반응표면분석법을 이용한 모수 및 공차설계 통합모형[1)]

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Response Surface Approach to Integrated Optimization Modeling for Parameter and Tolerance Design

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Abstract

Since the inception of off-line quality control, it has drawn a particular attention from research community and it has been implemented in a wide variety of industries mainly due to its extensive applicability to numerous real situations. Emphasizing design issues rather than control issues related to manufacturing processes, off-line quality control has been recognized as a cost-effective approach to quality improvement. It mainly consists of three design stages: system design, parameter design, and tolerance design which are implemented in a sequential manner. Utilizing experimental designs and optimization techniques, off-line quality control is aimed at achieving product performance insensitive to external noises by reducing process variability. In spite of its conceptual soundness and practical significance, however, off-line quality control has also been criticized to a great extent due to its heuristic nature of investigation. In addition, it has also been pointed out that the process optimization procedures are inefficient. To enhance the current practice of off-line quality control, this study proposes an integrated optimization model by utilizing a well-established statistical tool, so called response surface methodology (RSM), and a tolerance-cost relationship.

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1. Introduction

Since Taguchi [1] introduced the concept of off-line quality control, it has drawn a great deal of attention from the research community and it has widely been implemented in a variety of industries mainly due to its applicability to numerous real situations. Emphasizing design optimization issues rather than control issues related to manufacturing processes, off-line quality control has been recognized as a cost-effective approach to quality improvement. It consists of three main design stages, such as system design, parameter design, and tolerance design, which are implemented in a sequential manner. The system design stage is to establish an overall product architecture and identify related techniques. Parameter design is mainly concerned with finding optimal settings of design parameters so that the product performance would be insensitive to external noises. Finally, tolerance design is to further capture and reduce variability in product performance by imposing tighter tolerances on the individual design parameters. The basic assumption behind the tolerance design principle is that a design engineer may control variability of design parameters.

It is usually assumed that tolerances on design parameters are pre-specified when determining the optimal settings of design parameters during the parameter design stage. After implementing tighter tolerances on design parameters through tolerance design, however, the optimal settings found in parameter design may not be optimal. Bisgaard and Ankenman [2] discussed that the optimal solution in parameter design depends on tolerances of design parameters and they also pointed out that these two design modules need either to be performed simultaneously or solved through several iterations. Along this line, there have been several research efforts, including Chan and Xiao [3] and Li and Wu [4], to incorporate parameter and tolerance designs into an integrated framework. Chan and Xiao [3] suggested an iterative combined design optimization scheme while Li and Wu [4] suggested an integrated optimization model by introducing tolerance-related costs into the objective function.

Based on our careful examination on the literature, two observations are drawn as follows: First, it has been assumed that the tolerances would be selected from a discrete set of values, i.e., so-called discrete tolerances. Further, all the possible combinations of individual tolerances on design parameters are to be examined. When the number of design parameters are small, it may not cause any difficulties to evaluate the cost associated with imposing tolerances. When the number of design parameters gets larger, however, it may not be feasible to explore all the combinations of individual tolerances exhaustively. Thus, the selection of tolerances needs to be determined more systematically. Fortunately, a problem involving discrete tolerances can be modeled as an integer programming problem. Second, the Taylor series expansion has been used when a functional relationship between design parameters and product performance, which is called a transfer function, is known. In many practical situations, however, an explicit form of the transfer function is usually unknown. To tackle this problem, Li and Wu [4] used a Monte Carlo simulation approach to the approximation of transfer function. The simulation approach may be quite inefficient since it requires a great deal of time and effort to re-run the experiments, especially when any changes are made to the manufacturing process. There is a need for developing more efficient procedures for estimating the transfer function so that experiments can easily be followed up. Response surface methodology (RSM) may provide a good alternative to the simulation approach in the sense that it can expedite the follow-up experimental procedures.

In these respects, this article proposes an integrated design optimization procedures with a minimum cost by modeling the problem as a mixed integer programming problem and using RSM to approximate the transfer function. This article is organized as follows: First, the RSM approach to parameter design problem by approximating the transfer function will be discussed. In the following section, how to incorporate the determination of discrete tolerances into the design optimization problem is examined. Then, an integrated design optimization procedure is proposed and followed by a numerical example. Conclusions are drawn in the last section.

2. Response Surface Approach to Parameter Design

Response surface methodology (RSM) is a statistical tool that is useful for modeling and analysis in situations where a response of interest is affected by several input variables. RSM is typically used to optimize this response by estimating an input-response functional form when the exact relationship is not known or very complicated. RSM is often viewed in the context of design of experiments (DOE), model fitting, and optimization. For a comprehensive presentation of RSM, see Box and Draper [5], Khuri and Cornell [6], and Myers and Montgomery [7]. More recently, Myers [8] provided his insightful presentations on the current status and future directions of RSM.

In most industrial problems, the exact

form of the functional relationship between design parameters and product performance is usually unknown. When the transfer function is not explicitly given, a well-established statistical tool such as RSM can efficiently be used to estimate it. Suppose that a design engineer is concerned with a manufacturing process involving a product performance Y that depends on the design parameters $\boldsymbol{X}=(X_1, X_2, \ldots, X_n)$. In many practical situations, a second-order polynomial model may be adequate to accommodate the curvature in the transfer function (Myers and Montgomery [7]). Thus, the postulated model for the transfer function can be written as

$$Y(\boldsymbol{X}) = \beta_0 + \sum_{i=1}^{n} \beta_i X_i + \sum_{i=1}^{n} \sum_{j=i}^{n} \beta_{ij} X_i X_j + \varepsilon, \quad (1)$$

where β_0, β_i, and β_{ij} are coefficients associated with constant, linear, and quadratic terms, respectively, and ε represents a random error not accounted for in the estimated transfer function. It is common assuming that ε follows the normal distribution with mean zero and variance σ_ε^2. Applying the method of least squares on the basis of experimental data at each design point, the fitted model for the transfer function is then given by

$$\hat{Y}(\boldsymbol{X}) = b_0 + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} \sum_{j=i}^{n} b_{ij} X_i X_j, \quad (2)$$

where b_0, b_i, and b_{ij} are the least squares estimates of β_0, β_i, and β_{ij}, respectively.

From the second-order polynomial representation of the transfer function, the mean and variance of the product performance can be approximated. Let σ_i denote the standard deviation of the design parameter X_i. It can then be shown that the estimated mean and variance of the product performance for given σ_i, denoted by $\hat{\mu}(\boldsymbol{X};\sigma_i)$ and $\hat{\sigma}^2(\boldsymbol{X};\sigma_i)$, respectively, are

$$\hat{\mu}(\boldsymbol{X};\sigma_i) = \hat{Y}(\boldsymbol{X}) + \frac{1}{2} \sum_{i=1}^{n} b_{ii} \sigma_i^2, \quad (3)$$

and

$$\hat{\sigma}^2(\boldsymbol{X};\sigma_i) = \sigma_\varepsilon^2 + \sum_{i=1}^{n} (b_i + 2b_{ii} X_i + \sum_{j \neq i} b_{ij} X_i X_j)^2 \sigma_i^2, \quad (4)$$

Based on the popular relationship between tolerance and process variance $t_i = 3\sigma_i$, the estimated functions in equations (3) and (4) can be written in terms of $\boldsymbol{X}$ and t_i's as follows:

$$\hat{\mu}(\boldsymbol{X};t_i) = \hat{Y}(\boldsymbol{X}) + \frac{1}{2} \sum_{i=1}^{n} \frac{1}{9} b_{ii} t_i^2, \quad (5)$$

and

$$\hat{\sigma}^2(\boldsymbol{X};t_i) = \sigma_\varepsilon^2 + \sum_{i=1}^{n} \frac{1}{9} (b_i + 2b_{ii} X_i + \sum_{i \neq j} b_{ij} X_i X_j)^2 t_i^2 \quad (6)$$

It has been pointed out that when the exact form of transfer function is known, the use of Taylor series expansion for approximating process mean and variance might be more effective than the use of experimental design. However, when the transfer function is not available or very complicated, one may need an alternative way to estimate the transfer function. Systematically evaluating the relationship between design parameters and product performance through a designed experiment, RSM provides an easy follow-up procedure for any changes made in the manufacturing process. Furthermore, RSM may also facilitate a deeper understanding of the process since the contribution of design parameters to the performance measure can be examined synthetically from the functional relationship between design parameters and performance measure.

The main concern of parameter design is to find the optimal settings of design parameters so that a quality loss would be minimized. A quality loss is incurred to the customer when the product performance deviates from its nominal target value. A quadratic representation has widely been used to approximate the quality loss on a monetary scale, which can be written as $L(Y)=k(Y-\tau)^2$, where k and τ represent a loss coefficient and the target value of the product performance, respectively. It is well known that the expected quality loss can be found as $E[L(Y)]=k[(\mu-\tau)^2+\sigma^2]$. Then, the expected quality loss can be approximated by replacing the process mean and variance with their estimates given in equations (5) and (6) when the transfer is unknown. If an explicit form of the transfer function is known, the estimates can easily be obtained by using the Taylor series expansion. Hence, the optimization model for the parameter design problem can simply be written as

Minimize

$$E[L(Y)]=k[(\hat{\mu}(\mathbf{X};t_i)-\tau)^2+\hat{\sigma}^2(\mathbf{X};t_i)] \qquad (7)$$

subject to $\mathbf{X}\in\Omega$,

where Ω represents a feasible region for design parameters.

As implied in the optimization model of equation (7), it has been a common practice performing parameter design with given tolerances on design parameters. If the variability of product performance is not satisfactory even after parameter design, tighter tolerances are imposed to design parameters. Since tightening tolerances may alter the optimal settings of design parameters, parameter and tolerance designs can not be considered separately but need to be performed in an iterative manner. Notwithstanding the fact that tighter tolerances incur a higher manufacturing cost, an obsessive attention to the concept of quality loss has shaded significant economic impacts of the manufacturing cost associated with imposing tolerances. Hence, considering the tolerances on design parameters along with the settings of design parameters as decision variables by incorporating the tolerance-related costs into the model, an integrated design optimization scheme may be constructed so that parameter and tolerance designs can be conducted simultaneously.

3. Integrated Optimization Modeling

The objective of the proposed approach is to find optimal tolerances as well as optimal settings of design parameters so that total cost would be minimized. The total cost includes the manufacturing costs associated with tolerances on the design parameters as well as a quality loss. The expected quality loss can be approximated by replacing the process mean and variance with their estimates in equations (5) and (6). The tolerances on design parameters are usually specified in terms of discrete tolerances. That is, there exist several grades for the tolerance of each design parameter and the manufacturing cost for each tolerance grade is also available. Let t_{ij} and c_{ij} be the j^{th} grade tolerance of the design parameter X_i and the manufacturing cost associated with t_{ij}, respectively. Since only one of the discrete tolerances needs to be selected for each design parameter, $\sum_j I_{ij}=1$ ($i=1,2,\ldots,n$), where I_{ij} represents an indicator variable which takes the value of 1 only if the j^{th} grade tolerance for X_i is selected. Hence, the tolerance of design parameter X_i can be expressed $t_i=\sum_j t_{ij}I_{ij}$. Further, the manufacturing cost associated with imposing tolerances on X_i, denoted by MC_i, can be written as $MC_i=\sum_j c_{ij}I_{ij}$, for $i=1,2,\ldots,n$. Now, the expected total cost ETC can be written as the sum of expected quality loss and manufacturing cost.

$$\begin{aligned} ETC &= E[L(Y)]+\sum_{i=1}^{n} MC_i \\ &= k[(\hat{\mu}(X;t_i)-\tau)^2+\hat{\sigma}^2(X;t_i)]+\sum_{i=1}^{n}\sum_j c_{ij}I_{ij} \end{aligned}$$

The lower and upper limits for individual tolerances are usually specified to meet the functional requirements. Furthermore, there may also exists a constraint related to the stack-up tolerances that can be written as

$$\sum_{i=1}^{n}\left(\frac{\delta y}{\delta x_i}\right)^2 t_i^2 \approx \sum_{i=1}^{n}(b_i+2b_{ii}x_i+\sum_{j\neq i}b_{ij}x_{ij})^2 t_i^2 \le T_{\max}$$

where $T_{\max}$ represents the maximum allowable stack-up tolerance. Then, the integrated optimization model for parameter and tolerance designs can be written as

Minimize

$$ETC = k[(\hat{\mu}(X;t_i)-\tau)^2+\hat{\sigma}^2(X;t_i)]+\sum_{i=1}^{n}\sum_j c_{ij}I_{ij}$$

Table 1. Design parameters for the polyamide resin example

Level	Design Parameters		
	Temperature (X_1)	Agitation (X_2)	Rate (X_3)
+1	200	10.0	25
0	175	7.5	20
-1	150	5.0	15

subject to $\sum_j I_{ij}=1$, for $i=1,2,\ldots,n$

$$t_i^L \le \sum_j t_{ij} I_{ij} \le t_i^U, \text{ for } i=1,2,\ldots,n$$

$$\sum_{i=1}^{n}\left(b_i+2b_{ii}x_i+\sum_{j\ne i} b_{ij}x_{ij}\right)^2 t_i^2 \le T_{\max}$$

$$X \in \Omega,$$

where t_i^L and t_i^U are the lower and upper limits for t_i, respectively, and $I_{ij}=$ 0 or 1. Note that the optimal solution to the optimization model yields to the optimal tolerances as well as the optimal settings of design parameters while meeting the functional requirements. Using the proposed integrated optimization model, there is no need for repeatedly conducting parameter and tolerance designs since the optimum tolerances and settings of design parameters can be found by solving a single optimization model instead of separately and sequentially optimizing the tolerances and settings.

4. Numerical Example

To demonstrate the proposed model, an example involving the design and manufacturing of a particular polyamide resin is taken from Myers and Montgomery [7]. The manner of adding amines has a significant effect on the viscosity of the resin. Three design parameters, such as the temperature at the time of addition (X_1), agitation (X_2), and rate of addition (X_3), presumably affect the viscosity of the resin of which target value is 55. The treatment levels of individual design parameters are shown in Table 1. A Box-Bohnken Design is used for the purpose of experimentation. The data including experimental format and response values, are shown in Table 2. The fitted transfer function in terms of design parameters is found to be (Myers and Montgomery [7]):

$$\begin{aligned}\hat{Y}(X) &= -58.875+2.650X_1-0.650X_2\\ &- 11.125X_3-0.012X_1^2+0.300X_2^2\\ &- 0.145X_3^2-0.032X_1X_2+0.088X_1X_3\\ &+ 0.140X_2X_3.\end{aligned}$$

Suppose that the study of the current manufacturing process reveals that $t_1=9.00$, $t_2=0.45$, $t_3=0.75$, $\sigma_\varepsilon=1.50$, and $k=1.00$. Using equations (5) and (6),

Table 2. Box-Behnken Design and experimental data

Design Point	Temperature	Agitation	Rate	Viscosity
1	-1	-1	0	53
2	+1	-1	0	58
3	-1	+1	0	59
4	+1	+1	0	56
5	-1	0	-1	64
6	+1	0	-1	45
7	-1	0	+1	35
8	+1	0	+1	60
9	0	-1	-1	59
10	0	+1	-1	64
11	0	-1	+1	53
12	0	+1	+1	65
13	0	0	0	65
14	0	0	0	59
15	0	0	0	62

the estimated functions for the process mean and variance can then be found as

$$\begin{aligned}\hat{\mu}(\boldsymbol{X};t_i) &= -58.930+2.650X_1-0.650X_2\\ &- 11.125X_3-0.012X_1^2+0.300X_2^2\\ &- 0.145X_3^2-0.032X_1X_2+0.088X_1X_3\\ &+ 0.140X_2X_3\end{aligned}$$

and

$$\begin{aligned}\hat{\sigma}^2(\boldsymbol{X};t_i) &= 73.197-1.266X_1-1.739X_2\\ &+ 4.597X_3+0.006X_1^2+0.019X_2^2\\ &+ 0.075X_3^2+0.015X_1X_2\\ &- 0.041X_1X_3-0.052X_2X_3.\end{aligned}$$

Cost analysis reveals that the manufacturing costs associated with the given values of t_1, t_2, and t_3 are $0.650, 0.880, and 0.775, respectively. Based on the optimization model presented in this study, the optimal settings of design parameters that minimize the quality loss for given tolerances turn out to be (X_1, X_2, X_3) =(177.82, 5.73, 25.00) with expected total cost of $6.270.

Suppose that there are three grades for each design parameter as shown in Table 3. Incorporating the tolerance-related costs into the objective function and treating the individual tolerances as decision variables, the integrated optimization model can be constructed. The maximum allowable stack-up tolerance is set to 12.00. The optimal settings of design parameters are found to be (X_1, X_2, X_3)=(177.80, 5.64, 25.00) with expected total cost of $5.642, and the optimal tolerances turn out to be (t_1, t_2, t_3)=(7.00, 0.60, 0.75). Implementing the integrated optimization model gives a

Table 3. Tolerance-related cost data

Grade (j)	Temperature (X_1)		Agitation (X_2)		Rate (X_3)	
	t_{1j}	c_{1j}	t_{2j}	c_{2j}	t_{3j}	c_{3j}
1	7.0	0.730	0.30	1.155	0.50	1.035
2	9.0	0.650	0.45	0.880	0.75	0.775
3	11.0	0.590	0.60	0.730	1.00	0.635

cost savings of 10.02 % (=(6.270-5.642)÷ 6.270). The results are summarized and compared with those of conventional approach in Table 4.

Advantages of the integrated approach can be observed in two ways. First, the settings of design parameters yield to the process mean closer to the target value. Further, the process variance is significantly reduced by adjusting the tolerances on design parameters. In these respects, the manufacturing process implemented by the integrated approach is apparently superior. Second, a significant amount of cost savings can be accrued from the proposed approach. It can be observed that even though the manufacturing cost related to tolerances may increase slightly, the expected total cost is notably reduced due to the significant savings in the expected quality loss.

5. Conclusions

Recognizing the importance of design optimization issues for quality improvement, off-line quality control has received a great deal of attention from researchers. It is recommended that parameter and tolerance designs be

Table 4. Comparison of results

	Conventional Approach	Integrated Approach
Optimal Settings	(177.82, 5.73, 25.00)	(177.80, 5.64, 25.00)
Tolerances	(9.00, 0.45, 0.75)	(7.00, 0.60, 0.75)
Process Mean	55.21	55.17
Process Variance	3.92	3.38
Stack-up Tolerance	15.05	10.15
$E[L(Y)]$	\$3.96	\$3.41
Tolerance Cost	(\$0.650, \$0.880, \$0.775)	(\$0.730, \$0.730, \$0.775)
ETC	\$6.270	\$5.642

conducted in a sequential manner. However, a sequential optimization of parameter and tolerance designs could be inefficient in the sense that it requires several iterations to achieve desirable settings of a manufacturing process since these two design modules are interdependent. This article proposes an integrated design optimization scheme so that parameter and tolerance designs can be performed simultaneously in a more efficient manner. Response surface methodology is utilized to approximate the transfer function and obtain the estimated functions of process mean and variance. Furthermore, the tolerances on design parameters are incorporated into the design optimization problem as decision variables by modeling a mixed integer programming problem. The objective function of the propose optimization model is to minimize the expected total cost including quality loss and manufacturing costs. Of course, there may be situations where either tight adherence to product quality or low manufacturing cost is essential. The proposed model can be modified to accomodate these situations by assigning weights to cost components.

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