ANALYSIS OF VELOCITY STRUCTURE OF WALL JET ORIGINATING FROM CIRCULAR ORIFICES IN SHALLOW WATER

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Abstract: In this study, breakwater model which has several outlet pipes to discharge water is settled in the experimental open channel and mean velocity distributions of multi wall jet are measured. The length of zone of flow establishment of wall jet is shorter than that of free jet and decay rate of jet centerline longitudinal velocity along x is linear in $0.3 \le x/l_q \le 17$. The rate of vertical width and lateral width spreading of multi wall jet is respectively 0.0753, 0.157 ~ 0.190.

Keywords: wall jet, length of zone of flow establishment, width spreading

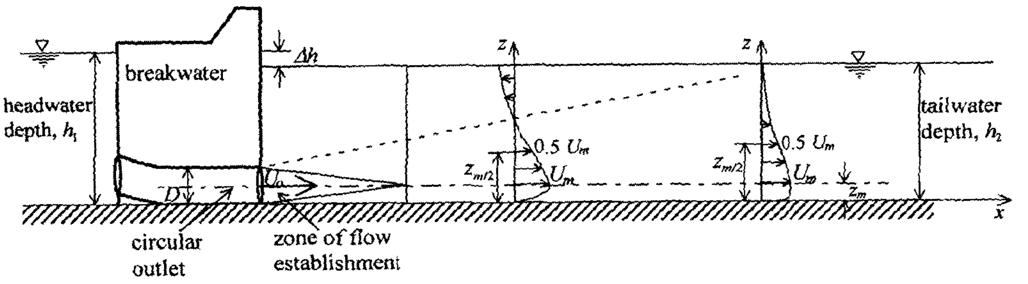
1. INTRODUCTION

The demand of electrical energy in Korea has been increasing rapidly concurrent with the rise in the standard of living and the intensive push toward industrialization. This increasing demand as well as economical considerations have resulted in the construction of large nuclear generating units of capacities of the order of 1000 MW. These plants reject large amounts of waste heat into the water body which acts as the source of condenser cooling water. Faced with ever-increasing difficulty in selecting proper new sites, installing additional generators to existing sites has emerged as a valid alternative. However, the plan may cause serious problem such as drop in power generation efficiency associated with the recirculation of heated water and increase in environmental effect. Since it is foreseeable that more stringent nationwide regulations are to be imposed in response to the heightened public awareness on environmental issues, more effective method of discharging heated water are needed at this point.

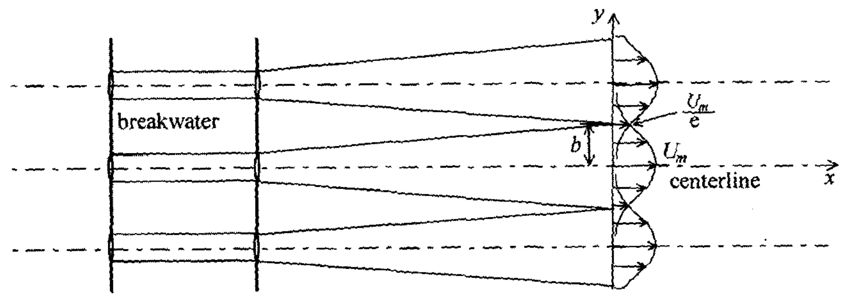
Surface discharge provides the easiest way of discharging cooling water into the receiving body of water, but it causes high temperature rise at the sea surface. To minimize ambient water temperature rise and the size of the thermal impact area, submerged discharge method by breakwater with discharge outlets is adopted for discharge of heated water in Wolsung nuclear power station, Wolsung, Korea. The velocity structure of this flow can be explained by round wall jet theory. Fig. 1 shows a typical mean velocity distribution of the flow discharged from breakwater outlets. D is diameter of discharge outlet. h_1, h_2 are respectively head water and tail water depth. U_o is discharge velocity at outlet. U_m is jet centerline longitudinal velocity of each section. z_m is vertical location on which U_m occurs.

The wall jet is described as a jet of fluid which impinges onto a wall, and has both a solid boundary and a free boundary of the same fluid as the jet. The free and solid boundaries generate considerable interest, since the flow will have both a jet-like property and also be influenced by the wall. Moreover, there is another free surface boundary in the upper place of the jet. Fig. 2 shows typical flow characteristics of the round wall jet. In this figure, $z_{m/2}$ is vertical location on which $U_m/2$ occurs. In case that infinite boundary is exists in z direction as shown in Fig. 2(a), vertical distribution of longitudinal velocity is composed of two layer which are typical turbulent boundary layer from bottom to z_m and free shear flow layer above z_m. Fig. 2(b) illustrates wall jet which has both bottom boundary and free surface boundary. In this case, to supply jet entrainment, back currents exist near the free surface in some distance, i.e., transition zone. Outer region of the zone, back currents do not exist any more like Fig. 2(a)

In this study, mean velocity distributions of the wall jet measured in the laboratory experiments are analyzed in detail in transition zone. The characteristics of round wall jet are compared with those of free jet. Vertical width and

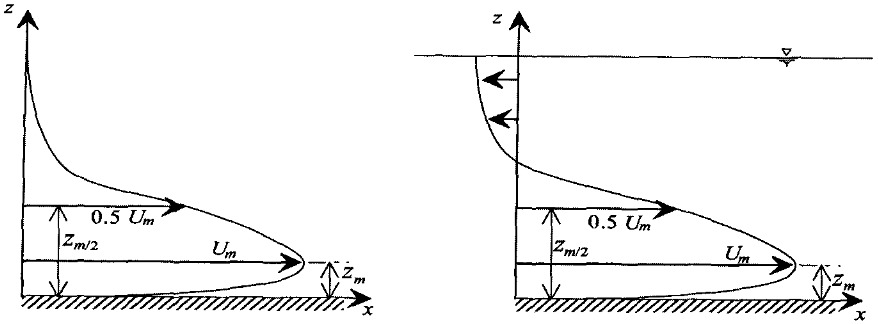


(a) side



(b) plan

Figure 1. Typical mean velocity distribution of the flow discharged from breakwater Outlets



(a) flow characteristics with infinite boundary

(b) flow characteristics with free surface boundary

Figure 2. Flow characteristics of round wall jet

lateral width spreading rate of multi wall jet are also investigated.

2. THEORETICAL BACKGROUND

2.1 Velocity Scale, U_m and Vertical Length Scale, $z_{m/2}$

Experimental studies of air jet with infinite boundary have been carried out to design submerged foil and air foil. Glauert (1956) has examined theoretically the similarity problem of the laminar and turbulent, radial and plane wall-jets. The velocity profiles showed a self-preserving character at both inner and outer layers. The velocity distribution of the turbulent radial wall-jet and the variation of velocity and length scales with downstream position has been measured by Bakke (1957). He found that the velocity scale varied as a power of x with exponent -1.12 and the vertical length scale as 0.94. The velocity distribution of the two-dimensional turbulent plane wall jet and the variation of velocity and vertical length scales with downstream position has been measured by Schwarz and Cosart (1961). They found that the velocity scale varied as a power of x with exponent -0.555 and the vertical length scale as 1.0. The dimensionless equation for $z_{m/2}$ can be written

$$\frac{z_{m/2}}{l_q} = f\left(\frac{x}{l_q}, \mathrm{Re}\right) \tag{1}$$

in which Re is outlet Reynolds number and discharge geometric length scale, l_q is defined as following

$$l_q = \frac{Q}{\sqrt{M}} = \sqrt{A} \tag{2}$$

in which Q is discharge volume flux, M is discharge momentum flux and A is cross- sectional area of discharge outlet. From the concept of self-preservation, Eq. (2) may be rewritten as

$$\frac{z_{m/2}}{l_q} = \frac{x}{l_q} f_1(\mathrm{Re}) \tag{3}$$

According to Schwarz and Cosart (1961), there is not any systematic change in $f_1(\mathrm{Re})$ as the outlet Reynolds number is varied. Lanuder and Rodi (1992) proposed linear relationship to describe the rate of vertical spread of the flow. They suggested the following relationship.

$$\frac{d z_{m/2}}{dx} = 0.073 \pm 0.002 \text{ for the plane two-dimensional wall jet} \quad (4a)$$

$$\frac{d z_{m/2}}{dx} = 0.090 \pm 0.005 \text{ for the round wall jet} \quad (4b)$$

The rate of spread of 0.090 for round wall jet is some 20 % higher than that observed for the plane wall jet. Newman et al. (1971) investigated the mean velocity distribution far downstream of a jet discharging from a circular nozzle adjacent to a plane, which showed a much more rapid diffusion parallel to the plane than at right angles to the plane. The decay of a jet discharging from a circular nozzle parallel to and displaced from a solid surface is investigated by Davis and Winarto (1980). They investigate the structure of the turbulent flow in more detail with a view to explaining the cause of the substantial difference in diffusion rates transverse to the mean flow and parallel to and perpendicular to the plane. The spatial distribution of the mean velocity in plane wall jet was measured for a variety of nozzle Reynolds numbers by Wygnanski et al. (1992) and Katz et al. (1992). They found that the velocity scale varied as a power of x with exponent -0.47 and the length scale as 0.88.

Examples of wall jets in hydraulic field are flow below a submerged sluice gate (Rajaratnam, 1965; Rajaratnam, 1976; Wu and Rajaratnam, 1995), diffusion of bluff wall jets in finite depth tailwater(Rajaratnam and Humpries, 1983) and flow discharge from power stations and water treatment (Johnston and Volker, 1993). Rajaratnam (1965) studied flow below a submerged sluice gate using a plane wall jet theory. He found that the vertical velocity profiles of the plane wall jet are similar if they are normalized with a velocity scale u_m and a vertical length scale $z_{m/2}$. Further more this normalized vertical velocity profiles are described well by the corresponding curve of the wall jet for $z/z_{m/2} \leq 1$. For $z/z_{m/2} > 1$, the experimental observations are deviated from theoretical curve, showing smaller values. He found that the velocity scale varied as a power of x with exponent -0.515 and the length scale as 1.0.

Based on the results of an experimental study on the diffusion of bluff turbulent wall jets in finite depth tailwater in a wide channel, Rajaratnam and Humphries (1983) postuated that the transverse expansion of the jet and the velocity profiles in the central plane depends on the ratio of the tailwater depth to the jet outlet thickness. They showed that the length of potential core decreases as the tailwater depth decreases. Jonston and Volker (1993) studied jet entering shallow water. They found that centerline velocity decay distribution showed that the length of the constant velocity core appears shorter in shallow conditions. At a section of 3.38 l_q, immediately downstream of this core, the velocity is considerably lower than that further downstream.

Wu and Rajaratnam (1995) classified flow below a submerged sluice gate into three categories, i.e., the free jumps (hydraulic jump), submerged jumps of varying degrees of submergence, and the plane wall jet. The free jump and the wall jet are shown to be the end states. The submerged jumps form the transition state in between. They investigated properties of wall jet which are similarity of the velocity distribution, decay of the velocity scale and the growth of the length scale. Summary of variation of velocity and vertical length scale are given in Table 1.

Table 1. Summary of variation of velocity and vertical length scale for wall jet

	Air		Water	
	Plane	Round	Plane	Round
Velocity scale U_m	$x^{-0.555}$ (Schwarz and Cosart, 1961) $x^{-0.47}$ (Wygnanski, et al., 1992)	$x^{-1.12}$ (Bakke, 1957) $x^{-1.0}$ (Davis and Winarto, 1980)	$x^{-0.515}$ (Rajaratnam, 1965)	Deep water : $x^{-1.0}$ (Wu and Rajaratnam, 1983) Shallow water : $x^{-0.5}$ (Wu and Rajaratnam, 1983)
Vertical length scale $z_{m/2}$	$x^{1.0}$ (Schwarz and Cosart, 1961) $x^{1.0}$ (Launder and Rodi, 1983) $x^{0.88}$ (Wygnanski, et al., 1992)	$x^{0.94}$ (Bakke, 1957) $x^{1.0}$ (Davis and Winarto, 1980) $x^{1.0}$ (Launder and Rodi, 1983)	$x^{1.0}$ (Rajaratnam, 1965)	$x^{1.0}$ (Wu and Rajaratnam, 1983)

2.2 Lateral Length Scale, b

In general, lateral velocity distribution of single jet can be expressed by using the self-similar velocity profile

$$\frac{U}{U_m} = \exp\left[-\left(\frac{y}{b}\right)^2\right] \tag{5}$$

in which y is lateral coordinate and b is the lateral width of a jet defined as the value of y at which U reduces to $0.37U_m$. Many experimental investigations of single round jet shows that width of a single jet has a following relation (Fischer, et. al., 1979).

$$\frac{db}{dx} = 0.107 \tag{6}$$

According to Wood et. al.(1993), within the planes of symmetry each jet has an initial region in which the flow is axisymmetric, a merging region or transition region and finally a two dimensional region of free multi jet. A reasonable velocity distribution of free multi jet in the transition region is obtained if the velocities from each jet are assumed additive. This yields

$$\frac{U}{U_m} = \frac{\exp\left(-\left(z/b\right)^2\right)\sum_{-\infty}^{\infty}\exp\left[-\left(\frac{y_l+n}{b_l}\right)^2\right]}{\sum_{-\infty}^{\infty}\exp\left[-\left(\frac{n}{b_l}\right)^2\right]} \tag{7}$$

in which U_m is now the velocity in the centerline of one of the merging jets, n is number of port and y_l, b_l are defined as following.

$$y_l = {}^{y}\!/_{l} \tag{8a}$$

$$b_l = {}^{b}\!/_{l} \tag{8b}$$

in which l is distance between orifice and orifice.

In single free jet, decay rate of longitudinal velocity along x is defined as following (Fischer et. al., 1979). This shows that diminution of velocity is not seen until dimensionless length arrives at 7 and from that region, velocity is declining as proportional to x^{-1}.

$$\frac{U_m}{U_o} = 7\left(\frac{l_q}{x}\right) \quad \text{for} \quad \frac{x}{l_q} > 7.0 \tag{9}$$

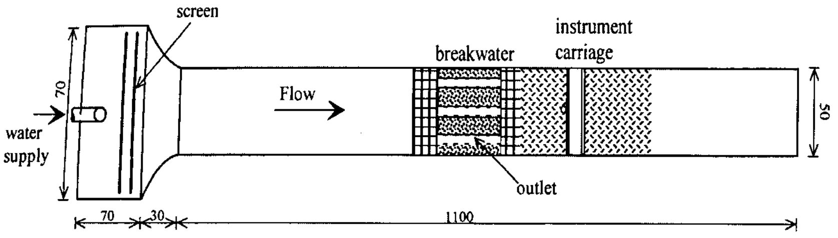
3. EXPERIMENTS

3.1 Laboratory Model

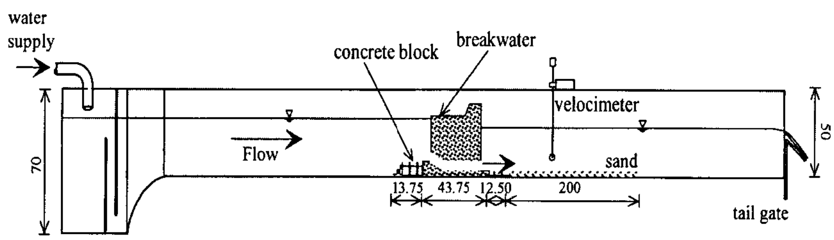
The geometric configuration of the laboratory model of breakwater with discharge outlets was based primarily on prototype breakwater under construction. Even though the characteristics identified in existing breakwater were used as the primary criteria to design the model breakwater, the principles of hydraulic similitude were also used as guidelines in determining the appropriate scale of the model.

The significant characteristics and dimension of the laboratory model constructed in a 12 m long, 0.5 m wide, and 0.5 m high flume in the Hydraulics Laboratory at Seoul National University are depicted in Fig. 3. Breakwater model with three discharge outlets is manufactured. Diameter of discharge outlet is 5 cm. Distance between two discharge outlets is 15 cm.

Water discharge is measured by using calibrated weir. Water stage is measured by standard steel point gage with a vernier reading to the nearest 0.01 cm. Velocity profiles are measured using propeller type Velocimeter. Discharge velocity is controlled by head difference of head water and tail water of breakwater model. Stage head of tail water is 20 cm.



(a) plan



(b) side

Figure 3. Schematic diagram of laboratory flume and experimental setup(unit : cm)

Table 2. Summary of experimental conditions

Set	Case No.	h_1 (cm)	h_2 (cm)	U_o (cm/sec)	Re_o $\left(=\frac{U_o D}{\nu}\right)$	Fr_o $\left(=\frac{U_o}{\sqrt{gD}}\right)$
I	S01	22.70	20.00	59.31	29,655	0.85
	B01	22.30		58.01	29,005	0.83
	R01	22.20		59.61	29,805	0.85
II	S02	21.75	20.00	50.60	25,300	0.72
	B02	21.50		49.41	24,705	0.71
	R02	21.45		48.51	24,255	0.69
III	S03	21.28	20.00	47.20	23,600	0.67
	B03	21.22		43.61	21,805	0.62
	R03	21.04		42.60	21,320	0.61

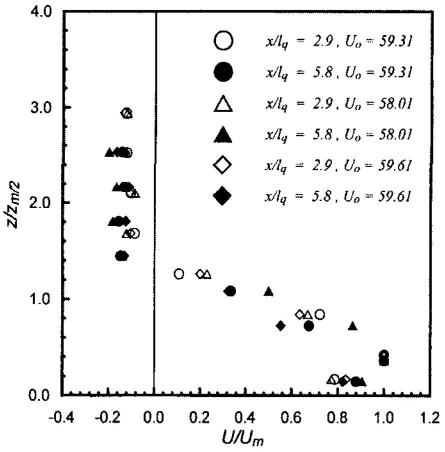


Figure 4. Vertical distributions of centerline velocity

3.2 Experimental Program

Three sets of data were collected as a part of the experimental program: Data Set I, discharge velocity is about 60 cm/s; Data Set II, discharge velocity is about 50 cm/s; Data Set III, discharge velocity is about 40 cm/s. Table 2 shows various experimental conditions. In this experiment, outlet Reynolds numbers and outlet Froude numbers are respectively about 20,000 ~ 30,000, 0.61 ~ 0.85.

4. ANALYSIS

4.1 Vertical Velocity Distribution

Fig. 4 shows that vertical distribution of longitudinal velocity of Data Set I. The distribution of data are quite identical to theoretical velocity distribution shown Fig. 2(b). The velocity profiles show self-preserving characteristics. Velocity data collected in this study shown that $f_1(\mathrm{Re})$ is 0.0753, and the Eq. (1) is written as

$$\frac{z_{m/2}}{l_q} = 0.0753\left(\frac{x}{l_q} + 14.91\right) \qquad (10)$$

Eq. (10) is approximate formula and are dependent on individual system. In particular, the location of the virtual origin (=14.91) is a function of many conditions such as the wall roughness, turbulence in the jet, and outlet Reynolds number. Eq. (10) shows that the vertical length scale varied as a power of x with exponent 1.0 and the rate of vertical spread of the flow is 0.0753. This result is almost identical to previous work(Schwarz and Cosart, 1961; Codazzi et al.; 1981; Launder and Rodi, 1992).

4.2 Decay of Centerline Velocity

Fig. 5 shows a decay rate of jet centerline longitudinal velocity. In this figure, continuous line is dimensionless longitudinal velocity along x of free single jet and dashed line is that of free multi jet (discharged from 3 circular orifice) non-affected boundary effect. Dashed line is considered superposition effect of additional two side jets and as x increase, due to effect of width spreading of side jets. Points are that of multi wall jet in variable conditions presented in Table 2. Dashed dot line is curve fitting result of these experimental data. The results of experiment of multi wall jet show that first, rapid diminution of velocity is seen; second, linear velocity diminution is occurred in transition

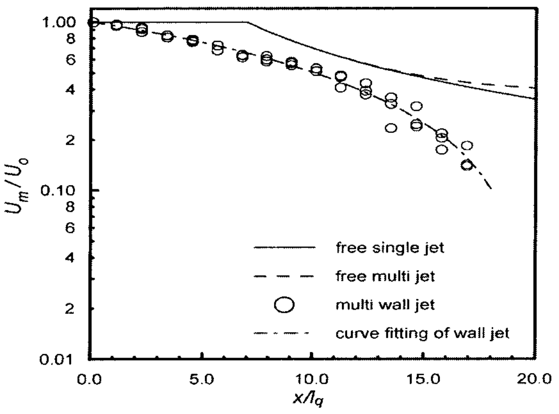


Figure 5. Decay rate of centerline longitudinal velocity

Table 3. Summary of spread constants with several dimensionless distance (Data Set I)

x/l_q	S01	B01	R01
5.64	0.200	0.186	0.215
6.77	0.190	0.169	0.204
7.90	0.199	0.161	0.200
9.03	0.178	0.149	0.186
10.20	0.164	0.128	0.190
11.30	0.202	0.154	0.148
Mean	0.189	0.157	0.190

zone if we judge from curve fitting equation of measured data - limit of curve fitting is $0.3 \le x/l_q \le 17$. The equation of curve fitting is following.

$$\frac{U_m}{U_o} = -0.0516\frac{x}{l_q} + 1.0166 \qquad (11)$$

So we can infer that jet centerline longitudinal velocity distribution of round wall jet has some differences which you can see in Fig. 5 comparing with free single and multi jet without effect of bottom boundary and free surface. In the transition zone, Eq. (11) is quite different from previous work in the point that the rate of velocity decay is very high.

4.3 Lateral Velocity Distribution

Fig. 6 is a graph of Eq. (7) multiplied by U_m/U_o of free multi jet in Fig. 5 with several longitudinal distance for this study. We can see that the jet is almost two dimensional in $x/l_q \ge 15.0$. Length scale of our study is almost located in transition zone. To study lateral shape of multi wall jet in the transition zone, by using experimental condition, Data Set I, optimal

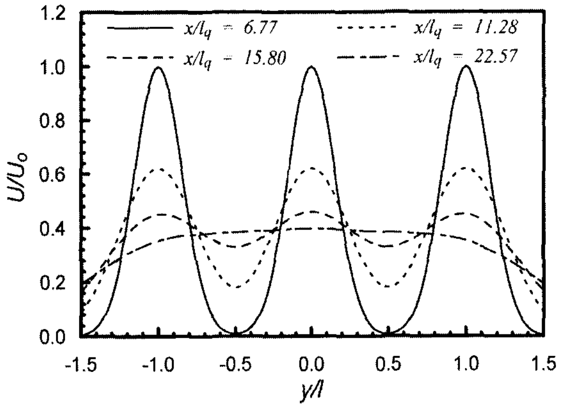


Figure 6. Lateral distributions of longitudinal velocity of free multi jet

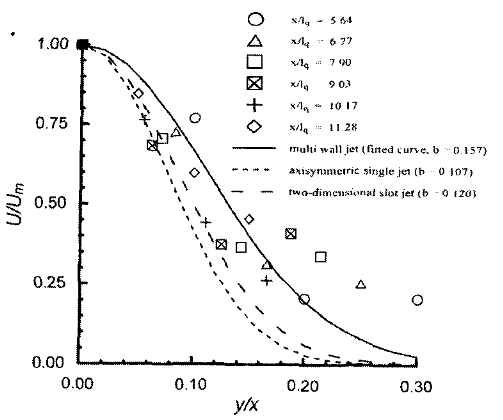


Figure 7. Lateral distributions of longitudinal velocity

spread constant (db/dx) which is best fitting Eq. (5) is found out by minimizing sum of square error between experimental data and Eq. (5). Table 3 is a summary of these spread constant with several dimensionless distance.

Fig. 7 shows lateral distribution of longitudinal velocity for several sections (Case No B01). Continuous line is a plot of Eq. (5) in which spread constant is 0.157 ~ 0.190 which is aver age of values in $x/l_q \geq 5.64$, in which is sufficient distance to attain gauss distribution in this round wall jet. Spread constants for a axisymmetric and two dimensional flow in a free jet are respectively 0.107 and 0.120 according to Wood et at., (1993). Dashed line is for axisymmetric single jet and dash dot line two dimensional slot jet. We can infer that due to bottom friction, spread constant of round wall jet is more bigger than those of free jets. The rate of lateral spreading is almost twice of the rate of vertical spreading.

5. CONCLUSION

Experimentally, velocity structures of wall jet originating from circular orifices are analyzed by using round wall jet theory. In this study, longitudinal, vertical and lateral velocity distributions of multi wall jet are compared with single and multi free jet. The length of zone of flow establishment of multi wall jet is shorter than that of free jet and decay rate of maximum longitudinal velocity along distance is linear in $0.3 \leq x/l_Q \leq 17$. The rate of vertical spreading of water jet is 0.0753 and it seems to be almost same to that of previous work. The rate of lateral spreading of multi wall jet is about 0.157 ~ 0.190 which is more bigger than axisymmetric single jet and two dimensional slot jet due to bottom friction. The rate of lateral spreading is almost twice of the rate of vertical spreading

6. LIST OF SYMBOLS

A	cross-sectional area of discharge outlet
b	lateral width of a jet
D	diameter of discharge outlet
Fr_0	outlet Froude number
h_1	head water depth
h_2	tail water depth
l	distance between orifice and orifice
l_q	discharge geometric length scale
M	discharge momentum flux
Q	discharge volume flux
Re_0	outlet Reynolds number
U_m	jet centerline longitudinal velocity of each section
U_o	discharge velocity at outlet
x	Cartesian coordinate in the longitudinal direction
y	Cartesian coordinate in the lateral direction
z	Cartesian coordinate in the vertical direction
z_m	vertical location on which U_m occurs
$z_{m/2}$	vertical location on which $U_m/2$ occurs

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