# Navier-Stokes 유체의 최적 제어

## Optimal Control of steady Incompressible Navier-Stokes Flows


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### 요 지

본 연구의 목적은 Navier-Stokes 유체의 최적 제어 문제의 해를 얻을 수 있는 효과적인 수치해석기법을 개발하고, 이를 물체의 항력(drag)을 최소화하는 문제에 적용하는데 있다. 본 연구는 항력을 줄인다는 산업적인 중요성과 함께 최적 제어를 위한 하나의 효과적인 최적화 기법의 모델을 제공하고 있다.

항력을 줄이기 위한 방법으로써 물체의 경계면에서 유체의 흡입(suction)과 방출(injection)이라는 기법을 사용하여 경계면에서 속도를 제어하였고, 목적함수로써 항력을 표현하기 위하여 에너지 소실의 변화율을 사용하였다. 컴퓨터 용량을 최소화하고 최적화에서의 해의 보장성과 경제성을 위하여, Navier-Stokes의 해석을 위하여 페널티 방법을 사용하였고 최적화 기법을 위해서는 SQP 방법을 사용하였다. 그리고 Navier-Stokes 유체는 대단히 비선형성을 나타내기 때문에 최적화를 수행하기에는 매우 힘들다. 이를 위하여 연속기법(continuation technique)을 사용하였다.



**핵심용어** : 최적제어, Navier-Stokes 유체, SQP 기법, 리듀스트 헤이시언 기법, 뉴턴법, 유사 뉴턴법, 흡입, 방출



### Abstract

The objective of this study is to develop efficient numerical method to enable solution of optimal control problems of Navier-Stokes flows and to apply these technique to the problem of viscous drag minimization on a bluff body by controlling boundary velocities on the surface of the body. In addition to the industrial importance of the drag reduction problem, it serves as a model for other more complex flow optimization settings, and allows us to study, modify, and improve the behavior of the optimal control methods proposed here.

The control is affected by the suction or injection of fluid on portions of the boundary, and the objective function represents the rate at which energy is dissipated in the fluid. This study shows how reduced Hessian successive quadratic programming method, which avoid converging the flow equations at each iteration, can be tailored to these problems.



***Keywords*** : *optimal control, navier-stokes flow, SQP method, reduced hessian method, newton method, quasi-newton method, suction, injection*



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## 1. Introduction

Control of the flow by suction and injection is one of the most promising method for controlling the boundary layer.[1] The first trial in this regard was reported in 1940.[2] They found that the drag on a cylinder could be reduced by sucking fluid out through a slit on the back side of the cylinder. The feasibility of using suction holes on a aircraft wings as a means of delaying seperation and reducing drag was demonstrated as early as the 1950s.[3)~5] The extreme example in this regard is the Northrop-21 flight test program. The concept of boundary layer control led flight engineers to expect that the performance of the aircraft could be greatly improved, particularly with respect to range and ecomomics of operation. Over 200 flight tests of two X-21 aircraft were performed to investigate laminar flow control on swept wings and to demonstrate its technical feasibility. Many thin and closely spaced spanwise suction slots were set on the surfaces of the X-21 wing. Suction was applied at each slit to maintain laminar flow, based on readings from flow monitors that provided information on stability of the boundary layer. Laminar flow had been maintained as high as Reynolds number $4.73\times10^7$, and airplane performance(as measured by the lift-to-drag ratio) was increased by 25%.

Until recently, flow control problems have been mainly solved by trial-and-error parameter studies based on experiments, analytical solutions, or numerical simulations. Reviews of such approaches can be found in references [22] and [23]. But these approaches cannot be guaranteed to obtain a minimum drag solution. The problem is inherently one in optimization of systems governed by partial differential equations, in particular the Navier-Stokes equations. To rigorously guarantee a optimum solution, one must consider the optimality conditions stemming from the optimization formulation of the drag mimimization problem, and develop methods for converging to solutions of these optimality conditions. Indeed, as has been pointed out by Gad-el-Hak in a recent review[6]:

"*Delaying transition using suction is a mature technology, where most of the remaining problems are in the maintainability and reliability of suction surfaces, and the optimization of suction rate and distribution.*"

Unfortunately, the problem of optimizing Navier-Stokes flows is very challenging. It leads to optimization problems with tens of thousands (or more) of nonlinear constraints, which until recently have been out of the reach of nonlinear numerical optimization methods. Furthermore, commonly-available computers have not been powerful enough to warrant such an approach.

However, with the development of Sequential Quadratic Programming(SQP) methods for solving nonlinearly-constrained optimization problems, and with the emergence of high-end workstations capable of performing tens of millions of operations per second, we are at a point at which it becomes possible to contemplate the formulation and solution of optimal flow control problems that are governed by Navier-Stokes equations. The SQP method is based on the iterative formulation and solution of quadratic programming subproblems. However, direct application of SQP methods to flow control problems is still not possible due to the size and complexity of the problem. These techniques must be tailored to the structure of flow problems. The goal of this research is to develop SQP method that exploit the structure and nature of optimization problems constrained by discrete Navier-Stokes equations, and to demonstrate the success of these methods by applying them to optimal boundary control problems in two dimensions. In this work we limit ourselves to time-independent flows, so that the optimal control is found for steady-state conditions.

In the flow control optimization problem, we may have tens of thousands of variables and constraints. By eliminating the flow constraints (discrete form of the Navier-Stokes equations) at each control iteration, as well as the flow variables(velocities), we reduce the size of the optimization problem considerably. The remaining optimization problem is then of dimension of the control variables(the suction/injection velocities on the boundary). The price that is paid for this dimensional reduction is that the objective function then becomes an implicit function of the control variables through solution of the state equations, and one needs special techniques to find its derivatives(known as *sensitivity analysis*). As a result, the nonlinearity of the objective function increases. Furthermore, the requirement of having to solve the Navier-Stokes equations (which we refer to as the analysis problem) at each iteration is quite onerous. In this study, we develop method to overcome these difficulties.

## 2. Selected previous work

After Prandtl reported in 1904 that the drag on a cylinder could be efficiently reduced by sucking stagnated fluid away from the boundary[2], considerable research has been conducted and is still continuing on developing devices for affecting suction and injection, as well as methods for controlling these devices. We will not discuss the design of these devices here, but will give an overview of methods for their control.

These control methods are mainly divided into two types, passive control and active control. Passive control includes methods that modify a flow without power input. Examples of passive control include permanent features of the geometry such as riblets[7),8)], as well as the use of a compliant wall that deforms in response to the flow[9)]. On the other hand, active control techniques include methods that manipulate an external boundary layer flow using some external power source, with the goal of achieving transition delay, separation postponement, recirculation elimination, or drag reduction. Examples include the suction and/or injection of fluid through orifices on a flow boundary, or micro-electromechanical devices that deploy in response to particular flow conditions.[1)] Our primary concern here is with active control methods, as they are generally regarded as being the most effective. Therefore, we will review several selected active control techniques. We further subdivide these active control techniques into sensor-based and model-based methods, depending on whether or not they are driven by a mathematical model of the flow field.

### 2.1 Sensor-based scheme

Sensor-based methods use various measured flow quantities to determine values of suction/injection. As such, they must rely on physical intuition or asymptotic solutions to suggest levels of appropriate boundary control.

#### 2.1.1 Direct cancellation scheme

The basic idea of a direct cancellation scheme is to cancel the fluid moving toward the wall (sweep) with injection, and fluid moving away from a wall(ejection) using suction. The intention is to use sensors to determine fluid velocities a distance away from the wall, and generate suction/injection based on the sensed velocities. Choi *et al*. investigated the possibility of this scheme using a fully-developed channel flow.[10)] Because of the difficulties in constructing and deploying such sensors, the study was conducted numerically; that is, the Navier-Stokes equations were solved to produce a numerical velocity field, which was then used to drive the suction/injection process according to the criteria mentioned above. Skin-friction was used to characterize the drag, and was measured in terms of the change in the mean pressure gradient necessary to drive the flow with a fixed mass flow rate. When a "sensor" detected sweep or ejection at a distance $y_d$ from the wall, an opposing velocity of the same magnitude was imposed on the wall in an effort to cancel it, i.e. when a sensor detected sweep, an equal amount of opposing velocity(injection) was added and when ejection was detected, the same amount of opposing velocity(suction) was applied. Figure 1 shows the basic idea of this scheme.

Several computations were performed for different $y_d$ to examine the effect of the sensor location. The best location yielded about 30% drag reduction. Direct cancellation schemes have two primary difficulties: (1) they rely on velocity sensors within in the flowfield, which are difficult to deploy, and (2) they cannot guarantee an optimum control.

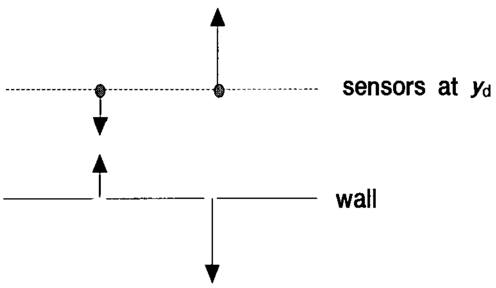


Figure 1 Cancellation scheme

2.1.2 Indirect cancellation scheme

In response to the impracticality of velocity-based sensing methods, other techniques have been developed that sense indirect quantities at the wall, such as wall pressure, streamwise wall-velocity gradient, and higher-order terms in Taylor series expansions of the normal velocity component near the wall. Choi *et al*. also examined these schemes. The difficulty here is finding good correlations between sensed wall quantities and sweep and ejection. In fact, these schemes yielded only about 6% reduction of drag, which is comparable to drag reduction by such passive control methods as riblets. Other types of sensors have been also been investigated: hot films by Alfredsson *et al*.,[11] floating element sensor by Haritonidis *et al*.,[12] piezo-electric foils by Nitsche *et al*.,[13] and surface acoustic wave sensor by Varadan *et al*..[14),15] The difficulty with all these methods is in making a global decision on drag reduction given only local information.

## 2.2 Model-based methods

Certainly any of the sensor-based methods described above can be converted into a model-based method as follows: replace the measurement of flow parameters by numerical solution of the governing fluid equations. Thus, the flowfield and derived quantities are available, and this can be used to drive the rules for specifying boundary controls. However, the power of model-based methods is that they can be used to find the optimum control set, and we will focus on such a use below.

2.2.1 Mathematical analysis

At this point, we have sufficient experimental evidence for the viability of drag reduction via suction/injection. Furthermore, there are calls in the literature for developing truly optimal methods for controlling flows.[16] What would be nice is to have some mathematical results concerning existence and regularity of solutions, with which we can proceed with confidence to develop numerical methods for determining these optimal solutions. Indeed, such results have recently been established by Gunzburger *et al*..[16),17] They have, for the stationary problem, shown the existence of optimal controls and states, derived first-order necessary conditions that must be satisfied by a continuous optimum, and given error estimates for finite element approximations of the optimal control and states.

2.2.2 First-order optimal control

It has recently been recognized that sensor-based methods "fail to provide us with a rigorous theory to determine the most efficient feedback control law for a given flow control problem".[12] Therefore, in the past several years, researchers have begun to formulate and solve the control problem as an optimization problem. First results towards this end have been obtained by Bewley *et al.*.[18] The approach taken here is to compute the gradient of the objective(e.g. drag on a body). The gradient of the objective function at any point is a vector in the direction of the greatest local increase in objective function.

Therefore they move in a direction opposite to the gradient. This is known in the optimization literature as the *steepest descent method*,[19] and is actually the poorest choice for an optimization technique. In practice steepest descent techniques converge too slowly to be effective and, on poorly-conditioned problems(i.e. when the ratio of the maximum to minimum eigenvalue of the Hessian matrix at any point is large), may fail to converge at all, a situation encountered by the authors.

On the other hand, SQP methods are generally regarded as being the most efficient optimization techniques, since they make use of higher order information in addition to avoiding satisfaction of the constraints at each iteration.[20] As discussed in the introduction, it is not straightforward to apply SQP methods to flow optimization problems, since the constraint sets produced are very large and nonlinear a fact that motivates the present work.

## 3. Problem definition

We consider flow around a bluff body immersed in a stream of fluid. At low Reynolds number, the flow divides and reunites smoothly but with increasing Reynolds number the flow separates and recirculates on the downstream side, and the wake behind the body becomes unstable. Our aim in this study is to inhibit this boundary layer separation and flow recirculation numerically by controlling the velocities on the surface of the body, using optimization method.

We restrict ourselves to the case of the time-independent flows. An outline of the our approach to flow control is as follows:

1) Initially, the velocities on the surface of the bluff body are assumed to be zero, i.e. there is a no-slip boundary condition on the surface.
2) Define $n$ disjoint holes on the surface of the body. Since flow separation and recirculation occur on the back side of the body, we confine the holes that region. Suction or injection can be applied at each hole by controlling the two velocity components $(u, v)$ in two dimensions, or the three velocity components $(u, v, w)$ in three dimensions.
3) Find the optimal velocity vector at each hole, representing the optimal injection or suction, that minimizes the objective function, subject to the flow equations.

As an objective function, we will use the rate of dissipation of energy due to viscosity, which is equivalent to the drag force on the body, in the case of an external incompressible flow. The flow is modeled by the incompressible steady-state Navier-Stokes equations.

Therefore the optimization problem can be expressed as :

$$\text{minimize } 2\mu\int_{\Omega}[\,\mathbf{D}:\mathbf{D}\,]\,d\Omega \quad (1)$$

$$\text{subject to : } -\mu\triangle\,\mathbf{u}+\rho(\mathbf{u}\cdot\nabla)\,\mathbf{u}+\nabla p=\mathbf{0} \quad (2)$$

$$\nabla\cdot\mathbf{u}=\mathbf{0} \quad (3)$$

where $D=D(\mathbf{u})=(\nabla\mathbf{u}+\nabla\mathbf{u}^T)/2$, $\mu$ is the dynamic viscosity, $\rho$ is the density, $\mathbf{u}$ is the flow velocity vector, $p$ is the pressure. Here, the symbolic " : " represents the scalar product of two tensors, so that

$$\nabla\mathbf{u} : \nabla\mathbf{v} = \sum_{i,j}\frac{\partial u_i}{\partial x_j}\frac{\partial v_i}{\partial x_j}$$

We reduce the size of the problem by employing a *penalty method* : let us relax equation (3) by replacing it with

$$\nabla \cdot \mathbf{u} = -\varepsilon p \tag{4}$$

Cleary as $\varepsilon \to 0$, we recover the original equation; in fact, the error in the derivative of $\mathbf{u}$ is of order $\varepsilon$.[19] By introducing the pressure in the mass equation, we can eliminate it from the problem by solving for $p$ in equation (4) and substitution expression into equation (2).

In general, it is not possible to solve infinite dimensional optimization problems such as equations (1)~(3) in closed form. Thus, we seek numerical approximations. Here, we use a Galerkin finite element method.

## 4. Formulation of the optimization problem

The continuous optimization problem has been defined in equations (1)~(2), that is: given a set of disjoint holes, find values of velocities at each of those holes that minimize the dissipation function, i.e. the rate at which heat energy is conducted into the fluid due to viscosity, subject to incompressible Navier-Stokes equations. Using the finite element approximation, we arrive at a discretized form of the optimal control problem, which in symbolic form is:

$$\begin{aligned}&\text{minimize} && \Phi(\mathbf{u}, \mathbf{b})\\ &\text{subject to} && \mathbf{h}(\mathbf{u}, \mathbf{b})=\mathbf{0}\end{aligned} \tag{5}$$

Here, the constraints $\mathbf{h}=\mathbf{0}$ are the discrete form of the Navier-Stokes equations. We have partitioned the velocities $\mathbf{u}$ into the *state variables* $\mathbf{u}$(i.e. the velocities at all nodes other than those that lie on suction/injection holes), and the *control variables* $\mathbf{b}$(i.e. the velocities of nodes where suction/injection is applied). The objective function $\Phi$ is related to velocities by

$$\Phi = \frac{1}{2}\mathbf{u}^T\mathbf{J}\mathbf{u} \tag{6}$$

and again depends on both state velocities $\mathbf{u}$ and control velocities $\mathbf{b}$. Here $\mathbf{J}$ is the portion of the Jacobian matrix that depends on viscosity.

We may also choose to augment the constraints by bounds on the control variables.

The problem is then one in nonlinearly constrained smooth optimization.

In general it is not straightforward to apply SQP methods to flow optimization problems of the equation (5), since the constraint sets produced are very large and nonlinear. Therefore in this chapter we pursue a decomposition of the problem into the state space and the control space, as follows: solve at each iteration the discrete Navier-Stokes equations ($\mathbf{h}(\mathbf{u}, \mathbf{b})=\mathbf{0}$) for the state variables ($\mathbf{u}$) given values of the control variables ($\mathbf{b}$). Thus, we have eliminated the state equations from the constraint set, and we have eliminated the state variables from the set of optimization variables. As a result of this decomposition, the state variables become an implicit function of the control variables, the implicit function being the flow solution itself.

Thus, we can write the optimization problem as the unconstrained optimization problem:

$$\text{minimize } \Phi(\mathbf{u}(\mathbf{b}), \mathbf{b}) \tag{7}$$

The dimension of the optimization problem is now greatly reduced.

## 5. Solution method for discrete Navier-Stokes equations

The discrete form of the Navier-Stokes equations are a system of nonlinear algebraic equation, i.e. $\mathbf{h}(\mathbf{u})=\mathbf{0}$, where $\mathbf{h}$ represents the residuals and $\mathbf{u}$ represents the vector of unknown velocities. A very effective method for solving these equations is Newton's method. It is well-known that this method is locally quadratically convergent, that is that close to the solution, the error is squared between subsequent iterations, i.e.the number of correct digits is doubled.

Following is a summary of the steps of Newton's method:

1) update $\mathbf{h}_k$ and $\mathbf{J}_k$
2) check convergence criterion : if $\|\mathbf{h}_k\| \leq \gamma$, then terminate; otherwise go to Step 3
3) solve $\mathbf{J}_k\ \mathbf{p} = -\mathbf{h}_k$
4) $\mathbf{u}_{k+1} = \mathbf{u}_k + \mathbf{p}$
5) go to Step 1

where $\mathbf{h}_k$ and $\mathbf{J}_k$ indicate evaluation of $\mathbf{h}$ and $\mathbf{J}$ at $\mathbf{u}_k$.. We use $\gamma = 10^{-7}$.

Because of the large dimensions involved, we must use an efficient method to solve this system of equations. Later, sensitivity analysis also requires the repeated solution of linear systems having the same coefficient matrix, but different right-hand sides, each corresponding to a different control variable. This, as well as the fact that $\mathbf{J}$ is unsymmetric, favors sparse direct methods for solution. Perhaps the most efficient code for factorization of sparse unsymmetric matrices is the unsymmetric-pattern multifrontal sparse LU factorization code UMFPACK,[21] and we use this code for solution of the linear system arising at each step of Newton's method.

Despite its excellent convergence rate, Newton's method is only locally convergent. In particular for Navier-Stokes equations, an upper bound on the diameter of the convergence "ball" for Newton's method varies as 1/Reynolds number.[19] The consequence is that as the Reynolds number increases, one needs better and better initial guesses to guarantee convergence to a solution; otherwise, divergence may occur. Thus, we use a *continuation strategy* to solve a sequence of problems leading to the Reynolds number of interest, as follows:

1) solve the problem with low Reynolds number.
2) increment Reynolds number.
3) solve the problem using the results of previous step as initial guesses.
4) repeat above steps until final Reynolds number is reached.

Density $\rho$ is used as a parameter to increase Reynolds number. Within each step, we iterate until convergence is achieved, then the converged solution is used as the initial guess for the next step.

## 6. SQP method for solving the optimization problem

The basic SQP iteration can be viewed as:

1) do analysis(obtain $\mathbf{u}_k$ knowing $\mathbf{b}_k$)
2) do sensitivity analysis
3) construct approximate Hessian matrix $\mathbf{B}$
4) check convergence criterion: if OK, then terminate; otherwise go to Step 5
5) find $\mathbf{p}_k$ using QP
6) $\mathbf{b}_{k+1} = \mathbf{b}_k + \alpha \mathbf{p}_k \quad \alpha \in (0,1]$
7) go to Step 1

As mentioned before, the continuation technique is used on the analysis problem.

The straightforward way to integrate this continuation technique into an optimization method is as follows(see Figure 2);

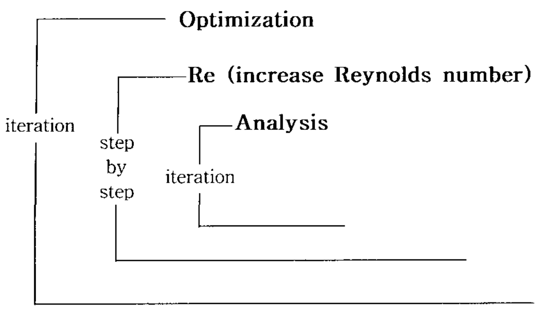


Figure 2 Proposed optimization algorithm

1) use continuation to solve the analysis problem for the given Reynolds number.
2) update the control variables by doing one optimization iteration.
3) repeat the above for each optimization iteration until the optimum is reached.

This method is superior to the *steepest descent method* that we reviewed in chapter 2, because it uses(approximate) curvature information in solving the optimization problem.

## 7. Numerical Example

The method and technique developed here are tested on a problem of flow around an infinite cylinder in two dimensions. However, our methodology and code can be applied to problems of arbitrary geometry, provided that an appropriate mesh is supplied.

Without boundary control, separation for a cylinder is evident already for small Reynolds numbers($Re < 10$); the flow field exhibits two symmetric standing eddies up to around $Re=50$; and beyond this range, the flow becomes increasingly unstable and vortices begin to be spun off asymmetrically. However, based on the experimental results of Prandtl, we expect that application of suction/injection would be capable of keeping the flow more-or-less attached for Reynolds number as high as at least 400. Thus, the steady Navier-Stokes equations are used to model the flow, with the knowledge that they may not be consistent with suboptimal solutions at this value of Re, but at the optimum we expect steady flow. In addition, we make use of symmetry about the midplane of the cylinder to reduce problem size; again, without boundary controls this assumption is not valid, but with boundary controls we expect that it should be.

These assumptions will be validated at the end of this section by showing that the flow field computed using a time-dependent simulation at $Re=400$ is both symmetric and steady when the optimal controls are applied.

Of course, when they are not applied, the flow field is unsymmetric and unsteady.

A problem description is shown in Figure 3.

Using symmetry about the midplane, we obtain the computational domain and boundary conditions shown in Figure 4.

The Reynolds number is determined by

$$Re = \frac{\rho U d}{\mu}$$

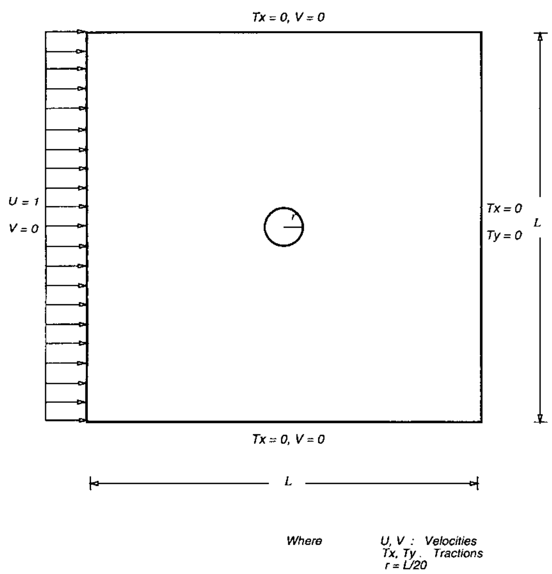


Figure 3 Flow around cylinder

where ***d*** is the diameter of a cylinder. Our target Reynolds number is 400.

Using symmetry and solving the steady Navier-Stokes equations, the flow field shown in Figures 5~7 is obtained.

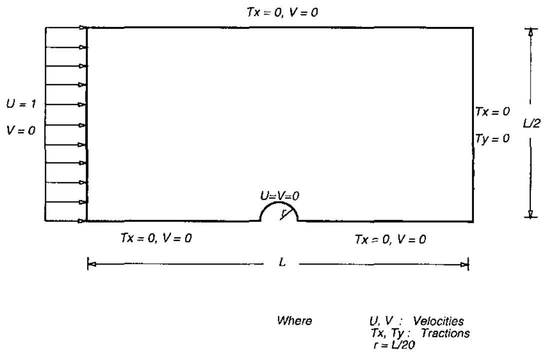

Figure 4 Flow domain

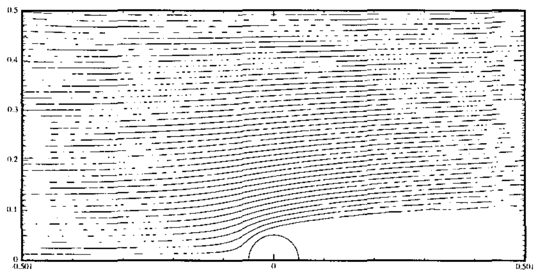
Figure 5 Streamlines, without control

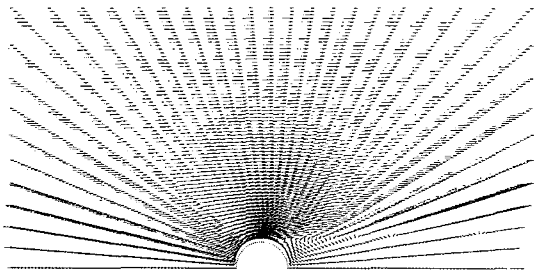
Figure 6 Velocity vectors, without control

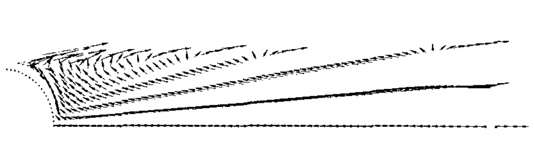
Figure 7 Velocity vectors of back side of the cylinder, without control

Figure 5 shows the streamlines around the cylinder without any control, i.e. with the no-slip condition enforced on the surface of the cylinder.

Figure 6 depicts the velocity vector field again without control. The flow separation and recirculation behind the cylinder are evident from the figures. A detail of the flow field behind the cylinder is shown in Figure 7.

To demonstrate optimal control, we choose five points on the back side of the cylinder.

Fluid is injected into the flow or sucked away from it at these chosen five points with the objective of minimizing the rate of viscous energy dissipation. These points are chosen equally spaced. Each point has two control variables which are the independent components of velocity.

Thus this example has a total ten optimization variables.

Figures 8~9 show the flow that results from optimal suction/injection. The streamlines corresponding to the optimal solution are shown in Figure 8.

The flow pattern appears to be very similar to that of a potential flow. The velocity field corresponding to the optimum is depicted in Figure 9. The direction and magnitude of suction/injection is apparent from the figure.

Suction works to keep the streamlines from separating on the back side of the cylinder, while injection at the base keeps the flow from stagnating and recirculating.

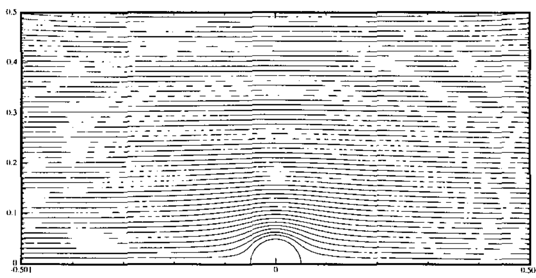
Figure 8 Optimal streamlines

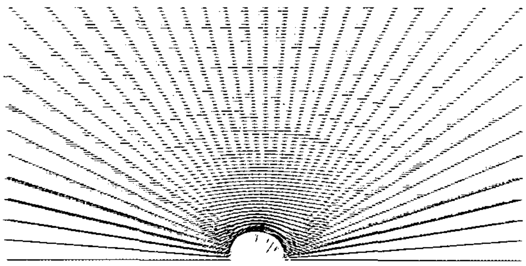

Figure 9 Optimal velocity vectors

Table 1 optimal control velocities and objective functions

| Re | | 100 | 200 | 300 | 400 |
|---|---|---|---|---|---|
| objective | no control | 6.53 | 8.13 | 7.42 | 11.83 |
| | optimal | 3.70 | 3.45 | 3.56 | 4.26 |
| control variables | $b_1$ | 1.59 | 1.43 | 1.20 | 1.06 |
| | $b_2$ | .45 | .48 | .39 | .32 |
| | $b_3$ | 1.48 | .90 | .33 | -.03 |
| | $b_4$ | .41 | .21 | .00 | -.14 |
| | $b_5$ | .92 | -.24 | -.55 | -.66 |
| | $b_6$ | .91 | -.37 | -.81 | -.98 |
| | $b_7$ | -.75 | -.55 | -.35 | -.21 |
| | $b_8$ | -1.05 | -.93 | -.64 | -.41 |
| | $b_9$ | -.22 | -.18 | -.16 | -.15 |
| | $b_{10}$ | -1.63 | -1.24 | -.92 | -.70 |

Table 1 shows the dependence of the optimal solution on different values of Reynolds number. The table includes both initial and final objective function values as well as the optimal values of controls. Note the smooth variation in values of the optimal controls with Reynolds number, suggesting that continuation may be useful.

We have assumed the optimal flow field to be both steady-state and symmetric for Reynolds number as high as 400. Thus, the steady Navier-Stokes equations were used to model the flow. To check the validity of this assumption, we solved our problem with time dependent analysis code over the whole domain.

Figures 10~11 are a snapshot of the flow field at 1 second, showing respectively the velocity vectors and streamlines without any suction or

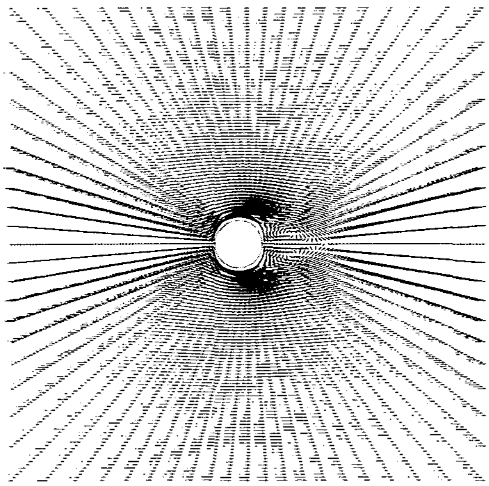

Figure 10 Time dependent velocity vectors, without control, t=1sec

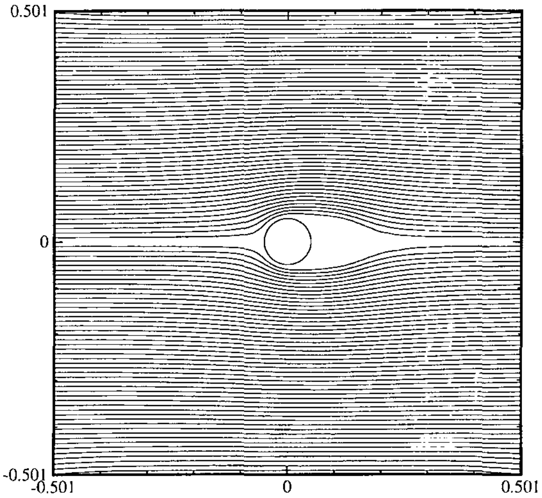


Figure 11 Time dependent streamlines, without control, t=1sec

injection for a flow of Re=400.

Figures 12~13 show velocity vectors and streamlines at 3.5 second.

A Karman vortex street is evident from the figures. Clearly the flow is time dependent and unsymmetric.

Now when the optimal controls are applied at five points on the backside of the cylinder, the flow field in Figures 14~17. Figures 14~15 show velocity vectors and streamlines at 1sec. At 3.5sec, Figures 16~17 show that the flow field is unchanged.

Furthermore, it is symmetric. Thus, our assumptions of steadiness and symmetry at the optimal solution are valid, and allow us to *simplify* the formulation of the optimization problem.

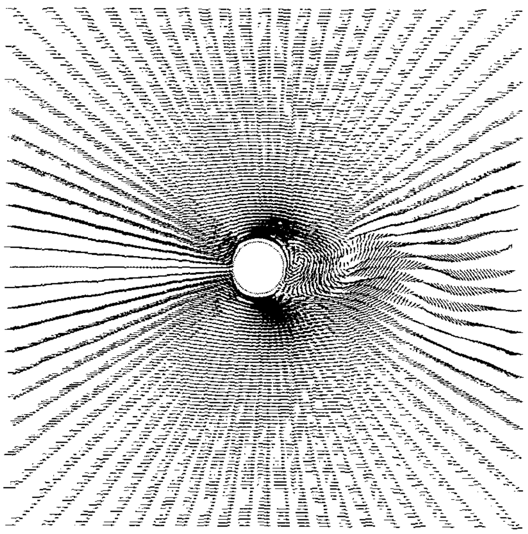

Figure 12 Time dependent velocity vectors, without control, t=3.5sec

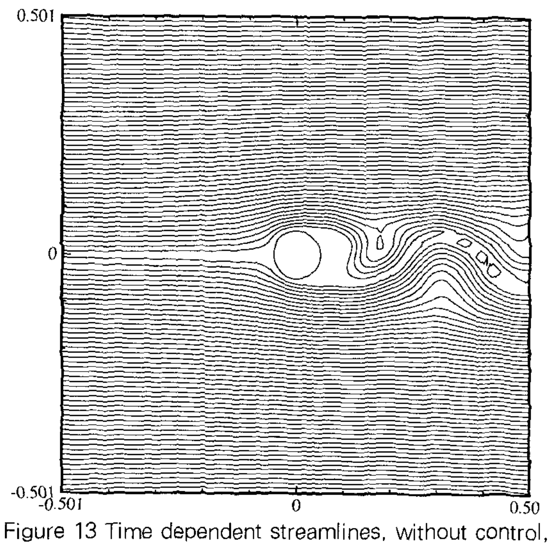


Figure 13 Time dependent streamlines, without control, t=3.5sec

## 8. Conclusions

In this study, we have addressed the problem of the optimal control of a viscous, incompressible fluid. The objective function considered is the drag on a solid body immersed within the flow field, which is equivalent to the rate of dissipation of energy due to the viscosity of the fluid.

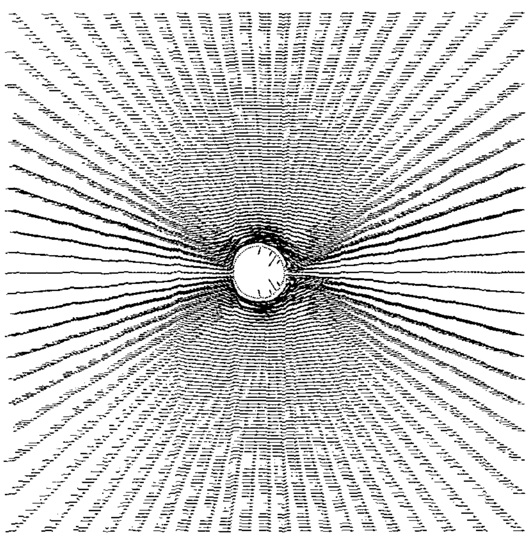

Figure 14 Time dependent velocity vectors, with control, t=1sec

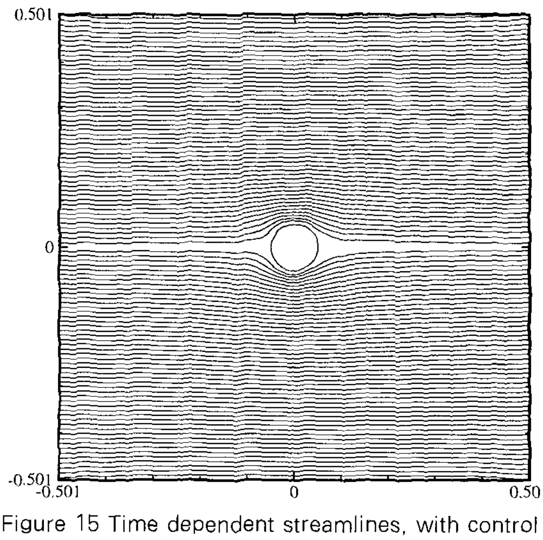


Figure 15 Time dependent streamlines, with control t=1sec

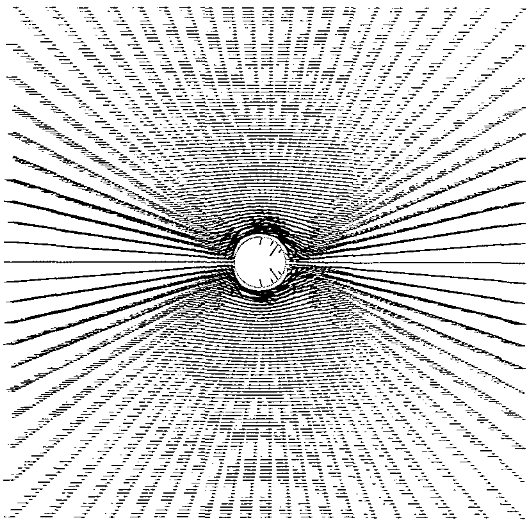

Figure 16 Time dependent velocity vectors, with control, t=3.5sec

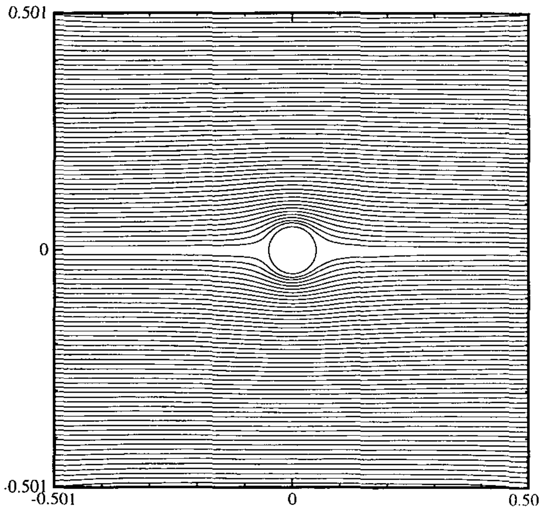


Figure 17 Time dependent streamlines, with control, t=3.5sec

The control mechanism is the rate of application of suction/injection at boundary holes.

The main contributions of this research are:

1) The development of a methodology for solving optimal flow control problems. The methodology has proven to be robust and efficient on the problems that were solved, never failing to converge to at least a locally-optimal solution. We feel that if optimal flow control is to prove useful industrially, the considerations of robustness and efficiency must be addressed.
2) The use of the optimal control methodology to assess the effectiveness of a current heuristic method, for the model problem of flow around an infinite cylinder. Because the study of these issues requires large amounts of computational resources, they could not have been performed without the development of methods as efficient as the ones presented here.

However, in order for the methodology presented in this study to become useful for industrial flows, several enhancements should be made to the flow model and solution methodology:

1) The incorporation of an efficient iterative solver for the linear systems that arise at each step of the Newton solver and for each right-hand side of the sensitivity equations. This would reduce storage and operation counts considerably, especially in three dimensions. This would most likely require moving to a mixed approximation of the Navier-Stokes equations: our experiments indicated that ill-conditioning created by the penalty method rendered a Krylov iterative method(the quasi-minimum residual method) all but hopeless. The choice of the iterative method/preconditioner pair should take into account the need in sensitivity-based optimization for solving multiple right-hand side linear systems, one corresponding to each control variable (unless an adjoint formulation is used). Thus one can invest in a good preconditioner, amortizing its cost over the multiple right-hand sides.
2) A turbulence model must be incorporated.

Most industrial flows are not laminar. While this is a fairly straightforward thing to do generally, it does bring up a difficulty in the context of optimal flow control: for proper first and second derivatives with respect to control variables, we require exact Jacobians matrices of the state equations with respect to fluid velocities. The addition of a turbulence model makes the viscosity depend on the velocity gradient, complicating the computation of the Jacobian. It would be interesting to see if the standard Navier-Stokes Jacobian(without a turbulence model) is a sufficiently good approximation of the true Jacobian. If true, it would substantially simplify the addition of a turbulence model.

3) It would be desirable to extend the model to a compressible fluid, because of the interest in aerodynamic flow control. Again this is ordinarily a straightforward(albeit time-consuming) task, but the need in sensitivity analysis for exact Jacobians makes things more complicated in compressible flows, one rarely couples all equations together and computes their Jacobian matrix.

4) It would also prove useful to extend the methodology to time-dependent problems, since this usually characterizes industrial flows. Here one would have to define an appropriate horizon of time, over which the objective function would be integrated, given an approximate solution of the time-dependent Navier-Stokes equations over this horizon. The major challenge would be overcoming the enormous computational complexity associated with having to do time-dependent simulation at each optimization iteration. In the near future this appears to be an intractable problem on anything but the most powerful parallel supercomputers.

Ultimately, we feel that the main contribution of this study is the demonstration that, through the use of sophisticated method, one can solve model optimal control problems governed by Navier-Stokes flows. Thus, we can contemplate the solution of such problems for more complex industrial flows in the not-too-distant future.

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