma 1 : Consider the system (1). Assume that

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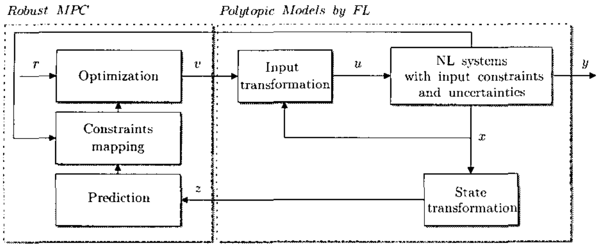


Fig. 1. Configuration of LMI-based RMPC using FL.

1. the nominal part of the system (1) is fully feedback linearizable, i.e., the relative degree $r = n$.
2. the uncertain term $\Delta g(\cdot, \theta)$ satisfies that

$$L_{\Delta g} L_{f_0}^{i} h(\cdot) = 0 \text{ for } 1 \le i \le n-2. \tag{2}$$

Then, by the nominal input-output linearization, the uncertain nonlinear system (1) is transformed as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 + \Delta\bar{\alpha}_1(x,\theta) \\ &\vdots \\ \dot{z}_n &= \Delta\tilde{\alpha}(x,\theta) + (1 + \Delta\tilde{\beta}(x,\theta))v. \end{aligned} \tag{3}$$

Proof : Firstly, by differentiating the nominal co-ordinate transformation

$$z = T_s(x, \theta_0) = \begin{bmatrix} h(x) \\ L_{f_0} h(x) \\ \vdots \\ L_{f_0}^{n-1} h(x) \end{bmatrix}, \tag{4}$$

and by assumptions of lemma 1, the system(1) can be written as

$$\begin{aligned} \dot{z}_i &= z_{i+1} + \Delta\bar{\alpha}_i(x,\theta), \quad 1 \le i \le n-1, \\ \dot{z}_n &= \bar{\alpha}(x) + \bar{\beta}(x)u + \Delta\bar{\alpha}_n(x,\theta) + \Delta\bar{\beta}(x,\theta)u, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \bar{\alpha}(x) &= L_{f_0}^{n} h(x), \bar{\beta}(x) = L_{g_0} L_{f_0}^{n-1} h(x), \\ \Delta\bar{\alpha}_i(x,\theta) &= L_{\Delta f} L_{f_0}^{i-1} h(x), \\ \Delta\bar{\beta}(x,\theta) &= L_{\Delta g} L_{f_0}^{n-1} h(x). \end{aligned} \tag{5.a}$$

Secondly, by choosing the nominal feedback law as below to cancel the nominal nonlinearity in (5)

$$u = \frac{-\bar{\alpha}(x) + v}{\bar{\beta}(x)}, \tag{6}$$

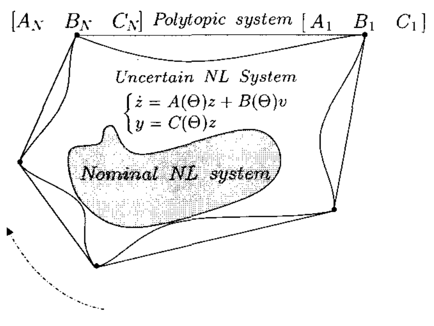


Fig. 2. Polytopic models.

we have

$$\begin{aligned} \dot{z}_i &= z_{i+1} + \Delta\bar{\alpha}_i(x,\theta), \quad 1 \le i \le n-1, \\ \dot{z}_n &= \Delta\tilde{\alpha}(x,\theta) + (1 + \Delta\tilde{\beta}(x,\theta))v, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \Delta\tilde{\alpha}(x,\theta) &= \Delta\bar{\alpha}_n(x,\theta) - \Delta\bar{\beta}(x,\theta)\frac{\bar{\alpha}(x)}{\bar{\beta}(x)} \\ \Delta\tilde{\beta}(x,\theta) &= \frac{\Delta\bar{\beta}(x,\theta)}{\bar{\beta}(x)}. \end{aligned} \tag{7.a}$$

Hence, by (4) and (6) the uncertain nonlinear system (1) is transformed to feedback linearized system(7) with uncertain terms.

However, we can observe that the feedback linearized system of (7) is still nonlinear. The uncertain terms $\Delta\bar{\alpha}_i(\cdot,\theta), \Delta\tilde{\alpha}(\cdot,\theta)$ and $\Delta\tilde{\beta}(\cdot,\theta)$ are nonlinear functions of the states x and uncertain parameter θ. For the use of linear robust control techniques these nonlinearities have to be linearized. Furthermore, these uncertain terms occur at levels of differentiation different from that of the control input v, i.e., these uncertain terms do not satisfy the matching condition. Therefore, as shown in lemma 2, we overcome these restrictive conditions by characterizing the uncertainties in a suitable form to design linear robust controller.

Lemma 2 : Consider the system (3). If the uncertain terms satisfy

$$\begin{aligned} \Delta\bar{\alpha}_i(x,\theta) &= M_i(x,\theta)z, \ 1 \le i \le n-1, \\ \Delta\tilde{\alpha}(x,\theta) &= M_n(x,\theta)z, \\ |\Delta\tilde{\beta}(x,\theta)| &\le |\theta\delta_{\tilde{\beta}}|, \end{aligned} \tag{8}$$

where $M_i(x,\theta), M_n(x,\theta)$ are row vectors with bounded elements and are affine in θ, and $\delta_{\tilde{\beta}}$ is a constant value, then the transformed system (3) by feedback can be reduced to polytopic systems in the form as follows:

$$\dot{z} = A(\theta)e + B(\theta)v. \tag{9}$$

Proof : Using (3) and (8), (9) is easily obtained. ■

If the uncertain parameter vector θ is defined as polytope given by

$$\theta \in \text{Co}\{\theta^i\} = \sum_{i=1}^{N} \alpha_i \theta^i, \quad \alpha_i \ge 0, \ \sum_{i=1}^{N} \alpha_i = 1, \tag{10}$$

where θ^i denotes vertex vectors of θ, Co denotes convex hull, and $N = 2^\kappa$, κ is the number of uncertain parameters, then, the linearized polytopic systems (9) can be represented as a convex combination of the vertices of system matrices. From assumption of Lemma 2, $A(\theta), B(\theta)$ is affine in θ. Hence,

$$\begin{aligned} A(\theta) &\in \sum_{i=1}^{N} \alpha_i A(\theta^i) = \alpha_1 A_1 + \cdots + \alpha_N A_N \\ &= \text{Co}\{A_1, \cdots, A_N\} \\ B(\theta) &\in \sum_{i=1}^{N} \alpha_i B(\theta^i) = \alpha_1 B_1 + \cdots + \alpha_N B_N \\ &= \text{Co}\{B_1, \cdots, B_N\}. \end{aligned} \tag{11}$$

As a result of the nominal co-ordinate, input transformation and the characterization of uncertainties, we can obtain convex combinations of the linearized polytopic system for $A(\theta), B(\theta)$ as follows:

$$[A(\theta)\ \ B(\theta)] \in \Omega_c = \mathrm{Co}\{[A_1\ \ B_1], \cdots, [A_N\ \ B_N]\}. \quad (12)$$

Since the control law involves the discretization for discrete controller design, the continuous-time vertex models of (12) need to be expressed in discrete-time. By selecting an appropriate sampling interval that maintains feedback linearizability in the discrete-time domain, (12) can be discretized using a Euler method. The discretized polytopic models of (12) is given as follows:

$$z(k+1) = A_d(\theta)z + B_d(\theta)v, \quad (13)$$

where

$$[A_d(\theta)\ \ B_d(\theta)] \in \Omega_d = \mathrm{Co}\{[A_{d,1}\ \ B_{d,1}], \cdots, [A_{d,N}\ \ B_{d,N}]\}.$$

The input and co-ordinate transformation is performed at each sampling time using the discrete controller and current sampled states. Based on the vertex systems in (13), it is possible to synthesize linear robust controller to realize performance specifications such as stabilization.

III. Robust predictive control law

1. Unconstrained case

Consider the discretized polytopic systems via feedback linearization as follows:

$$\begin{aligned} z(k+1) &= A_{d,i}z(k) + B_{d,i}v(k), \\ y &= C_d z(k), \\ [A_{d,i}\ B_{d,i}] &\in \Omega_d, \quad i \geq 0. \end{aligned} \quad (14)$$

The MPC problem in this paper is to obtain state feedback controller K_k which minimizes, at each sampling time k, a robust performance objective over infinite prediction horizon as follows:

$$\min_{v(k+i|k), i=0,1,\ldots,m} \quad \max_{[A_{d,i}\ B_{d,i}] \in \Omega_d, i \geq 0} \quad J_\infty(k), \quad (15)$$

with

$$J_\infty(k) = \sum_{i=0}^{\infty} \left(z(k+i|k)^T Q_w z(k+i|k) + v(k+i|k)^T R v(k+i|k) \right),$$

where $Q_w > 0, R > 0$ are symmetric weighting matrix and factor, m is control horizon, $z(k+i|k)$ is state at time $k+i$, predicted based on the measurements at time k, and $v(k+i|k)$ is control input at time $k+i$ computed by optimization problem (15) at time k, respectively. Above min-max problem(15) can be solved in the following two steps: Firstly, an upper bound on the robust performance objective is derived, and then minimize this upper bound with a constant state feedback control law $v(k+i|k) = K_k z(k+i|k), i \geq 0$. It is assumed that exact measurement of the state of the system is available at each sampling time k, i.e. $z(k|k) = z(k)$.

Consider a quadratic function $V(z) = z^T P z, P > 0$ of the state $z(k|k) = z(k)$ of the system (14) with $V(0) = 0$. From the result of [11], we obtain

$$-V(z(k|k)) \leq -J_\infty(k).$$

Thus,

$$\max_{[A_{d,i}\ B_{d,i}\ C_{d,i}\ 0] \in \Omega_d, i \geq 0} J_\infty(k) \leq V(z(k|k)). \quad (16)$$

This gives an upper bound on the performance objective(for details, see [11]). From (16), the control problem in (15) is equivalent to

$$\min_{v(k+i|k), i=0,1,\ldots,m} V(z(k|k)). \quad (17)$$

Thus, the goal of robust predictive control law via feedback linearization is redefined to synthesize, at each time step k, a constant state-feedback control law $v(k+i|k) = K_k z(k+i|k)$ to minimize this upper bound $V(z(k|k))$. As is standard in MPC, only first computed control $v(k|k) = K_k z(k|k)$ is implemented. At the next sampling time the state $z(k+1)$ is measured or calculated, and the optimization is repeated to recompute K_k. Conditions for existence of state feedback gain K_k in minimization problem(17), are derived from the following theorem.

Theorem : 1 ([11]) Let $z(k) = z(k|k)$ be the state of the uncertain system (14) measured at sampling time k. Suppose the uncertainty set Ω_d is defined by a polytopic system as in (14). Then, the state feedback matrix K_k in the control law $v(k+i|k) = K_k z(k+i|k),\ i \geq 0$ which minimizes the upper bound $V(z(k|k))$ on the robust performance objective function at sampling time k is given by

$$K_k = YQ^{-1}, \quad (18)$$

where $Q > 0$ and Y are obtained from the solution to the following linear objective minimization problem:

$$\min_{\gamma, Q, Y} \gamma \quad (19)$$

subject to

$$\begin{bmatrix} 1 & z(k|k)^T \\ z(k|k) & Q \end{bmatrix} \geq 0 \quad (20)$$

and for $i = 1, \ldots, N$

$$\begin{bmatrix} Q & QA_{d,i}^T + Y^T B_{d,i}^T & QQ_w^{1/2} & Y^T R^{1/2} \\ A_{d,i}Q + B_{d,i}Y & Q & 0 & 0 \\ Q_w^{1/2}Q & 0 & \gamma I & 0 \\ R^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0. \quad (21)$$

2. Constrained case

Consider the system (1) with hard constraint on original input

$$u_{lb} \leq u \leq u_{ub}, \quad (22)$$

where u_{lb}, u_{ub} are lower and upper bound, respectively. Then, we can define a new control variable v as an input of linearized system. Introducing the nominal co-ordinate and input transformation and discretizing the continuous-time polytopic system(9), we can define a new discrete linearized system as (14). Note that (14) holds the linear properties only if the constraint condition on its input for $0 \leq j \leq m$ is satisfied

$$v_{lb}(k+j|k) \leq v(k+j|k) \leq v_{ub}(k+j|k), \quad (23)$$

where $v(k+j|k)$ is the value of the input $v(k+j)$ computed at time step k, $v_{lb}(k+j|k)$ and $v_{ub}(k+j|k)$ are the constraints computed at time step k, respectively. The new input v is related to the actual control input u through a mapping of nonlinear scalar functions $\bar{\alpha}(x), \bar{\beta}(x)$, as given by (6). It is clear that, due to the feedback linearization, original hard constraint (u_{lb}, u_{ub}) are mapped into the MPC constraint $(v_{lb}(\cdot), v_{ub}(\cdot))$ on v which is in general nonlinear and state dependent. The input constraint mapping is performed using the feedback linearization law(24) and the current state measurement $x(k)$. This mapping can be written as follows:

$$v(k) = \bar{\alpha}(x(k)) + \bar{\beta}(x(k))u(k). \tag{24}$$

For $0 \le j \le m$, the transformed constraints at time step k are determined by solving the following optimization problem

$$\begin{aligned} v_{lb}(k+j|k) &= \min_{u(k+j|k)} v(k+j|k) \\ v_{ub}(k+j|k) &= \max_{u(k+j|k)} v(k+j|k) \end{aligned} \tag{25}$$

subject to original hard constraints

$$u_{lb} \le u(k+j|k) \le u_{ub}. \tag{26}$$

Because exact mapping of future input constraints is impractical, it is necessary to approximate the constraints $v_{lb}(k+j|k)$ and $v_{ub}(k+j|k)$ for $j \ge 1$. The predicted values of the transformed state variable is obtained.

$$\begin{aligned} \hat{z}(k+j|k) &= A_{d,i}\hat{z}(k+j-1|k) + B_{d,i}v(k+j-1|k-1) \\ \hat{z}(k|k) &= z(k). \end{aligned} \tag{27}$$

The state sequences and the inverse transformation $T_s^{-1}(\hat{z})$ are used to compute future values of the state vector. The solution of optimization problem in (25) yields the transformed constraints. These variable constraints are used in the linear MPC design. The procedure is repeated at the next time step with the input sequences and the measurement $x(k+1)$. Next, we show how constraint on input can be incorporated into control algorithm as sufficient LMI constraints. The basic idea of this problem can be found in *Boyd et al.*[10]. Let $\mathcal{S}$ define the ellipsoid as follows:

$$\mathcal{S} = \{\xi \in R^n | \xi^T Q^{-1} \xi \le 1\}. \tag{28}$$

If there exists Q such that (21) holds, and $z(k|k) \in \mathcal{S}$, then the predicted states $z(k+j|k)$ of uncertain system belong to $\mathcal{S}$ for all j [10], [11]. Thus, $\mathcal{S}$ is an invariant ellipsoid for the predicted states $z(k+j|k)$. At time step k, consider the Euclidean norm constraint on input

$$\|v(k+j|k)\| \le v_b(k), \quad j \ge 0. \tag{29}$$

The constraints $v_b(k)$ are computed as follows, at time step k, by solving the optimization problem in (25)

$$v_b(k) = \min(|v_{lb}(k+j|k)|, |v_{ub}(k+j|k)|). \tag{30}$$

The constraint $v_b(k)$ is imposed on the present and the entire horizon of future manipulated variables, although only the first control move $v(k|k) = v(k)$ is implemented. From [10], we have

$$\begin{aligned} \max_{j\ge 0} \|v(k+j|k)\| &= \max_{j \ge 0} \|(YQ^{-1}z(k+j|k))\| \\ &\le \max_{\xi \in \mathcal{S}} \|(YQ^{-1}\xi)\| \\ &= \bar{\sigma}(YQ^{-1/2}) \end{aligned} \tag{31}$$

Thus, if

$$\bar{\sigma}(YQ^{-1/2}) \le v_b(k), \tag{32}$$

or

$$(Q^{-1/2}Y^TYQ^{-1/2}) \le v_b^2(k)I, \tag{33}$$

then $\|v(k+j|k)\| \le v_b(k)$, $j \ge 0$ for any $[A_{d,i}\ B_{d,i}] \in \Omega_d$, $i \ge 0$. Using Schur compliment after dividing both side of (33) by $v_b^2(k)$ and multiplying on left and right by $Q^{1/2}$, we obtain

$$\begin{bmatrix} Q & Y^T \\ Y & v_b^2(k)I \end{bmatrix} \ge 0. \tag{34}$$

Corollary 1: Suppose that the assumption in theorem 1 is satisfied. Then, the state feedback matrix K_k in the control law $v(k+i|k) = K_k z(k+i|k)$, $i \ge 0$ which minimizes the upper bound $V(z(k|k))$ on the robust performance objective function and satisfies a specified original input constraint at sampling time k is given by

$$K_k = YQ^{-1}, \tag{35}$$

where $Q > 0$ and Y are obtained from the solution to the following linear objective minimization problem:

$$\min_{\gamma, Q, Y} \gamma \tag{36}$$

subject to (20), (21) and (34).

IV. Simulations

A flexible single link manipulator system with a parameter uncertainty and input constraint is considered as a design example and its mechanism is given in Fig. 3. Consider the differential equation for the flexible single link manipulator with uncertainty in parameter I given by

$$\begin{aligned} \dot{x} &= f_0(x) + g_0(x)u + \Delta f(x,\theta) + \Delta g(x,\theta)u, \\ y &= h(x), \end{aligned} \tag{37}$$

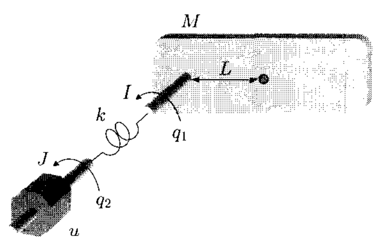


Fig. 3. Flexible single link manipulator.

with

$$
\begin{aligned}
f_0(x) &= \begin{bmatrix} x_2 \\ -\frac{1}{I}(MgL\sin x_1 + k(x_1 - x_3)) \\ x_4 \\ \frac{k}{J}(x_1 - x_3) \end{bmatrix}, \\
g_0(x) &= \begin{bmatrix} 0 & 0 & 0 & \frac{1}{J} \end{bmatrix}^T, \quad h(x) = x_1, \\
\Delta f(x,\theta) &= \begin{bmatrix} 0 \\ \frac{\theta}{I}(MgL\sin x_1 + k(x_1 - x_3)) \\ 0 \\ 0 \end{bmatrix}, \\
\Delta g(x,\theta) &= 0,
\end{aligned} \tag{38}
$$

where x_1, x_3 are the angles of the link q_1 and motor shaft q_2, u is torque applied to the motor shaft, J, I are inertia moments of the motor and the link, L is distance from the motor shaft to the center of mass of the link, M is mass of the link, g is acceleration due to gravity, k is torsional spring constant. The parameter θ is an uncertainty factor used to represent changeable inertia moment I of the link. And it is assumed that $-9 \leq \theta \leq 0.4737$ corresponding to $-90\% \sim 90\%$ uncertainty in I. The set parameters are chosen in this simulation as follows: $I = 0.04[Nms^2/rad], J = 0.04[Nms^2/rad], k = 0.8[Nm/rad], L = 0.1[m], M = 6[kg], g = 9.8[m/s^2]$. And the input torque u is limited $|u| \leq 20[Nm]$. Using the nominal feedback linearization in (4),(6) for the system in (37), we have

$$
\begin{aligned}
z = T_s(x,0) &= \begin{bmatrix} x_1 \\ x_2 \\ -\frac{1}{I}(MgL\sin x_1 + k(x_1 - x_3)) \\ -\frac{1}{I}(MgLx_2\cos x_1 + k(x_2 - x_4)) \end{bmatrix}, \\
u &= \frac{-\bar{\alpha}(x) + v}{\bar{\beta}(x)},
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\alpha}(x) &= \frac{MgL}{I} x_2^2 \sin x_1 + \frac{k^2}{IJ}(x_1 - x_3) \\
&\quad + \frac{1}{I^2}(MgL\cos x_1 + k)(MgL\sin x_1 + k(x_1 - x_3)) \\
\bar{\beta}(x) &= \frac{k}{IJ}.
\end{aligned}
$$

From $(5.a)$ and $(7.a)$, we have

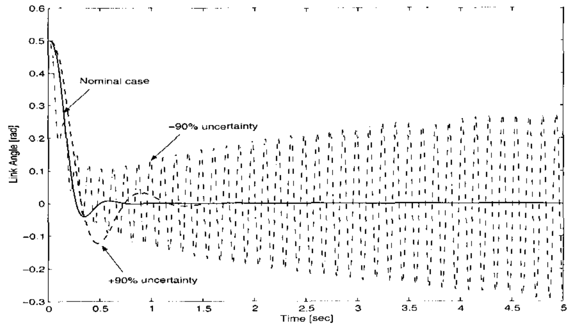
$$
\begin{aligned}
&\Delta\bar{\alpha}_1(x,\theta) = 0, \Delta\bar{\alpha}_2(x,\theta) = -\theta z_3, \Delta\bar{\alpha}_3(x,\theta) = 0, \\
&\Delta\tilde{\alpha}(x,\theta) = \frac{\theta}{I}(MgL\cos z_1 + k)z_3, \\
&\Delta\tilde{\beta}(x,\theta) = 0.
\end{aligned} \tag{39}
$$

As a result of feedback linearization, the original uncertain nonlinear system is expressed to uncertain polytopic systems as follows.

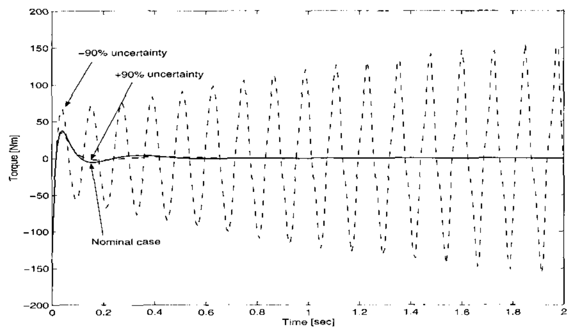
$$
\begin{aligned}
\dot{z}_1 &= z_2, \quad \dot{z}_2 = (1-\theta)z_3 \\
\dot{z}_3 &= z_4, \quad \dot{z}_4 = \frac{\theta}{I}(MgL\cos z_1 + k)z_3 + v,
\end{aligned}
$$

or

$$
\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1-\theta & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{\theta}{I}(MgL+k) & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v. \tag{40}
$$

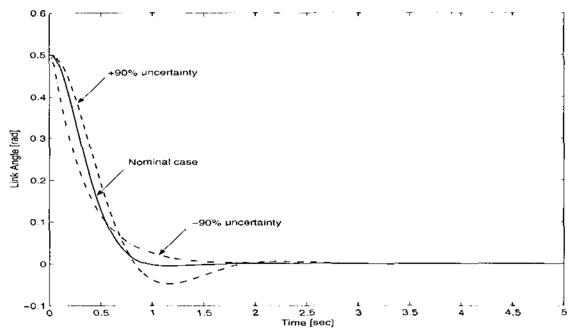


(a) Link angle

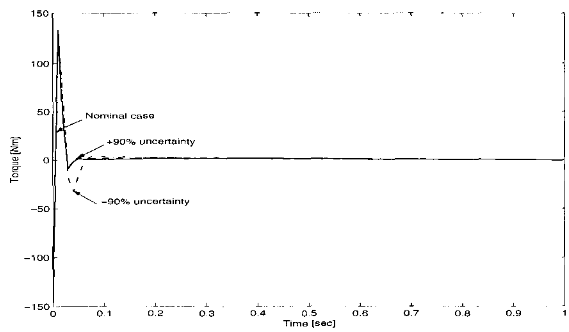


(b) Input torque

Fig. 4. The performances of controller based on nominal model.



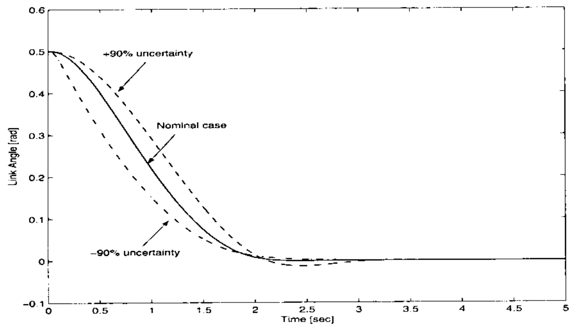
(a) Link angle



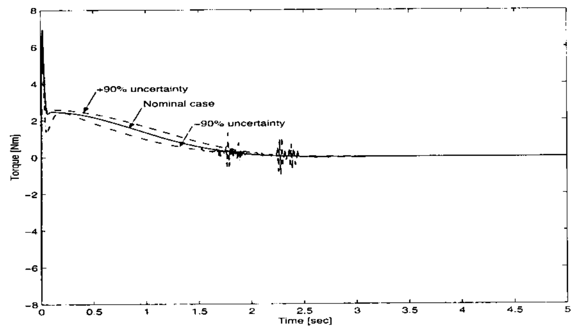
(b) Input torque

Fig. 5. The performances of unconstrained robust controller based on Theorem 1.

And the polytopic system(40) in continuous-time is discretized with sampling time $0.01[sec]$ to design the controller. By (25) and (30), the hard constraint (u_{lb}, u_{ub}) on input u is mapped to constraint $v_b(k)$ on input v of linearized system at the time step k. With $Q_w = I_{4\times4}, R = 0.01$, all LMIs-related computations



(a) Link angle



(b) Input torque

Fig. 6. The performances of constrained robust controller based on Corollary 1.

are performed using LMI control toolbox[12].

The effect of uncertainties on regulation performances for the case of controller based on only nominal model and for the case of controller based on polytopic models are shown from Fig. 4 to Fig. 6, respectively. We can see from Fig. 4 that the control performances are degraded significantly unless the uncertainty is considered to the system. Fig. 6 shows that the input torque u satisfies the constraint well on the constrained input. We can clearly see that if the constraint on input is not considered in the control law, then the improved performance cannot be obtained.

V. Conclusions

In this paper, based on convex optimization with LMIs (Linear Matrix Inequalities) and FL (Feedback Linearization), robust feedback linearizing predictive control scheme for fully feedback linearizable nonlinear systems is proposed for regulation problem. The main contribution of this paper is to consider the problem of control of a nonlinear system with uncertain parameters and input saturation. First a linearized system with uncertain terms via Feedback Linearization is characterized and discretized in uncertain polytopic systems. The constraint mapping scheme is also applied to relation between original input $u(\cdot)$ of nonlinear system and new input $v(\cdot)$ of linearized and discretized system to preserve the linearity established by Feedback Linearization. The stability of controlled system with uncertain parameter and input constraint is guaranteed by Corollary 1. Finally, the effectiveness and performance of the proposed control scheme are illustrated and analyzed via some simulation applied to flexible single link manipulator with uncertainty in inertia moment of motor shaft and constraint on input torque.

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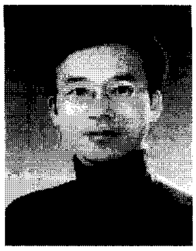

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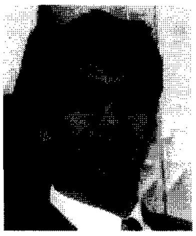

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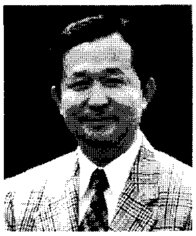

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