Fault Diagnosis and Accommodation of Linear Stochastic Systems with Unknown Disturbances

Jong-Hyo Lee and Joon Lyou

Abstract: An integrated robust fault diagnosis and fault accommodation strategy for a class of linear stochastic systems subjected to unknown disturbances is presented under the assumption that only a single fault may occur at a given time. The strategy is based on the fault isolation and estimation using a bank of robust two-stage Kalman filters and introduction of the additive compensation input for cancelling out the fault's effect on the system. Each filter is set up such that the residual is decoupled from unknown disturbances and fault with the influence vector designed in the filter. Simulation results for the simplified longitudinal flight control system with parameter uncertainties, process and sensor noises demonstrate the effectiveness of the present approach.

Keywords: robust fault diagnosis and accommodation, unknown disturbance, two-stage Kalman filter

I. Introduction

In the past two decades, a model-based fault diagnosis (FDI, fault detection and isolation) methods have been studied under the requirement of improving the reliability and maintainability of control systems [1][2][3]. If the mathematical model is a very accurate representation of the system behavior, model-based FDI methods can be used to diagnose faults correctly and reliably. However, in practice, the system may not be free from unknown disturbances, modeling errors and noises, requiring the FDI algorithms robust enough.

One of the most successful robust FDI approaches for stochastic systems is the use of disturbance decoupling principle and Kalman filtering technique, in which the residual is designed to be decoupled to unknown disturbances, modeling errors and noises, whilst sensitive to faults. In the disturbance decoupling design, the distribution matrix for disturbances must be known a priori although the actual disturbances remain unknown. However, the robustness problem in practice is difficult to solve because its distribution matrix is normally unknown. For this, an approach has been suggested, where unknown disturbances were represented approximately with an estimated distribution matrix [4][5]. In this way, an optimally robust solution is achievable. This approaximate strategy has extended the application domain of disturbance decoupling based robust FDI approaches. Recently, some progress has been made in the design of optimal filtering, by an optimal observer [6] and a special structure of the full-order Kalman filter [7], for stochastic systems with unknown disturbances. Darouach et al. [8] proposed two-stage Kalman filter for systems affected by unknown inputs and constant biases, and optimal two-stage Kalman filter in the presence of random biases in [9].

Fault accommodation (FA) method is provided to make the system stable and retain acceptable performance unter the fault. All present FA approaches can be classified into two groups. The first group is based on FDI and the second one is independent of FDI. The essence of the FDI-based FA approaches is to detect and isolate the fault on-line, and then to modify or redesign the control law by using fault estimate from FDI stage to make the faulty system stable, as shown in [10]. The second group of FA approaches uses fixed controllers without consideration for whether the fault has occurred or not. In [11], Frank et al. adopted a fixed FA strategy against sensor faults by feedback the estimated state instead of the measured output.

However, most of present model-based fault diagnosis approaches have considered the systems without uncertainties [10][11], and although system uncertainties are considered, it has been remained at fault detection level [6] or treated only for a part, actuator faults [7] or sensor faults [11], among system faults.

This paper extends Darouach's two-stage Kalman filter [8][9] for the stochastic systems with unknown disturbances and random biases, and proposes the integrated robust FDI-based FA strategy against actuator and sensor faults. It is assumed that only a single fault may occur at a given time. The FDI module is constructed based on simultaneous generation of the residual set and estimation of the bias through a bank of the robust two-stage Kalman filters. Each filter is set up such that the residual is decoupled from unknown disturbances and fault with the influence vector designed in the filter, and that effects of process and sensor noises are minimized. All components of the residual set are evaluated by using the hypothesis statistical test, and the fault is declared according to the prepared decision logic. Once the fault is indicated, the FA module is activated, and additive fault compensation input is computed by using the fault estimate from the filter with the influence vector of the indicated fault and is combined to the nominal control law so that its effect on the faulty system is cancelled out.

II. System model and fault description

Consider the following discrete-time stochastic system described by

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ed_k + w_k^x \\ y_k &= Cx_k + v_k \end{aligned} \tag{1}$$

where $x_k \in R^n$ is the state vector, $y_k \in R^m$ is the output vector, $u_k \in R^p$ is the known input vector, and $d_k \in R^q$ is

Manuscript received: Oct. 31, 2001., Accepted: July. 22, 2002.

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the unknown disturbance vector, Matrices A, B, C and E are known matrices with appropriate dimensions. Without loss of generality, it is assumed that E is of full column rank. w_k^x and v_k are the process and the measurement noise sequences, respectively.

Unknown disturbances Ed_k in system (1) may involve additive disturbances and modeling errors such as nonlinear terms in the system dynamics, linearization and model reduction errors, and parameter variations. The disturbances may also appear in the output equation, however this case is not considered here because disturbances can be null by simply using a transformation of the output signal y_k.

In the system modeling, faults are described in two different types; 1) additive faults, characterizing actuator or sensor faults, 2) multiplicative faults, designating plant faults. The typical faults treated are actuator and sensor faults, and they are usually described in the way of adding directly on the dynamics or on the measurements of the system.

Thus, actuator faults consider a loss in the actuator effectiveness and are represented by changing the matrix B as

$$B_f = B(I + diag(\xi_k^a)) \tag{2}$$

with $\xi^a = [\xi^{a_1} \cdots \xi^{a_i} \cdots \xi^{a_p}]^T$, and when the ith actuator breaks down, $\xi^{a_i} = -1$. Since B_f is an unknown matrix, representation of the faulty system requires the introduction of an unknown fault f_k^a, which is equal to zero in the fault-free case

$$x_{k+1} = Ax_k + Bu_k + Ed_k + F^a f_k^a + w_k^x \tag{3}$$

where $F^a = [F^{a_1} \cdot\cdot F^{a_i} \cdot\cdot F^{a_p}]$, $f_k^a = \left[f_k^{a_1} \cdot\cdot f_k^{a_i} \cdot\cdot f_k^{a_p}\right]^T$, and F^{a_i} denotes the influence vector of the ith actuator fault $f_k^{a_i}$ on the state x_k. Likewise, sensor faults characterize a scaling change in the state measurement and are represented by modifying the matrix C as

$$C_f = (I + diag(\xi_k^s))C \tag{4}$$

with $\xi^s = [\xi^{s_1} \cdots \xi^{s_i} \cdots \xi^{s_m}]^T$, and expression of the faulty system is

$$y_k = Cx_k + F^s f_k^s + v_k \tag{5}$$

where $F^s = [F^{s_1} \cdot\cdot F^{s_i} \cdot\cdot F^{s_m}]$, $f_k^s = [f_k^{s_1} \cdot\cdot f_k^{s_i} \cdot\cdot f_k^{s_m}]^T$, and F^{s_i} denotes the influence vector of the ith sensor fault $f_k^{s_i}$ on the output y_k.

We assume that only a single fault may occur at a given time and the fault is treated as the random bias. The system with an actuator fault is thus modeled by replacing the state equation in (1) as

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ed_k + F^{a_i} b_k + w_k^x \\ b_{k+1} &= b_k + w_k^b \end{aligned} \tag{6}$$

and the one with a sensor fault is modeled by substituting the output equation in (1) as

$$\begin{aligned} y_k &= Cx_k + F^{s_i} b_k + v_k \\ b_{k+1} &= b_k + w_k^b \end{aligned} \tag{7}$$

where the process noise w_k^x, the bias noise w_k^b and the measurement noise v_k are zero mean uncorrelated random sequences with covariance matrices $W^x \geq 0$, $W^b \geq 0$ and $V > 0$, respectively. The initial values x_0 and b_0 are assumed to be uncorrelated with the white noises w_k^x, w_k^b and v_k. Let x_0 and b_0 be the Gaussian random variables with $E\{x_0\} = \bar{x}_0$, $E\{(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\} = P_0^x > 0$, $E\{b_0\} = \bar{b}_0$, $E\{(b_0 - \bar{b}_0)(b_0 - \bar{b}_0)^T\} = P_0^b > 0$, $E\{(x_0 - \bar{x}_0)(b_0 - \bar{b}_0)^T\} = P_0^{xb}$,

III. On-line FDI algorithm

Fault diagnosis generally divided into the two tasks; 1) fault detection, deciding whether or not a fault has occurred, 2) fault isolation, deciding which element of the system is faulty. To achieve these tasks, the basic approach mainly depends on the residuals composed of the state and the bias estimation error, where the state and the bias estimate are generated using a robust filter for the system subjected to unknown disturbances. For estimation of both the state and the bias, the natural approach is to augment the bias as a part of the state, and to apply the Kalman filter. But, the augmented Kalman filtering technique may not work effectively in case that the actuator fault occurs because it acts on the system in the same way as the unknown disturbances do. Thus, we use two parallel reduced-order Kalman filter, called "two-stage Kalman filter", which is composed of the bias-free filter and the bias filter [8][9].

1. Robust two-stage Kalman filter

From the robust filtering context [6][7][8], the necessary and sufficient condition for decoupling unknown disturbances is given by

$$rank(E) = rank(CE) = q, \; q \leq m \tag{8}$$

meaning that the maximum number of disturbances cannot be larger than the number of independent measurements.

To take into account the unknown disturbances in [9], the gain matrices, $\bar{L}_{k+1}^x$ in the bias-free filter and L_{k+1}^b in the bias filter of the robust two-stage Kalman filter must be satisfied with the following constraints

$$\begin{aligned} \bar{L}_{k+1}^x CE &= E \\ L_{k+1}^b CE &= 0 \end{aligned} \tag{9}$$

Eq. (9) have solutions if the conditions in (8) is satisfied, then proceeding as in [8], the solutions can be explained easily and the robust two-sage Kalman filter for the stochastic systems with unknown disturbances and random biases is finally obtained by substituting the expressions of gain matrices $\bar{L}_{k+1}^x$ and L_{k+1}^b into the gain matrices $\bar{K}_{k+1}^x$ and K_{k+1}^b in [9]. Table 1 presents the entire algorithm for the robust two-stage Kalman filter.

To be specific in the robust two-stage Kalman filter for system (6), in case of the actuator fault ($F^{s_i} = 0$), (8) leads to

$$\begin{aligned} &rank(E) = rank(CE) = q \\ &rank([CE \;\; CF^{a_i}]) = q + 1, \; q \leq m - 1 \end{aligned} \tag{10}$$

while for the system (7), in case of the sensor fault ($F^{a_i} = 0$),

$$\begin{aligned} &rank(E) = rank(CE) = q \\ &rank([CE \;\; F^{s_i}]) = q + 1, \; q \leq m - 1 \end{aligned} \tag{11}$$

2. Residual generation and fault detection

To detect a fault, the residual can be generated using the output estimation, $y_{k/k} = Cx_{k/k} + F^{s_i} b_{k/k}$, as

$$r_k = y_k - y_{k/k} = Ce_k^x + F^{s_i} e_k^b + v_k \tag{12}$$

Table 1. The algorithm of the robust two-stage Kalman filter

$$x_{k+1/k+1} = \bar{x}_{k+1/k+1} + \beta_{k+1/k+1} b_{k+1/k+1}$$
$$P^x_{k+1/k+1} = \bar{P}^x_{k+1/k+1} + \beta_{k+1/k+1} P^b_{k+1/k+1} \beta^T_{k+1/k+1}$$

with

$$\bar{x}_{0/0} = x_0 - \beta_{0/0} b_0,\ \beta_{0/0} = P^{xb}_0 (P^b_0)^{-1},$$
$$\bar{P}^x_{0/0} = P^x_0 - \beta_{0/0} P^b_0 \beta^T_{0/0}$$

Bias-free Filter

$$\bar{x}_{k+1/k+1} = \bar{x}_{k+1/k} + \bar{L}^x_{k+1} \bar{\gamma}_{k+1}$$
$$\bar{P}^x_{k+1/k+1} = (I - \bar{K}^x_{k+1} C) \bar{P}^x_{k+1/k} + \eta^x_{k+1} \Pi^x_{k+1} G^x_{k+1} \Pi^{x^T}_{k+1} \eta^{x^T}_{k+1}$$

with

$$\bar{L}^x_{k+1} = \bar{K}^x_{k+1} + \eta^x_{k+1} \Pi^x_{k+1}$$
$$\eta^x_{k+1} = E - \bar{K}^x_{k+1} CE$$
$$\Pi^x_{k+1} = [(CE)^T G^{x^{-1}}_{k+1} CE]^{-1} (CE)^T G^{x^{-1}}_{k+1}$$

where

$$\bar{\gamma}_{k+1} = y_{k+1} - C\bar{x}_{k+1/k}$$
$$\bar{x}_{k+1/k} = A\bar{x}_{k/k} + Bu_k + \alpha_k b_{k/k} - \beta_{k+1/k} b_{k/k}$$
$$\bar{K}^x_{k+1} = \bar{P}^x_{k+1/k} C^T G^{x^{-1}}_{k+1}$$
$$\bar{P}^x_{k+1/k} = A\bar{P}^x_{k/k} A^T + W^x + \alpha_k P^b_{k/k} \alpha^T_k - \beta_{k+1/k} P^b_{k+1/k} \beta^T_{k+1/k}$$
$$G^x_{k+1} = C\bar{P}^x_{k+1/k} C^T + V$$

Bias Filter

$$b_{k+1/K+1} = b_{k/k} + L^b_{k+1} \gamma^b_{k+1}$$
$$P^b_{k+1/k+1} = (I - K^b_{k+1} H_{k+1/k}) P^b_{k+1/k} + \eta^b_{k+1} \Pi^b_{k+1} G^b_{k+1} \Pi^{b^T}_{k+1} \eta^{b^T}_{k+1}$$

with

$$L^b_{k+1} = K^b_{k+1} + \eta^b_{k+1} \Pi^b_{k+1}$$
$$\eta^b_{k+1} = K^b_{k+1} CE$$
$$\Pi^b_{k+1} = [(CE)^T G^{b^{-1}}_{k+1} CE]^{-1} (CE)^T G^{b^{-1}}_{k+1}$$

where

$$\gamma^b_{k+1} = \bar{\gamma}_{k+1} - H_{k+1/k} b_{k/k}$$
$$K^b_{k+1} = P^b_{k+1/k} H^T_{k+1/k} G^{b^{-1}}_{k+1}$$
$$P^b_{k+1/k} = P^b_{k/k} + W^b$$
$$G^b_{k+1} = H_{k+1/k} P^b_{k+1/k} H^T_{k+1/k} + G^x_{k+1}$$

Coupling Equations

$$H_{k+1/k} = F^{s_i} + C\beta_{k+1/k} \quad ; F^{s_i} = 0 \text{ for the model (6)}$$
$$\beta_{k+1/k+1} = \beta_{k+1/k} - \bar{L}^x_{k+1} H_{k+1/k}$$
$$\alpha_k = A\beta_{k/k} + F^{a_i} \quad ; F^{a_i} = 0 \text{ for the model (7)}$$
$$\beta_{k+1/k} = \alpha_k P^b_{k/k} P^{b^{-1}}_{k+1/k}$$

where $e^x_k = x_k - x_{k/k}$ and $e^b_k = b_k - b_{k/k}$ are the state and the bias estimation errors. Note that these errors have minimum variances. As we assumed, the noise sequences w^x_k, w^b_k and v_k are white Gaussian, the residual will also have the Gaussian distribution.

Now, using the residual and its covariance, a residual evaluation method is formulated. The well-known χ^2 based hypothesis test, which is referred in the related text and paper, can be used to examine the residual and, subsequently to detect the fault in case that the residual has Gaussian distribution. The two hypotheses to be tested can be identified as H_0, the normal or common fault mode which has no fault or only the fault with the influence vector designed in the filter, and H_1, the uncommon fault mode which have other faults with the exception of the common fault.

Under the normal or common fault condition, from eq. (12) and the relation of the initial value, $\beta_{0/0} = P^{xb}_0 (P^b_0)^{-1}$, in Table 1, the statistics of the residual can be represented as

$$H_0 = \begin{cases} E\{r_k\} = 0 \\ cov\{r_k\} = W_k = CP^x_{k/k} C^T + F^{s_i} P^b_{k/k} \beta^T_{k/k} C^T \\ \qquad + C\beta_{k/k} P^b_{k/k} F^{s_i^T} + F^{s_i} P^b_{k/k} F^{s_i^T} + V \end{cases} \tag{13}$$

When an uncommon fault occurs in the system, the residual statistics will be different from the normal or common fault mode.

The task of fault detection is to distinguish which one between two hypotheses H_0 and H_1. Since the residual is of Gaussian distribution, the test statistic λ_k forms χ^2 distribution of Ml degrees of freedom.

$$\lambda_k = \sum_{i=k-M+1}^{k} r^T_i W^{-1}_i r_i \tag{14}$$

with M being the window size and l the dimension of r_k. The remaining problem is to set a threshold value to which λ_k is compared for fault detection. Taking the reasonably low false alarm rate into account, the threshold level T_D is chosen from the χ^2 distribution table such that

$$prob[\lambda_k \geq T_D / H_0] = P_f \tag{15}$$

where P_f is the prescribed probability of false alarm. Then, we can detect the fault using the following detection rule.

$$S(r_k) = \begin{cases} 0 & \text{if } \lambda_k < T_D \\ 1 & \text{otherwise} \end{cases} \tag{16}$$

3. Fault isolation

Fault isolation requires the generation of a residual set insensitive to one fault and sensitive to the others. Thus, a bank of two-stage Kalman filters is set up according to the system model (6) with actuator fault and (7) with sensor fault. The ith filter corresponds to the ith fault among p actuators and m sensors, and uses the influence vector of the fault, F^i for $i = a_1, \cdots, a_p$ (actuator index), $s_1, \cdots, s_m$ (sensor index). So, the ith residual is generated from the ith filter with F^{a_i} ($F^{s_i} = 0$) or F^{s_i} ($F^{a_i} = 0$) in the algorithm of Table 1.

The residuals generated from the bank of two-stage Kalman filters in case of an actuator or sensor fault summarize as follows.

1) The residual of the ith filter in case of the jth actuator fault ($i = a_1, \cdots, a_p, s_1, \cdots, s_m,\ \ j = a_1, \cdots, a_p$)

$$r^{a_i}_k = Ce^x_k + v_k \tag{17}$$

where

$$\begin{aligned} e^x_{k+1} &= (I - L_{k+1} C) F^{a_j} b_k - (I - L_{k+1} C) F^{a_i} b_{k/k} \\ &\quad + (I - L_{k+1} C) A e^x_k + (I - L_{k+1} C) w^x_k \\ &\quad - L_{k+1} v_{k+1} \\ e^b_{k+1} &= e^b_k - L^b_{k+1} C F^{a_j} b_k + L^b_{k+1} C F^{a_i} b_{k/k} \\ &\quad - L^b_{k+1} C A e^x_k + w^b_k - L^b_{k+1} (C w^x_k + v_{k+1}) \end{aligned} \tag{18}$$

with

$$L_{k+1} = \bar{L}^x_{k+1} + \beta_{k+1/k+1} L^b_{k+1} \tag{19}$$

$$r^{s_i}_k = Ce^x_k - F^{s_i} b_{k/k} + v_k \tag{20}$$

where

$$\begin{aligned} e^x_{k+1} &= (I - L_{k+1}C)F^{a_j} b_k + L_{k+1} F^{s_i} b_{k/k} \\ &\quad +(I - L_{k+1}C)Ae^x_k + (I - L_{k+1}C)w^x_k \\ &\quad -L_{k+1} v_{k+1} \\ e^b_{k+1} &= e^b_k - L^b_{k+1} C F^{a_j} b_k + L^b_{k+1} F^{s_i} b_{k/k} \\ &\quad -L^b_{k+1} CAe^x_k + w^b_k - L^b_{k+1}(Cw^x_k + v_{k+1}) \end{aligned} \tag{21}$$

2) The residual of the ith filter in case of the jth sensor fault $(i = a_1, \cdots, a_p, s_1, \cdots, s_m,\ \ j = s_1, \cdots, s_m)$

$$r^{a_i}_k = Ce^x_k + F^{s_j} b_k + v_k \tag{22}$$

where

$$\begin{aligned} e^x_{k+1} &= -L_{k+1} F^{s_j} b_k - (I - L_{k+1}C)F^{a_i} b_{k/k} \\ &\quad +(I - L_{k+1}C)Ae^x_k + (I - L_{k+1}C)w^x_k \\ &\quad -L_{k+1}(F^{s_j} w^b_k + v_{k+1}) \\ e^b_{k+1} &= e^b_k - L^b_{k+1} F^{s_j} b_k + L^b_{k+1} C F^{a_i} b_{k/k} \\ &\quad -L^b_{k+1} CAe^x_k + (I - L^b_{k+1} F^{s_j}) w^b_k \\ &\quad -L^b_{k+1}(Cw^x_k + v_{k+1}) \end{aligned} \tag{23}$$

$$r^{s_i}_k = Ce^x_k + F^{s_j} b_k - F^{s_i} b_{k/k} + v_k \tag{24}$$

where

$$\begin{aligned} e^x_{k+1} &= -L_{k+1} F^{s_j} b_k + L_{k+1} F^{s_i} b_{k/k} \\ &\quad +(I - L_{k+1}C)Ae^x_k + (I - L_{k+1}C)w^x_k \\ &\quad -L_{k+1}(F^{s_j} w^b_k + v_{k+1}) \\ e^b_{k+1} &= e^b_k - L^b_{k+1} F^{s_j} b_k + L^b_{k+1} F^{s_i} b_{k/k} \\ &\quad -L^b_{k+1} CAe^x_k + (I - L^b_{k+1} F^{s_j}) w^b_k \\ &\quad -L^b_{k+1}(Cw^x_k + v_{k+1}) \end{aligned} \tag{25}$$

From eq. (17)-(25), it is noted that, after a fault occurs, only the filter corresponding to the fault (i.e. $i = j$) can estimate the state and the bias correctly, and the residual from this filter will be maintained the residual statistics in eq. (13), but the residual statistics from the other filter will be deviated from eq. (13) due to the wrong estimation of the state and bias. And it is also noted that unknown disturbances Ed_k do not affect all of the residuals. It means that these residuals are robust against unknown disturbances.

Now, each residual is examined through the hypothesis statistical test as in section III.2, and the resultant vectors, $S^a(r_k)$ for residuals from the filters with F^{a_i} and $S^s(r_k)$ for residuals from the filters with F^{s_i} , are produced as

$$\begin{aligned} S^a(r_k) &= [S^{a_1}(r_k) \cdots S^{a_i}(r_k) \cdots S^{a_p}(r_k)]^T \\ S^s(r_k) &= [S^{s_1}(r_k) \cdots S^{s_i}(r_k) \cdots S^{s_m}(r_k)]^T \end{aligned} \tag{26}$$

Then for fault isolation, $S^a(r_k)$ and $S^s(r_k)$ in (26) are compared to the fault signatures $S^a(ref, f_i)$ and $S^s(ref, f_i)$ which are the column vectors of the fault signature matrices defined in Table 2. Note that the "0" element of the fault signature matrices are designated based on the underlying principle that only the filter associated with the fault occurred can estimate the state and the fault correctly. Now, if $S^a(r_k)$ coincides with a column of the actuator fault signature matrix in Table 2(a), the corresponding fault indicator $I(f_{a_i})$ or $I(f_s)$ is set to "one". If $I(f_{a_i}) = 1$, the ith actuator is declared to be faulty. If $I(f_s) = 1$, we guess a sensor fault. Further, by checking if $S^s(r_k)$ is same as which column of the sensor fault signature matrix in Table 2(b), the corresponding fault indicator $I(f_{s_i})$ set to 1, and the ith sensor is declared to be faulty.

IV. Integrated FDIA strategy

After the fault is indicated, its effect on the system is compensated using the fault(bias) estimate $b_{k/k}$ from the filter associated with the fault. The concept of this approach is depicted in Fig. 1. It should be noted the ability of the FA to compensate for the fault is closely related to the result given by the FDI module concerning the decision of whether an actuator or a sensor fault has occurred.

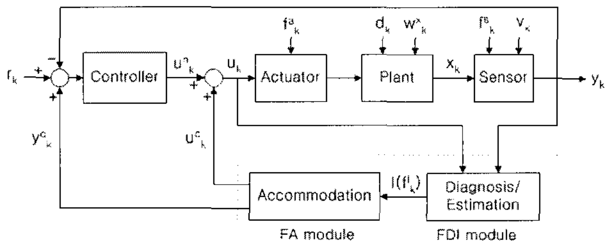


Fig. 1. Integrated FDIA scheme

In case of the actuator fault, the additive control input u^c_k is computed such that

$$B(u^n_k + u^c_k) + F^{a_i} f^{a_i}_k = Bu^n_k \tag{27}$$

where u^n_k is the nominal input without fault. Note that u^c_k is introduced to cancel out the unwanted input by the fault, $f^{a_i}_k$. If the matrix B is of full rank, u^c_k is obtained as

$$u^c_k = -B^+ F^{a_i} f^{a_i}_k \tag{28}$$

where B^+ is the pseudo-inverse of B, and actually $b_{k/k}$ is used replacing $f^{a_i}_k$. Even if B is not of full rank, there is a solution [8]. If the control input is single, then $F^{a_1} = B$, and (28) reduces to

$$u^c_k = -f^{a_i}_k \tag{29}$$

If a sensor fault occurs, the corresponding measured output may be different from its nominal value without fault. So, in this case, rather than trying to modify the nominal control law, we include a correcting term y^c_k in the measurement equation such that

$$y_k + y^c_k = Cx_k + F^{s_i} f^{s_i}_k + v_k + y^c_k = Cx_k + v_k \tag{30}$$

Then, using the fault (bias) estimate $f^{s_i}_k$, the faulty output is compensated by

$$y^c_k = -F^{s_i} f^{si}_k \tag{31}$$

Table 2. Fault signature matrices

(a) for actuator faults

$S^a(r)$	$S^a(ref, nofault)$	$S^a(ref, f_1)$	$S^a(ref, f_i)$	$S^a(ref, f_p)$	$S^s(ref, f)$
$S^{a_1}(r)$	0	0	1	1	1
$S^{a_i}(r)$	0	1	0	1	1
$S^{a_p}(r)$	0	1	1	0	1

(b) for sensor faults

$S^s(r)$	$S^s(ref, nofault)$	$S^s(ref, f_1)$	$S^s(ref, f_i)$	$S^s(ref, f_m)$
$S^{s_1}(r)$	0	0	1	1
$S^{s_i}(r)$	0	1	0	1
$S^{s_m}(r)$	0	1	1	0

V. Simulation

The effectiveness of the present FDIA strategy will be demonstrated through computer simulations for the simplified longitudinal flight control system [6] whose discrete-time model is given by

$$A = \begin{bmatrix} 0.9944 & -0.1203 & -0.4302 \\ 0.0017 & 0.9902 & -0.0747 \\ 0 & 0.8187 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4252 \\ -0.0082 \\ 0.1813 \end{bmatrix}, \; C = I_3, \; x = \begin{bmatrix} \eta_y \\ \omega_z \\ \delta_z \end{bmatrix} \quad (32)$$

where the state variables are pitch angle δ_z, pitch rate ω_z, and normal velocity η_y, and the control input is elevator control signal. The covariance matrices for process and measurement noise sequences are $W^x = diag\{0.02^2, 0.02^2, 0.002^2\}$ and $V = 0.02^2 I_3$. The unknown disturbances usually stem from perturbations in aerodynamic related coefficients, and it is assumed that there are parameter perturbations like $\Delta a_{2i} = -0.5a_{2i}$ and $\Delta b_2 = 0.5b_2$. Thus, disturbances Ed_k is represented as

$$\begin{aligned} E &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T \\ d_k &= [-0.0009 \; -0.4951 \; 0.0374] x_k - 0.0041 u_k \quad (33) \end{aligned}$$

The purpose of the nominal control is to make the pitch angle δ_z track the reference input r. For this, the integral, $z_{k+1} = z_k + T_s(r_k - y_{3k})$, of the tracking error is first defined and appended into the state vector, $\tilde{x}_k = \begin{bmatrix} x_k^T & z_k \end{bmatrix}^T$, where $T_s = 0.1s$ is the sampling interval, y_{3k} is the $3rd$ element of the output vector y_k, and $\tilde{x}_k$ is the augmented state. Then, under the assumption that the state variables are available for measurements, the LQ(Linear Quadratic) state feedback control law $u_k = -K\tilde{x}_k$ can be implemented by choosing the weighting matrices $Q = diag\{1, 1, 1, 300\}$ and $R = 1$.

As an actuator fault, we consider a loss in the actuator effectiveness, abruptly(step wise), $f_k^a = -\rho u_k^n, 0 < \rho < 1$, or incipiently(ramp wise), $f_k^a = -\delta k T_s$, δ is small, with the influence pattern, $F^a = B$. Likewise, a sensor fault is modeled by abrupt changes, $f_k^{s_i} = \Delta x_{ik}$, or incipient variations, $f_k^{s_i} = \delta k T_s$, δ is small, for the output measurement with $F^{s_i} = C^i$.

Now, the FDI and FA capability has been evaluated under various fault conditions. Fig. 2 shows the hypothesis test statistics λ in (14) under step fault in the actuator, and ramp fault in the pitch angle sensor, respectively. The threshold on the fault detection rule (16) is selected as $T_D = 12.84$ from χ^2 distribution table with the window size $M = 1$, the residual dimension $l = 3$ and the false alarm probability $P_f = 0.005$. Observing this figure and many simulation results under other faults, all of the faults have been detected, and isolated without false alarm or missed alarm. The abrupt faults can be detected very fast, usually it needs only one or two steps to detect the fault after occurrence. The incipient faults can also be detected at an early step.

Fig. 3 displays profiles of the fault estimate f^i, the state x and the control input u with and without FA under step fault in the actuator and step fault in the pitch angle sensor, respectively. This figure and many simulation results under other faults clearly demonstrate that the present approach has FA ability against such faults, whilst the nominal control has no FA capability even under the sensor fault. In case of the actuator fault, the system response reach their nominal value, but very slowly, even without fault compensation because the actuator fault affect the system as a perturbation, and mainly due to the presence of the integral action in the controller.

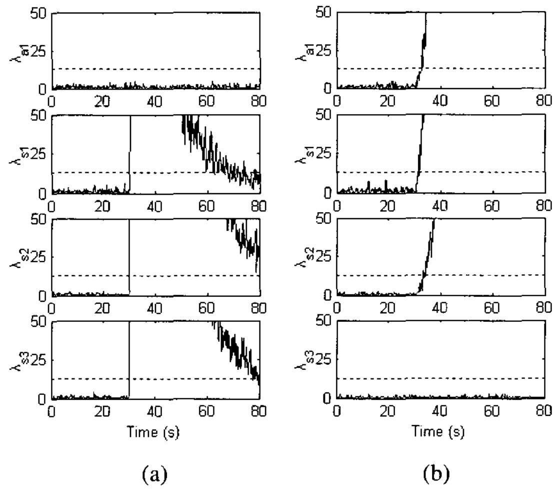
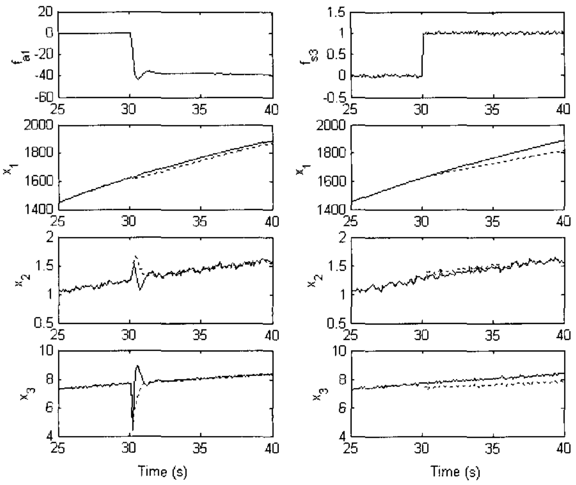


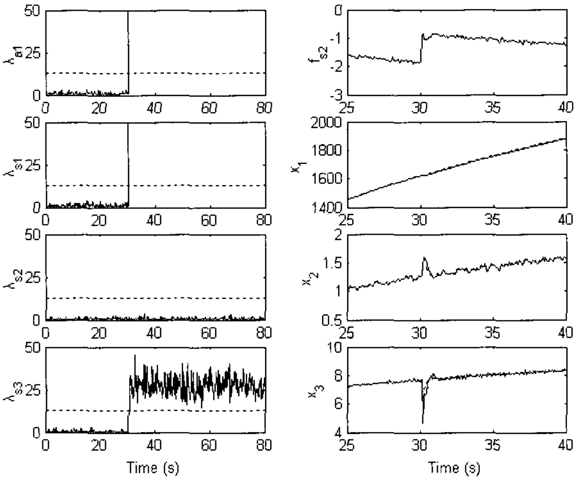
Fig. 2. Hypothesis test statistics λ under the fault (a) in the actuator(f^a, step, $\rho = 0.05$) (b) in the pitch angle sensor(f^{s_3}, ramp, $\delta = 0.05$)

One remark is in order, let us check the necessary and sufficient conditions (10) and (11) for decoupling unknown distur-



(a) (b)

Fig. 3. Trajectories of the fault estimate f^i, state x and control input u without($\cdots$) and with(—) FA under the fault (a) in the actuator(f^a, step, $\rho = 0.5$) (b) in the pitch angle sensor(f^{s_3}, step, $\Delta x_{3k} = 1.0$)



(a) (b)

Fig. 4. Under the fault in the pitch rate sensor(f^{s_2}, step, $\Delta x_{2k} = 1.0$) (a) Hypothesis test statistics λ (b) Trajectories of the fault estimate f^i, state x and control input u without($\cdots$) and with(—) FA

bances. Since

$$\begin{aligned} rank(E) &= rank(CE) = q = 1 \\ rank([CE \;\; F^{s_2}]) &= 1 \neq 2(= q+1) \end{aligned} \tag{34}$$

it follows that (11) is not satisfied for the pitch rate sensor(S_2) case. In this case, the fault influence vector of the S_2 sensor may be represented as $F^{s_2} = \alpha CE$, $\alpha = constant$. So, according to the relations (9) for decoupling unknown disturbances, the residual from the filter with F^{s_2} in case of the fault in the S_2 sensor represents as follow.

$$\begin{aligned} r_k^{s_2} &= C(I - (L_k + F^{s_2} L_k^b)C)Ae_{k-1}^{\bar{x}} \\ &\quad + C(I - (L_k + F^{s_2} L_k^b)C)w_{k-1}^{\bar{x}} - (L_k + F^{s_2} L_k^b)v_k \end{aligned} \tag{35}$$

where

$$\begin{aligned} e_{k+1}^{\bar{x}} &= -\bar{L}_{k+1}^{x} F^{s_2} e_k^b + (I - L_{k+1}C)Ae_k^{\bar{x}} \\ &\quad + (I - L_{k+1}C)w_k^{\bar{x}} - \bar{L}_{k+1}^{x} F^{s_2} w_k^b - L_{k+1}v_{k+1} \\ e_{k+1}^b &= e_k^b - L_{k+1}^b CAe_k^{\bar{x}} + w_k^b - L_{k+1}^b(Cw_k^{\bar{x}} + v_{k+1}) \end{aligned} \tag{36}$$

From the expression of bias estimation error in (36), we can know that gain matrix L_k^b in the bias filter isn't operated completely. Hence, although FDI ability is good as shown in fig. 4 under step fault in the S_2 sensor, we can expect that the fault estimate and FA ability will not be good enough.

VI. Conclusion

An integral framework has been given for the robust fault diagnosis and accommodation of linear stochastic systems subjected to unknown disturbances and actuator or sensor faults. A bank of two-stage Kalman filters is adapted to estimate both the state and the fault as well as to generate the residual set decoupled to unknown disturbancese but discernible by a fault. Also, a simple decision logic based on hypothesis test statistics is provided to identifiy the fault correctly. By the theoretical explanations and interpreting the simulation results, the present approach turns out to be effective enough no matter whether there are abrupt or incipient faults. Also, the inherent parallel structure makes it attractive for real-time and practical applications.

It has been recognized that the fault isolation problem of actuators and sensors is very difficult. A related further works is to relax the rather restrictive decoupling condition and extend to a plant fault of uncertain stochastic systems.

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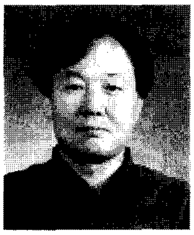

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