International Journal of Ocean Engineering and Technology Vol. 5, No. 1, pp 40~45, December 2002 (ISSN 1229-1668)

A Study on the Pit Excavation Volume Using Cubic B-Spline

DU-YEOUL MUN*
*Deptartment of Civil Engineering, Dongeui University, Busan, Korea
(Received 14 May 2002, accepted 30 July 2002)

ABSTRACT: *The calculation of earthwork plays a major role in the planning and design phases of many civil engineering projects, such as seashore reclamation; thus, improving the accuracy of earthwork calculation has become very important. In this paper, we propose an algorithm for finding a cubic spline surface with the free boundary conditions, which interpolates the given three-dimensional data, by using B-spline and an accurate method to estimate pit-excavation volume. The proposed method should be of interest to surveyors, especially those concerned with accuracy of volume computations. The mathematical models of the conventional methods have a common drawback: the modeling curves form peak points at the joints. To avoid this drawback, the cubic spline polynomial is chosen as the mathematical model of the new method. In this paper, we propose an algorithm of finding a spline surface, which interpolates the given data, and an appropriate method to calculate the earthwork. We present some computational results that show the proposed method, of the Maple program, provides better accuracy than the method presented by Chen and Lin.*

KEY WORDS: Pit-Excavation Volume, Spline, B-Spline, Spline Surface

1. Introduction

Surveyors are often called upon to measure volumes of earthwork that needs to be moved for construction of seashore reclamation, highways, railroads, canals, earth dams, pipelines, and similar projects. Earthwork quantities, in the types of construction projects described herein, are frequently of such magnitude as to make up appreciable percentages of the total project cost. Several methods have been developed for estimating the pit excavation volume, ranging from a simple formula to more complicated formulas and numerical methods. The standard methods can be characterized with three basic ideas: the trapezoidal rule, the Simpson rule, and the cubic spline function. The trapezoidal method, which is the simplest method, approximates the ground profile of each grid cell using a plane, and estimates the pit excavation volume as the product of the area of the grid cell and the average excavation heights of the grid cell corners (Anderson et al, 1985; Schmidt and Wong, 1985; Wolf and Brinker, 1989; Moffit and Bossler, 1998). This method is the most commonly used, but the interfaces between the approximating planes are sharp, and it may not properly describe the ground surface. The Simpson-based methods improve the accuracy of the volume estimation for the approximation of the ground surface by considering a second-degree polynomial or a third-degree polynomial in each direction of the grid (Easa, 1988; Chambers, 1989).

The first author : Du-Yeoul Mun
Email : dymun@dongeui.ac.kr

In Easa (1988), it was assumed that the rectangles formed by the grid were of equal size; that is, the grid was formed by taking equal size intervals along each axis. Chambers (1989) generalized Easa's result by allowing grids in which the rectangles were of unequal sizes; that is, the grids were formed by partitioning the axis into intervals of unequal sizes. However, both methods have a common drawback: the interfaces of the approximating surfaces are too sharp. To eliminate this drawback, Chen and Lin (1991) proposed the cubic spline method, which provides smooth connections between the approximating cubic spline polynomials with the natural boundary conditions. Also, Easa (1998) developed the cubic Hermite polynomial method, which guarantees smooth connections between the approximating cubic Hermite polynomials. In this paper, we propose a method of finding a cubic spline surface, which interpolates the given three dimensional data, by using cubic B-splines. Note that our developed method is different from Chen and Lin's result. Chen and Lin (1991) approximate the ground surface with the cubic spline along one direction, and with the linear function along the other direction. But our proposed method approximates the ground surface with the cubic spline polynomial along both x and y directions. The method is based on the cubic B-spline, and the interpolating cubic spline surface can be obtained by those B-splines. Computational results of the proposed method and some comments are presented in section 4. I have also compared the proposed method with the spot height method, line of best fit method, and the Chen and Lin method to earthwork volume.

2. Methods of Pit Excavation Volume

2 1 Spot height method

Consider the rectangular grid of whose sides into m and n intervals. The excavation depths f(xi ,yi) at the intersection points f(xi ,yi), with i = 0, 1, ⋯, m and j = 0, 1, ⋯, n are known. Then, the composite formular for calculating the volume of total grid, V, is given by

$$V=\frac{h^2}{4}\sum_{i=0}^{m}\sum_{j=0}^{n}a_{ij}f_{ij} \tag{1}$$

in which aij = the corresponding elements of the following matrix

$$a_{ij}=\begin{vmatrix} 1 & 2 & \cdots & 2 & 1 \\ 2 & 1 & \cdots & 4 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 2 & 4 & \cdots & 4 & 2 \\ 1 & 2 & \cdots & 2 & 1 \end{vmatrix} \tag{2}$$

2 2 Line of best fit method

In this method, a simplified cross secton is formed by fiting a straight line to predetemined cross section points using the theory of least squares(see equa. 3).

$$a=\frac{n\sum_{i=1}^{n}x_i y_i-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}x_i^2-(\sum_{i=1}^{n}x_i)^2} \tag{3}$$

$$b=\frac{-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}x_i y_i+\sum_{i=1}^{n}x_i^2\sum_{i=1}^{n}y_i}{n\sum_{n=1}^{n}x_i^2-(\sum_{i=1}^{n}x_i)^2} \tag{4}$$

Value of parameters a and b in Eqs.(3), (4) and vary from one cross section to another due to the variation in elevations.

2.3 Chen and Lin method

Considering the area between the x-axis and curve ab, the cubic spline polynomial Sj(x) in the interval(xj, xj+1) may be written in the form

$$S_j(x)= a_j+b_j(x-x_j)-c_j(x-x_j)^2+d_j(x-x_j)^3 \tag{5}$$

and the area Aj can be computed with the integral

$$A_j=(a_j-b_jc_j+c_jx_j^2-d_jx_j^3)(x_{j+1}-x_j)+$$
$$+(\frac{b_j}{2}-c_jx_j-\frac{3d_jx_j^2}{2})(x_{j+1}^2-x_j^2)$$
$$+\left(\frac{c_j}{3}-d_jx_j\right)(d_{j+1}{}^3-x_j^3)+\frac{d_j}{4}(x_{j+1}^4-x_j^4) \tag{6}$$

Using Eq. 6, we can compute the area between the curve ab and the x-axis in the interval (xj, xj+1), i.e., the area Aj. In similar manner we can compute the area between the curve ab and the x-axis in the interval (xo, xn) using the following equation:

$$A=\int_{x_0}^{x_n}S(x)dx=\sum_{j=0}^{n-1}A_j \tag{7}$$

Eq. 7 is defined as the cubic spline area frmular. we can derive n cubic spline polynomias, Si,0, Si,1,...., Si,n-1 in the x =xi direction.

$$(S_{i,j})_{j=0}^{n-1}=a_{i,j}+b_{i,j}(y-y_j)+c_{i,j}(y-y_j)^2$$
$$+d_{i,j}(y-y_j)^3 \tag{8}$$

Using Eq. 6, the formular for calculating the area between the curve that passes through the points f0,k, f1,k,...., fm,k and the baseline y = yk(z=0) in the interval [(x0, yk), (xm, yk)], denoted as Ayk, is given by

$$A_{yk}=\int_{x_0}^{x_1}S_{0,k}(x)dx+\int_{x_1}^{x_2}S_{1,k}(x)dx+\cdots\cdots$$
$$+\int_{x_{m-1}}^{x_m}S_{m-1,k}(x)dx \tag{9}$$

the points f0,k, f1,k,...., fm,k+1 and the baseline y = yk+1(z=0) in the interval [(x0, yk+1), (xm, yk+1)], denoted as Ayk, is given by

$$A_{yk+1}=\int_{x_0}^{x_1}S_{0,k+1}(x)dx+\int_{x_1}^{x_2}S_{1,k+1}(x)dx+\cdots\cdots$$
$$+\int_{x_{m-1}}^{x_m}S_{m-1,k+1}(x)dx \tag{10}$$

It is reasonable to use the "end area" method to calculate the excavation volume between Ayk and Ayk+1 as follows:

$$V=\frac{(A_{yk}+A_{yk+1})\triangle y}{2} \tag{11}$$

2.4 Proposed method(spline surface interpolation and its induced linear systems)

Let two points $c_1=(x_1, y_1)$ and $c_2=(x_2, y_2)$ be given. Then the line segment joining above two points can be expressed as

$$p(t|c_1, c_2; t_2, t_3)=\frac{t_3-t}{t_3-t_2}c_1+\frac{t-t_2}{t_3-t_2}c_2,\ t\in[t_2, t_3] \tag{12}$$

The two parameters t_2 and t_3 are arbitrary real numbers with $t_2 < t_3$.

If we introduce the piecewise constant functions

$$B_{i,0}(t)=\begin{cases}1, & t_i\le t< t_{i+1},\\ 0, & \text{otherwise}\end{cases} \tag{13}$$

and set $p_{i,1}(t)=p(t|c_{i-1}, c_i; t_i, t_{i+1})$, we can write

$$f(t)=\sum_{i=2}^{n} p_{i,1}(t)B_{i,0}(t) \tag{14}$$

For the quadratic spline curve, let three control points c_1, c_2, c_3 be given and set the knots with $t_2 \le t_3 < t_4 \le t_5$. Then we can obtain the quadratic spline curve by using two straight lines passing through c_1 and c_2, c_2 and c_3 in the following way.

$$p(t|c_1,c_2,c_3;t_2,t_3,t_4,t_5)=\frac{t_4-t}{t_4-t_3}p(t|c_1,c_2;t_2,t_4)$$
$$+\frac{t-t_3}{t_4-t_3}p(t|c_2,c_3;t_3,t_5). \tag{15}$$

Here t is the parameter which is in $[t_3, t_4]$. For any n control points $(c_i)_{i=1}^{n}$, we can define the piecewise quadratic spline curve by using the formula (15) and the knot vector $(t_i)_{i=2}^{n+2}$ with $t_2 \le t_3 < \cdots < t_{n+1} \le t_{n+2}$. Set $p_{i,2}(t)=p(t|c_{i-2},c_{i-1},c_i;t_{i-1},t_i,t_{i+1},t_{i+2})$. Then we can write the formula more precisely as

$$f(t)=\sum_{i=3}^{n} p_{i,2}(t)B_{i,0}(t) \tag{16}$$

Similarly, we can define the piecewise cubic spline curve.

$$f(t)=\sum_{i=4}^{n} p_{i,3}(t)B_{i,0}(t), \tag{17}$$

The formulas (14), (16), and (17) can also be written in the form of

$$f(t)=\sum_{i=1}^{n} c_i B_{i,d}(t) \tag{18}$$

where $B_{i,d}(t)$ is given by the recurrence relation

$$B_{i,d}(t)=\frac{t-t_i}{t_{i+d}-t_i}B_{i,d-1}(t)+\frac{t_{i+1+d}-t}{t_{i+1+d}-t_{i+1}} \times B_{i+1,d-1}(t), \quad d=1,2,3. \tag{19}$$

Here the function $B_{i,d}$ is called a B-spline of degree $d(d=1,2,3)$ with knots t_i. The B-spline $B_{j,d}$ depends only on the knots $(t_i)_{i=1}^{j+d+1}$. To understand the nature of B-splines, $B_{j,d}(t)=B(t|t_j,\cdots,t_{j+d+1})$ is sometimes useful. For example, if $d \ge 2$ and if we set $(t_j,\cdots,t_{j+d},t_{j+d+1})=(a,b,\cdots,c,d)$, then (19) can be written

$$B(t|a,b,\cdots,c,d)(t)=\frac{t-a}{c-a}B(t|a,b,\cdots,c)(t)$$
$$+\frac{d-t}{d-b}B(t|b,\cdots,c,d)(t). \tag{20}$$

We consider an interpolation problem at a set of gridded data $(x_i, y_i, f_{ij})_{i=1,j=1}^{m_1,m_2}$, where $a=x_1<x_2<\cdots<x_{m_1}=b$ and $c=y_1<y_2<\cdots<y_{m_2}=d$. For each i,j, we can think of f_{ij} as the value of an unknown function $f=f(x,y)$ at the point (x,y). We think of S_1 and S_2 as two univariate piecewise cubic spline spaces $S_1=$ span $\{\phi_1,\cdots,\phi_{m_1}\}$ and

$S_2=$ span $\{\varphi_1,\cdots,\varphi_{m_2}\}$, where the ϕ's and φ's are bases of cubic B-splines for the two spaces. With g in the form $g(x,y)=\sum_{p=1}^{m_1}\sum_{q=1}^{m_2} c_{p,q}\varphi_q(y)\phi_p(x)$, the above interpolation conditions lead to a set of equations

$$\sum_{p=1}^{m_1}\sum_{q=1}^{m_2} c_{p,q}\varphi_q(y_j)\phi_p(x_i)=f_{ij} \tag{21}$$

for all $i=1,\cdots,m_1$ and $j=1,\cdots,m_2$. This double sum can be split into two sets of simple sums

$$\sum_{p=1}^{m_1} d_{p,j}\phi_p(x_i)=f_{ij} \tag{22}$$

$$\sum_{q=1}^{m_2} c_{p,q}\varphi_q(y_j)=d_{p,j}. \tag{23}$$

We can interpret (22) and (23) as follows:

$$\begin{pmatrix} \phi_1(x_1) & \phi_2(x_1) & \cdots & \phi_{m_1}(x_1) \\ \phi_1(x_2) & \phi_2(x_2) & \cdots & \phi_{m_1}(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ \phi_1(x_{m_1}) & \phi_2(x_{m_1}) & \cdots & \phi_{m_1}(x_{m_1}) \end{pmatrix} \begin{pmatrix} d_{1,j} \\ d_{2,j} \\ \vdots \\ d_{m_1,j} \end{pmatrix} = \begin{pmatrix} f(x_1,y_j) \\ f(x_2,y_j) \\ \vdots \\ f(x_{m_1},y_j) \end{pmatrix} \tag{24}$$

After solving the linear systems (23) and (24), we can determine the control points $c_{i,j}\ (i=1,2,\cdots,m_1, j=1,2,\cdots,m_2)$. Substituting the control points into $g(x,y)=\sum_{p=1}^{m_1}\sum_{q=1}^{m_2} c_{p,q}\varphi_q(y)\phi_p(x)$, we find a piecewise cubic spline surface $g(x,y)$ such that $g(x_i,y_j)=f_{ij}$ for

$$\begin{pmatrix} \varphi_1(y_1) & \varphi_2(y_1) & \cdots & \varphi_{m_2}(y_1) \\ \varphi_1(y_2) & \varphi_2(y_2) & \cdots & \varphi_{m_2}(y_2) \\ \vdots & \vdots & \vdots & \vdots \\ \varphi_1(y_{m_2}) & \varphi_2(y_{m_2}) & \cdots & \varphi_{m_2}(y_{m_2}) \end{pmatrix} \begin{pmatrix} c_{i,1} \\ c_{i,2} \\ \vdots \\ c_{i,m_2} \end{pmatrix} = \begin{pmatrix} d_{i,1} \\ d_{i,2} \\ \vdots \\ d_{i,m_2} \end{pmatrix} \tag{25}$$

$i=1,2,\cdots,m_1$ and $j=1,2,\cdots,m_2$. With this interpolating cubic spline surface $g(x,y)$, we can determine the approximate volume in the following way.

$$\text{Volume} \simeq \int_a^b\int_c^d \sum_{p=1}^{m_1}\sum_{q=1}^{m_2} c_{p,q}\varphi_q(y)\ \phi_p(x)\,dy\,dx \tag{26}$$

Note that there are many ways to determine an appropriate knot vector satisfying the Shoenberg-Whitney nesting conditions for a piecewise cubic spline surface interpolating the given data. In this paper, we only concentrate on the piecewise cubic spline without boundary conditions and suggest method for the choice of an appropriate knot vector which makes cubic B-splines. We consider nonuniform knot vector, and present some computational results of these two cases in the following section.

3. Some Computational Results

We use Maple software to implement our proposed algorithm. We test two examples with several cases given by Chen and Lin (1991), spot height method, line of best fit method(1987). The first example is f(x, y) = $\sqrt{400+y^2}/y$ for $1 \leq x \leq 121$ and $1 \leq y \leq 91$. There are three cases for the first example:

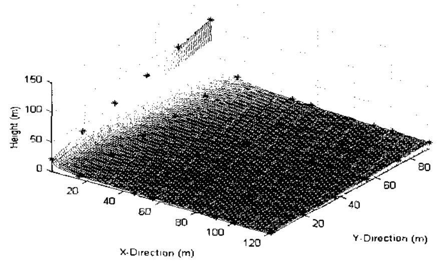

Fig. 1 An example terrain of $\sqrt{400+y^2}/y$

Table 1 The height data to case1 of $\sqrt{400+y^2}/y$

y \ x	1m	16m	41m	51m	91m	121m
1m	20.02	20.02	20.02	20.02	20.02	20.02
16m	1.60	1.60	1.60	1.60	1.60	1.60
31m	1.19	1.19	1.19	1.19	1.19	1.19
46m	1.09	1.09	1.09	1.09	1.09	1.09
61m	1.05	1.05	1.05	1.05	1.05	1.05
76m	1.03	1.03	1.03	1.03	1.03	1.03
91m	1.02	1.02	1.02	1.02	1.02	1.02

Case 1; a 5 X 6 grid, with unequal intervals [1 16 41 51 91 121] in the x- direction, but with equal intervals [1 16 31 46 61 76 91] in the y- direction, Case 2; a 5 X 6 grid, with equal intervals [1 25 49 73 97 121] in the x-direction, but with unequal intervals [1 11 31 41 71 81 91] in the y-direction,

Case 3; a 5 X 6 grid, with both unequal intervals [1 16 41 51 91 121] in the x-direction and [1 11 31 41 71 81 91] in the y-direction. The second example is f(x, y) = (20 +y)/ $\sqrt{x}$ for $1 \leq x \leq 121$ and $1 \leq y \leq 91$. There are three cases for the first example:Case 1; a 6 X 5 grid, with equal intervals [1 21 41 61 81 101 121] in the x-direction, but with unequal intervals [1 26 36 66 81 91] in the y-direction,

Table 2 The height data to case2 of $\sqrt{400+y^2}/y$

y \ x	1m	25m	49m	73m	97m	121m
1m	20.02	20.02	20.02	20.02	20.02	20.02
11m	2.08	2.08	2.08	2.08	2.08	2.08
31m	1.19	1.19	1.19	1.19	1.19	1.19
41m	1.11	1.11	1.11	1.11	1.11	1.11
71m	1.04	1.04	1.04	1.04	1.04	1.04
81m	1.03	1.03	1.03	1.03	1.03	1.03
91m	1.02	1.02	1.02	1.02	1.02	1.02

Table 3 The height data to case3 of $\sqrt{400+y^2}/y$

y \ x	1m	16m	41m	51m	91m	121m
1m	20.02	20.02	20.02	20.02	20.02	20.02
11m	2.08	2.08	2.08	2.08	2.08	2.08
31m	1.19	1.19	1.19	1.19	1.19	1.19
41m	1.11	1.11	1.11	1.11	1.11	1.11
71m	1.04	1.04	1.04	1.04	1.04	1.04
81m	1.03	1.03	1.03	1.03	1.03	1.03
91m	1.02	1.02	1.02	1.02	1.02	1.02

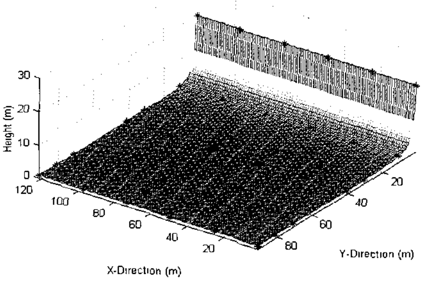

Fig. 2 An example terrain of (20 +y)/ $\sqrt{x}$

Case 2; a 6 X 5 grid, with unequal intervals [1 16 46 56 91 101 121] in the x-direction, but with equal intervals [1 19 37 55 73 91] in the y-direction, Case 3; a 5 X 6 grid, with both unequal intervals [1 16 46 56 91 101 121] in the x-direction and [1 26 36 66 81 91] in the y-direction. For the choice of a knot vector satisfying the Schoenberg-Whitney nest conditions, the uniform knot vector is the simplest method.

Table 4 The height data to case1 of (20 +y)/ $\sqrt{x}$

y \ x	1m	21m	41m	61m	81m	101m	121m
1m	21.00	4.58	3.28	2.69	2.33	2.09	1.91
26m	46.00	2.08	2.08	2.08	2.08	2.08	2.08
36m	56.00	12.22	8.75	7.17	6.22	5.57	5.09
66m	86.00	18.77	13.43	11.01	9.56	8.56	7.82
81m	101.00	22.04	15.77	12.93	11.22	10.05	9.18
91m	111.00	24.22	17.34	14.21	12.33	11.05	10.09

Table 5 The height data to case2 of $(20 + y)/\sqrt{x}$

y \ x	1m	16m	46m	56m	91m	101m	121m
1m	21.00	5.25	3.10	2.81	2.20	2.09	1.91
19m	39.00	9.75	5.75	5.21	4.09	3.88	3.55
37m	57.00	14.25	8.40	7.62	5.98	5.67	5.18
55m	75.00	18.75	11.05	10.02	7.86	7.46	6.82
73m	93.00	23.25	13.71	12.43	9.75	9.25	8.45
91m	111.00	27.75	16.37	14.83	11.64	11.05	10.09

Table 6 The height data to case3 of $(20 + y)/\sqrt{x}$

y \ x	1m	16m	46m	56m	91m	101m	121m
1m	21.00	5.25	3.10	2.81	2.20	2.09	1.91
26m	46.00	11.50	6.78	6.15	4.82	4.58	4.18
36m	56.00	14.00	8.26	7.48	5.87	5.57	5.09
66m	86.00	21.50	12.68	11.49	9.02	8.56	7.82
81m	101.00	25.25	14.89	13.50	10.59	10.05	9.18
91m	111.00	27.75	16.37	14.83	11.64	11.05	10.09

Therefore, we test the case of uniform knot vector. We take the uniform knot vector [0, 0, 0, 0, $\frac{122}{3}$, $\frac{244}{3}$, 122, 122, 122, 122,] in the x-direction and [0, 0, 0, 0, 23, 46, 69, 92, 92, 92, 92] in the y-direction for the first ex-ample. Also, for the second example, we have the uniform knot vector [0, 0, 0, 0, $\frac{122}{4}$, $\frac{244}{4}$, $\frac{366}{4}$, 122, 122, 122, 122] in the x-direction and [0, 0, 0, 0, $\frac{92}{3}$, $\frac{184}{3}$, 92, 92, 92, 92] in the,y-direction. For the nonuniform knot vector, we test the first example with [1, 1, 1, 1, 31, 61, 122, 122, 122, 122] in the x-direction and [1, 1, 1, 1, 7, 46, 69, 92, 92, 92, 92] in the y-direction. Also, we test the second example with [1, 1, 1, 1, 10, $\frac{121}{2}$, $\frac{363}{4}$, 122, 122, 122, 122] in the x-direction and [1, 1, 1, 1, $\frac{91}{3}$ $\frac{182}{3}$, 92, 92, 92, 92] in the y-direction. Here we can summarize some computational results. example with [1, 1, 1, 1, 10, $\frac{121}{2}$, $\frac{363}{4}$, 122, 122, 122, 122] in the x-direction and [1, 1, 1, 1, $\frac{91}{3}$ $\frac{182}{3}$, 92, 92, 92, 92] in the y-direction. Here we can summarize some computational results.

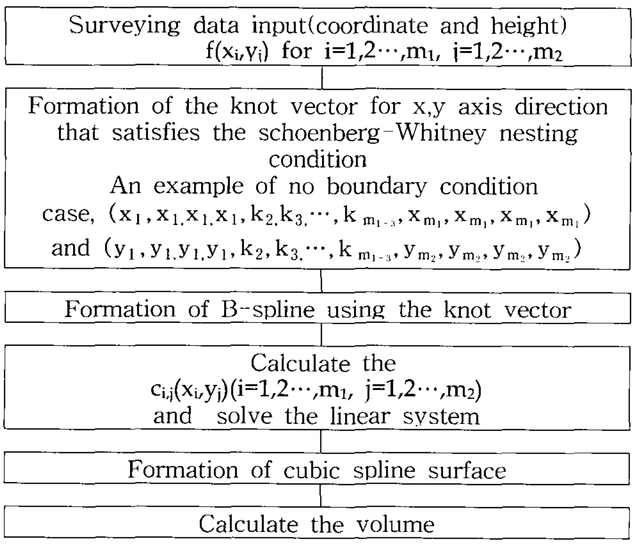

Fig. 3 Flow chart of a proposed method.

Table 7 Application results for example terrain of $\sqrt{400+y^2}/y$

Method	Case1		Case2		Case3	
	Volume (m^3)	Error (%)	Volume (m^3)	Error (%)	Volume (m^3)	Error (%)
Exact volume	17,109.13	-	17,109.13	-	17,109.13	-
Spot	29,664.00	73.381	24,906.00	45.571	24,018.00	40.381
Line	29817.00	74.280	24519.00	43.310	24519.00	43.310
Chen	26,178.85	53.011	18,076.02	5.651	18,076.02	5.651
Proposed	16,945.15	0.960	17,488.18	2.220	17,488.18	2.220

Table 8 Application results for example terrain of $(20 + y)/\sqrt{x}$

Method	Case1		Case2		Case3	
	Volume (m^3)	Error (%)	Volume (m^3)	Error (%)	Volume (m^3)	Error (%)
Exact volume	118,800.00	-	118,800.00	-	118,800.00	-
Spot	149,009.30	25.429	141,614.50	19.204	141,613.80	19.204
Line	178,025.76	49.850	190,359.50	60.240	190,374.40	60.250
Chen	139,567.80	17.481	122,008.90	2.701	121,859.90	2.576
Proposed	117,657.45	0.960	119,116.21	0.260	119,116.21	0.260

Since the ground surface is expressed mathematically, the exact volume can be determined using intergration as 17,109.13m^3 and 118,800m^3. From the results of cases 1, 2, 3 of example 1, as shown in Table 7, the error produced using a spot heigt method, line best of fit, Chen and Lin method is 2.5~77 times larger than that using a proposed method. Similarly, the results of cases 1, 2, 3 of example 2, as shown in Table 8, the error produced using a spot heigt method, line best of fit, Chen and Lin method is 9.9~232

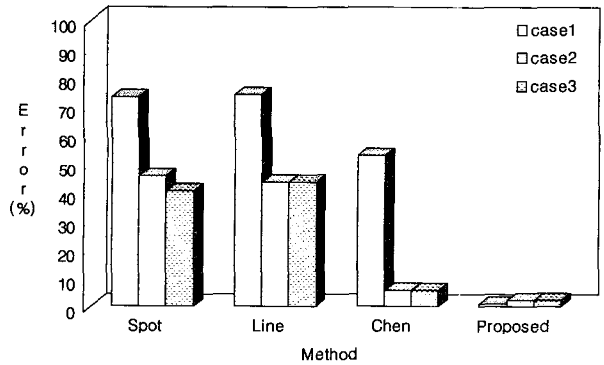

Fig. 4 An earthwork error of the terrain $\sqrt{400+y^2}/y$

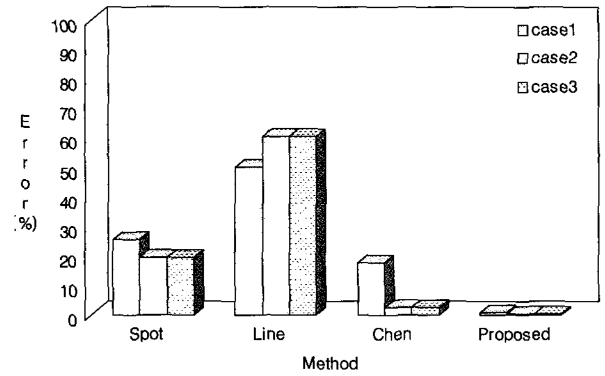

Fig. 5 An earthwork error of the terrain $(20 + y)/\sqrt{x}$

times larger than that using a proposed method. Also, from the results of cases 1, 2, 3 of example 1 and 2 as shown in Table 7, 8, the error produced using an improper grid is 2.3 ~ 9.4 times larger than that using a proper grid.

4. Conclusions

In this paper, a new formula has been developed for estimating the volume of a borrow pit excavation, based on an extension of the cubic spline polynomial, without boundary condition. Because the grid of a borrow pit is constructed by choosing the variational points of the ground profile and dividing the area of the pit into rectangles of unequal interval, the presented formula can be applied to the case in which the pit is divided into a grid with unequal intervals. From this study, the following comments may be made:

(1) The proposed method is applied to two examples, and the results show that it is generally better than the spot height method, line of best fit method, and the Chen and Lin method.

(2) We propose an algorithm of finding a spline surface, which interpolates the given data, and is an appropriate method to calculate the earthwork. We present some computational results showing that our proposed method provides better accuracy than Chen and Lin's method.

(3) For maximum accuracy in estimating the volume of a pit excavation, it is very important to select the proper points (the variational point of the ground profile) when constructing the grid.

(4) The mathematical models of the conventional methods have a common drawback: the modeling curves form peak points at the joints. To avoid this drawback, the cubic spline polynomial is chosen as the mathematical model of the new method. From the characteristics of the cubic spline polynomial, the modeling curve of the new method is smooth and matches the ground profile well.

Acknowledgement

This research was partially supported by Dongeui University (2002).

References

Mun, D.Y., (1995). "A study on the earthwork calculation for reclamation", Int. J. of KCORE., Vol 9, No 1, pp 61-70.

Anderson, J.M., Michael, E.M. and Woolnough, D.E. (1985). Introduction to surveying, McGraw-Hill Book Co., Inc., New York.

Chambers, D.W. (1989). "Estimating pit excavation volume using unequal intervals", J. of ASCE, Vol 115, No 4, pp 390-411.

Chen, C.S. and Lin, H.C. (1991). "Estimating pit excavation volume using cubic spline volume formula, J. of ASCE, Vol 117, No 2, pp 51-66.

Easa, S.M. (1993). "Smooth boundary approximation for computing area of irregular region", J. of ASCE, Vol 119, No 3, pp 86-101.

Easa, S.M. (1988). "Estimating pit excavation volume using nonlinear ground profile", J. of ASCE, Vol 114, No 2, pp 71-83.

Farin, G. (1988). Curves and surfaces in computer aided geometric design. Academic Press Inc, New York.

Moffit. F.H. and Bossler, J.D. (1998). Surveying, Harper & Row Publishers Inc, New York.