Combined Convection and Radiation in a Tube with Circumferential Fins and Circular Disks

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Combined convection and radiation heat transfer in a circular tube with circumferential fins and circular disks is investigated for various operating conditions. Using a finite volume technique for steady laminar flow, the governing equations are solved in order to study the flow and temperature fields. The P-1 approximation and the weighted sum of gray gases model (WSGGM) are used for solving the radiation transport equation. The results show that the total Nusselt number of combined convection and radiation is higher than that of pure convection. If the temperatures of the combustion gas and the wall in a tube are high, radiation becomes dominant. Therefore, it is necessary to evaluate the effect of radiation on the total heat transfer.

Key Words: Convection, Radiation, Nongray Radiation, P-1 Approximation, Weighted Sum of Gray Gases Model

Nomenclature

c_p	: Specific heat at constant pressure (J/kg·K)
D	: Diameter (m)
D_h	: Hydraulic diameter (m)
$e_{b\lambda}$	: Emissive power (W/m$^2\mu$m)
F	: Fin height (m)
I	: Radiation intensity (W/m^2)
G	: Integrated radiation intensity (W/m^2)
h	: Heat transfer coefficient (W/m^2·K)
H	: Pitch of the circumferential fin
k	: Thermal conductivity (W/m·K)
L	: Disk radius (m)
S	: Distance of fin and circular disk (m)
$\bar{s}$	: Unit length (m)
p	: Pressure (Pa)
r	: Radius (m)
q	: Heat flux (W/m^2)
u	: Axial velocity (m/sec)
v	: Radial velocity (m/sec)
T	: Temperature (K)
T_m	: Bulk mean temperature (K)
Pr	: Prandtl number
$\bar{q}_{rad}$	: Radiation flux vector
Re	: Reynolds number
Nu	: Nusselt number
μ	: Viscosity (kg/sec·m)
λ	: Wavelength (μm)
ρ	: Density (kg/m^3)
ω	: Solid angle (sr)

Subscript

Conv	: Convection
rad	: Radiation
tot	: Total

1. Introduction

The condensing boiler for domestic appliance has been developed to utilize latent heat of combustion gases. In order to recover latent heat of the combustion gases with a size of the heat exchanger similar to a conventional one, a highly efficient compact heat exchanger is required. Combination of circumferential fins and circular disks

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is suggested to increase the heat transfer rate of the heat exchanger for a condensing boiler. What these fins and disks do in a tube is to dramatically deflect the hot gas flow, which allows good mixing and strong impingement against the tube's wall. Therefore, the convection heat transfer coefficient of gases on the tube's wall becomes about more than ten times greater than that on a smooth tube's wall. On the other hand, the temperature of the combustion gas near a burner is so high that radiation heat transfer as well as convection heat transfer plays an important role in this particular heat exchanger. The combustion gas consists of carbon dioxide, water vapor, and nitrogen. Because carbon dioxide and water vapor absorb and emit thermal nongray radiation, the effects of nongray radiation heat transfer have to be thoroughly investigated to obtain the optimal design of the heat exchanger for the condensing boiler with circumferential fins and circular disks.

Berner et al. (1984), Webb and Ramadhyani (1985), Kelkar and Patankar (1987), and Habib et al. (1994) investigated convection heat transfer and pressure drop in parallel plate channels in which baffles were inserted to enhance heat transfer. They showed flow patterns around the baffles, the effects of the Reynolds number, the Prandtl number, and important geometric parameters on convection in channels. Although the characteristics of heat transfer and pressure drop might be qualitatively similar to these for a circular tube, it was difficult to use the results for the heat exchanger design for condensing boiler which was basically a circular tube. Rowley and Patankar (1984) studied heat transfer in a circular tube with circumferential fins. Unfortunately, they did not insert circular disks between two consecutive fins so that it was not possible to utilize their results, either. Jeon et al. (1999a, b) and Seo et al. (2000) investigated heat transfer enhancement in a circular duct with circumferential fins and circular disks numerically and experimentally, respectively. They considered laminar and turbulent flows, but the effect of radiation heat transfer was neglected.

Therefore, the characteristics of combined convection and radiation heat transfer of the laminar flow in a tube with circumferential fins and circular disks are numerically investigated in order to obtain design information of the circular tube heat exchanger for the condensing boiler. A variety of temperatures, the Reynolds numbers, and geometric arrangement are selected as the important design parameters, and the effects of radiation on the total heat transfer are studied.

2. Numerical Modeling

The geometry shown in Fig. 1 is used for the calculation and the computational domain is hatched in the figure. One module of the domain is shown in Fig. 1(a) in detail. The entire domain of the calculation consists of eight modules as shown in Fig. 1(b). Therefore, the heat transfer characteristics of developing flow as well as fully developed flow can be investigated. For the simplicity of calculation, the following assumptions are adopted.

① The flow is steady and laminar.

② Temperature and velocity at the inlet are uniform.

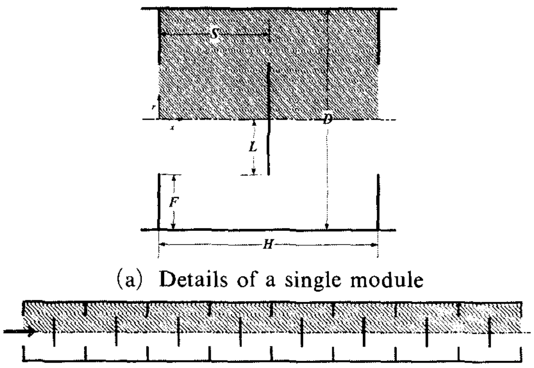
③ The wall is gray and the wall temperature is constant.

④ Energy loss caused by viscosity is ignored.

⑤ Combustion gas is incompressible and the properties are constant.

⑥ Gas does not scatter, but absorbs and emits radiation energy.

The conservation equations for mass, momentum, and energy with the constant properties can



(a) Details of a single module

(b) Entire domain of the calculation

Fig. 1 Considered disk and doughnut geometry be written as follows :

$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial(rv)}{\partial r}=0 \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial r}=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial^2 x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] \tag{2}$$

$$\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial r}=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial^2 v}{\partial^2 x}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)\right] \tag{3}$$

$$\rho c_p u\frac{\partial T}{\partial x}=k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)-\nabla\cdot\bar{q}_{rad} \tag{4}$$

and, the boundary conditions are given as follows :

$$\text{at } r=0\ ;\ \frac{\partial u}{\partial r}=0,\ \frac{\partial T}{\partial r}=0 \tag{5}$$

$$\text{at } r=R\ ;\ u=0,\ T=T_{wall} \tag{6}$$

$$\text{at } x=0\ ;\ u=u_{inlet},\ T=T_{inlet} \tag{7}$$

In order to obtain the radiation flux vector, $\bar{q}_{rad}$, the radiation intensity, I, is needed. In the present study, the P-1 approximation (Modest, 1993) is used to solve for I. And, the radiation flux vector is given by

$$\bar{q}_{rad}=\int_{4\pi}\int_0^\infty I_\lambda\bar{s}\,d\lambda d\omega \tag{8}$$

where the radiation heat transfer equation is as follows :

$$\frac{dI_\lambda}{ds}+a_\lambda I_\lambda=a_\lambda I_{b\lambda} \tag{9}$$

From Eqs. (8) and (9), we may write

$$-\nabla\cdot\bar{q}_{rad}=a_\lambda(G_\lambda-4e_{b\lambda}) \tag{10}$$

where the integrated radiation intensity is as follows :

$$G_\lambda=\int_{4\pi}I_\lambda d\omega \tag{11}$$

and the emissive power is defined by

$$e_{b\lambda}=\pi\cdot I_{b\lambda} \tag{12}$$

The P-1 approximation for

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{a_\lambda}\cdot\frac{\partial G_\lambda}{\partial r}\right)+\frac{\partial}{\partial x}\left(\frac{1}{a_\lambda}\cdot\frac{\partial G_\lambda}{\partial x}\right)=3a_\lambda(G_\lambda-4e_{b\lambda}) \tag{13}$$

The non-gray absorption coefficients of carbon dioxide and water vapor are calculated from WSGGM (Weighted Sum of Gray Gases Model) (Modest, 1993). And, the bulk mean temperature is given by (Patankar et al., 1977)

$$T_m=\int_0^{D/2}|u|\,Trdr\Big/\int_0^{D/2}|x|\,rdr \tag{14}$$

where the absolute values of velocity means that the main flow direction differs from the direction of the recirculation flow caused by circumferential fins and circular disks. And, the local heat transfer coefficient and the local convection Nusselt number are defined by

$$h=\frac{q_{conv}}{(T_w-T_m)} \tag{15}$$

$$\mathrm{Nu_{conv}}=hD/k \tag{16}$$

The average convection Nusselt number for a module is calculated as follows :

$$\overline{\mathrm{Nu}}_{\mathrm{conv}}=\int_{A_T}\mathrm{Nu}\,dA\Big/\int_{A_w}dA \tag{17}$$

where A_T is the total heat transfer area of the tube including circumferential fins and A_w is the area of the tube wall.

To calculate the radiation and the convection heat transfer rate, the total heat flux at each module is calculated as following :

$$q_{tot}=\frac{1}{R\cdot\Delta x}\left[\int_0^R\rho uc_pTrdr\Big|_{x+\Delta x}-\int_0^R\rho uc_pTrdr\Big|_x\right] \tag{18}$$

By substituting the total heat flux calculated from Eq. (18) into Eq. (19), the total Nusselt number is given as follows :

$$\mathrm{Nu_{tot}}=\frac{q_{tot}\cdot D}{k(T_w-T_m)}=\mathrm{Nu_{conv}}+\mathrm{Nu_{rad}} \tag{19}$$

where the local convection and the local radiation Nusselt number are defined by

$$\mathrm{Nu}_{rad}=\frac{q_{rad}\cdot D}{k(T_w-T_m)} \tag{20}$$

$$\mathrm{Nu_{conv}}=\mathrm{Nu_{tot}}-\mathrm{Nu_{rad}} \tag{21}$$

3. Numerical Methods

The governing equations were numerically solved for the developing and fully developed regions, respectively. The commercial finite volume code FLUENT is used for the present study.

All the computations are performed on a 102× 52 (x×r) nonuniform grid for each module. The relatively fine grid spacing is used to provide sufficient resolution of the critical areas in the velocity and temperature fields. In order to examine the grid sensitivity, the results for coarse grids are obtained and compared. It is shown that the results become saturated using finer girds and the overall convective heat transfer coefficient for a 122×62 grid is about 2.7% different from that for a 102×52 grid. In addition, the results for developing flow computation are compared to those for the fully developed flow calculations. Both the fully developed and developing results are almost identical and the difference is only 0.05%.

A mixed gas of CO_2 15%, H_2O 10%, and N_2 75% on a molar basis is used as the working fluid, and the calculations are performed with the different wall temperatures of 300K, 600K, and 900K at the fixed inlet temperature of 1,200K.

4. Results and Discussion

Before investigating the combined convection and radiation heat transfer, various dimensional parametric studies for pure convection heat transfer are carried out.

The effect of the Reynolds number on the flow and heat transfer field is shown in Fig. 2. The ratios $\overline{Nu}_{conv}/(Nu)_0$ and $f Re/(f Re)_0$ for $S/D=0.5$, $F/D=L/D=0.25$, and $H/F=1$ display a strong Reynolds number dependence. $(Nu)_0$ and $(f Re)_0$ are the Nusselt number and the friction factor times the Reynolds number for fully developed laminar flow in a circular smooth tube, which are 3.66 and 64.0, respectively. Both the average convection Nusselt number and the friction factor monotonously increase with the Reynolds number. As the Reynolds number increases, the main flow is distorted dramatically. Due to the flow distortion, the washing effect at the reattachment point increases significantly, and the strength of recirculation becomes greater with the size of fins and disks. On the other hand, the Nusselt number increase is accompanied by significant pressure drop increase. For example, while the average convection Nusselt number for Re=500 is approximately 10 times greater than that of a smooth tube, the corresponding pressure drop is 380 times greater than that for a smooth tube. However, it may not be a serious problem for this particular application. The typical tube diameter and length of the heat exchanger

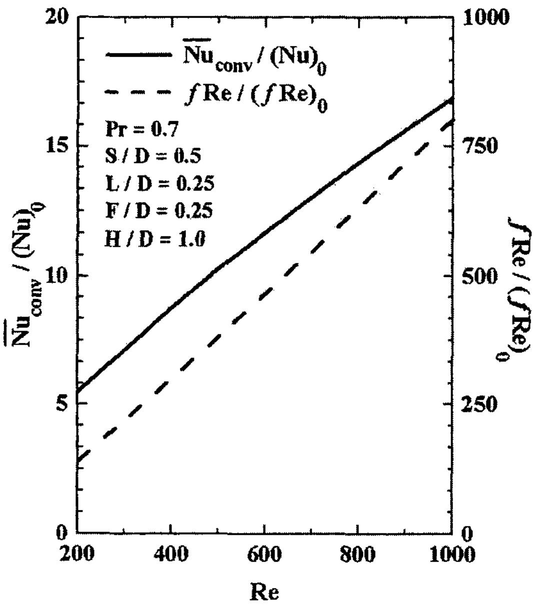


Fig. 2 Variation of friction factor and average convection Nusselt number with different Reynolds number for the fully developed region

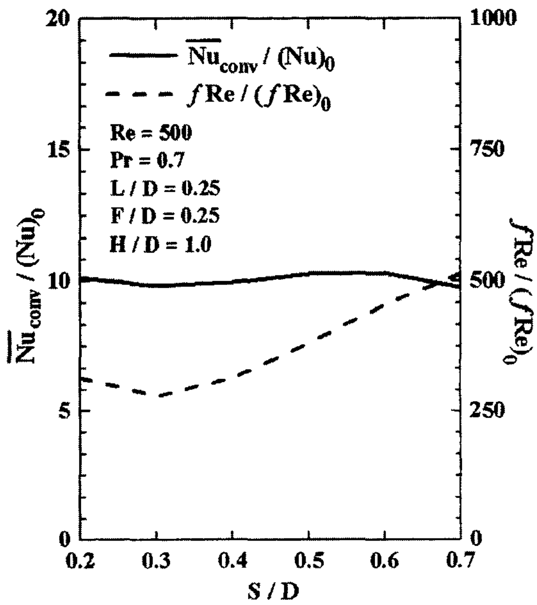


Fig. 3 Variations of friction factor and average convection Nusselt number as a function of fin-disk spacing for the fully developed region

for 23 kW capacity boiler are about 150 mm and 350 mm, respectively. The pressure drop for the heat exchanger is less than 1.0 Pa when the flow is laminar.

The effect of the disk location S on the pressure drop is shown in Figure 3. It is found that the pressure drop and the heat transfer enhancement does not change much when the disk location S/D varies from 0.2 from 0.7.

In Fig. 4., the effect of the circular disk size on heat transfer enhancement and pressure drop is shown. The convection Nusselt number and the friction factor increase slightly as the radius of the disk increases. It is because the flow patterns in the modules and the lengths of recirculation zones do not change much although the radius of the disk increases.

On the other hand, it is shown in Fig. 5 that the effect of the circumferential fin size on heat transfer and pressure drop is significant. The increasing rate of the friction factor is much greater than that of the average convection Nusselt number. And, the steep increase rate is shown after the F/D=0.25 around.

Figure 6 shows the velocity field of the developing flow for Re=500 and Pr=0.79 when S/D, L/D, F/D, and H/D are 1.0, 0.5, 0.25, and 1.0, respectively. It is clearly observed from the figure that the flow is fully developed after passing through the third module from the entrance. Then, the flow patterns become periodically identical with period H once it is fully developed. The flow-developing trend is consistent with the result in Berner et al. (1984), where the flow becomes periodic after only 3~5 modules in the entrance region. The circumferential

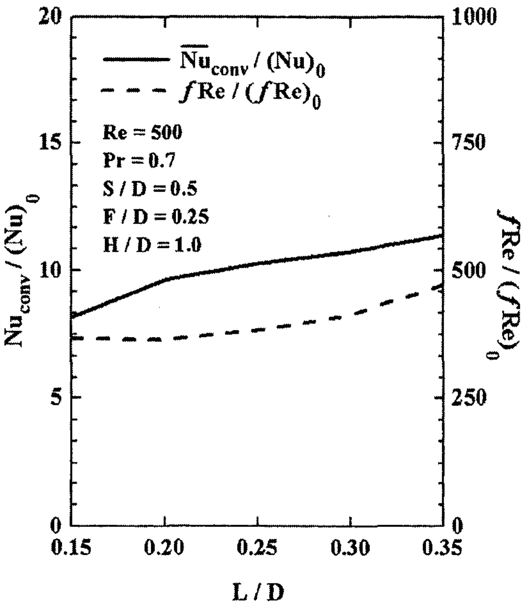


Fig. 4 Variations of friction factor and overall Nusselt number as a function of disk radius for the fully developed region

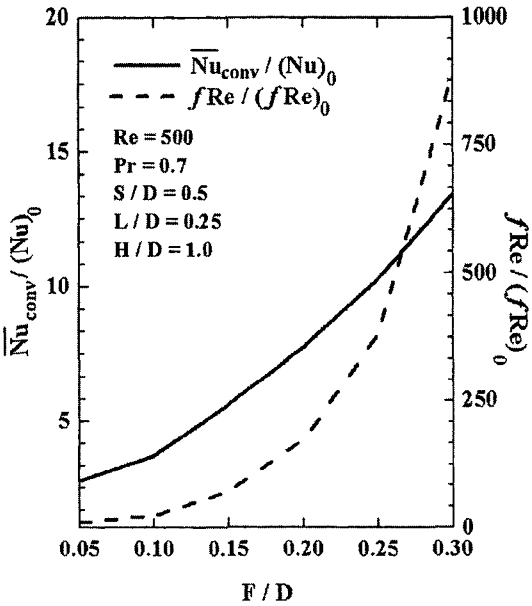


Fig. 5 Variations of friction factor and average convection Nusselt number as a function of fin height for the fully developed region

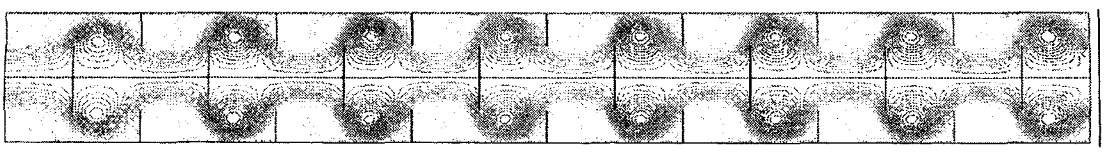


Fig. 6 Streamlines of developing and developed regions

fins reduce the cross section of the circular duct. The reduced cross section causes the acceleration of flow. Following sudden expansion, there is a large separation zone behind the fins. The core flow is deflected by the circular disk and impinges against the wall. After reattachment, the flow is accelerated by the bounded flow from the disk and reaches the region influenced by the next fin. A stream of the bounded flow creates a second vortex in front of the next fin.

Figure 7 shows the local convection Nusselt number variation along the axial direction of the tube for developing flow. In a smooth tube, the local convection Nusselt number decreases from infinity at the entrance and approaches to the fully developed Nusselt number as the working fluid goes downstream. In this study, however, the trend of the local convection Nusselt number variation is significantly different from that for a smooth tube. Although the local heat transfer coefficient strongly depends upon the location in a module, it increases gradually with the similar pattern until the flow is fully developed. And, the local convection Nusselt number is completely developed after the third module. The same trends are found from the other studies (Prakash and Liu, 1985). In general, a boundary layer is developed whenever fluid flow contacts a tube wall in the entrance region. In this geometry, however, the duct diameter is abruptly enlarged right after the inlet so that recirculation, which is not favorable to heat transfer, is generated instead of the regular boundary layer of a circular tube. The recirculation caused by the fins prevents the tube wall from directly contacting with the main flow. Therefore, the local convection Nusselt number at the recirculation region behind the fin is very low and the upturn of the local convection Nusselt number occurs as a result of the reattachment of the flow. The secondary recirculation of the upstream side of the fin decreases the local convection Nusselt number again, but it makes a small peak of the local convection Nusselt number at the reattachment point of the secondary recirculation.

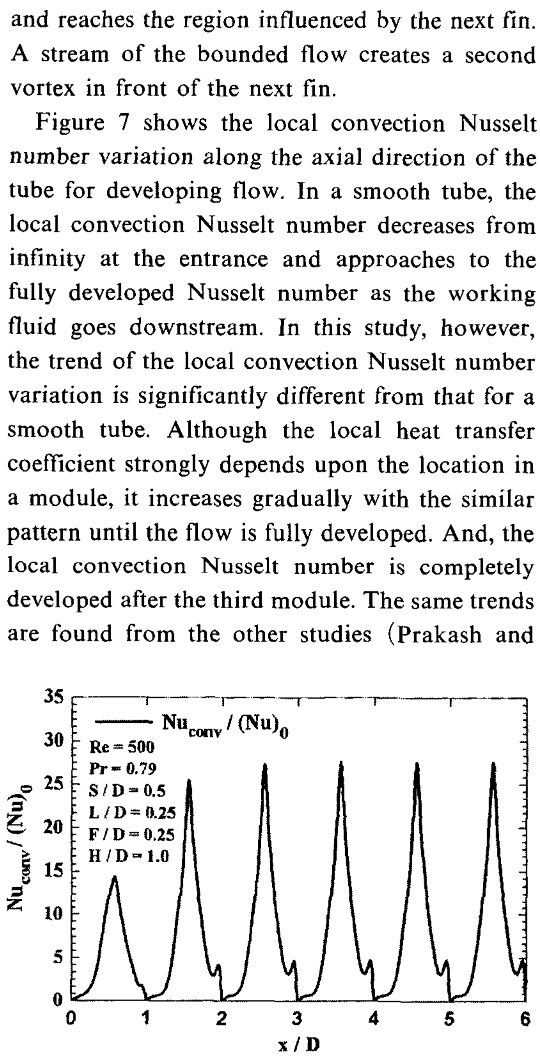


Fig. 7 Local convection Nusselt number variation

The isothermal lines of pure convection and combined convection and radiation are shown in Fig. 8 to compare the heat transfer characteristics. The upper part of the figure is for pure convection and the lower part for combined convection and radiation, respectively. In the first and the second modules the isothermal lines look similar, and it is difficult to identify the difference between the upper and lower parts. However, the differences of temperature fields near the center line become significant after the second module. The temperature gradient of pure convection near the center line is relatively steeper than that of combined convection and radiation. The temperature gradient of combined convection and radiation becomes gentle compared with that of pure convection because of radiation heat transfer. In the last two modules, the gas temperature of pure convection near the center line is not the same as that of the wall. On the other hand, the gas

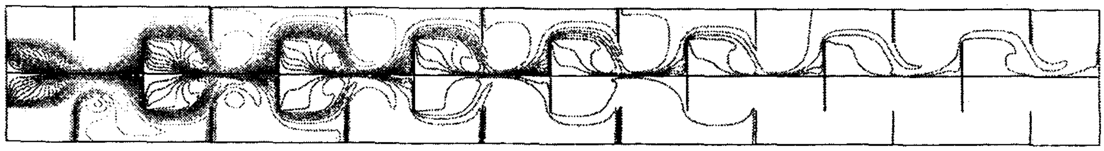


Fig. 8 Temperature contour comparison of pure convection (upper half) and combined convection and radiation (lower half)

temperature of combined convection and radiation is the same as that of the wall so that there is no temperature gradient in the duct. From the figure, the effect of radiation heat transfer can be clearly shown.

Variations of the average radiation Nusselt numbers of each module is shown in Fig. 9. The inlet temperature and the Reynolds number are 1,200K and 500, respectively. Three different wall temperatures, which are 300, 600, 900K, are used for the calculation. The average Nusselt numbers are compared to the fully-developed laminar flow Nusselt number in a smooth tube, which is known at 3.66, because it is easy to visualize the effect of radiation and the circumferential fins and disks on heat transfer enhancement. As the wall temperature increases, the radiation Nusselt number increases. This means that radiation heat transfer becomes relatively important when the

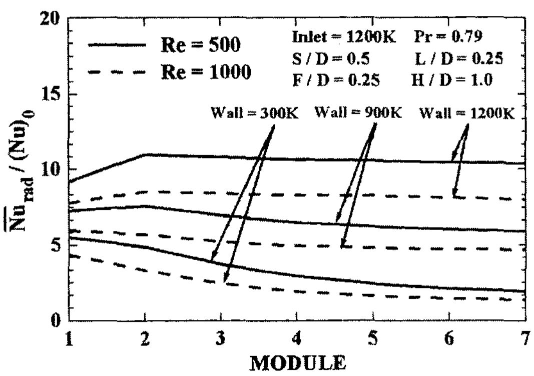


Fig. 9 Radiation Nusselt number variation for the Reynolds number

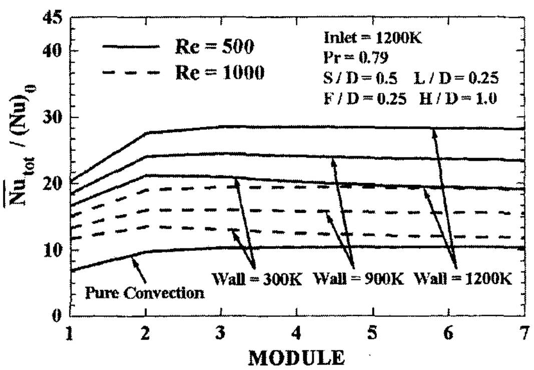


Fig. 10 Total Nusselt number variation for the Reynolds number

wall temperature is high. The average radiation Nusselt numbers decrease gradually as it goes downstream. As the wall temperature is lower, the decreasing rate of the average radiation Nusselt number becomes large.

The total Nusselt number variations for different wall temperatures are represented in Fig. 10. It is known that pure convection Nusselt number is fully developed after the third module. Based on the results, the convection and radiation contribution to total heat transfer can be obtained. The total Nusselt numbers are 14~84% greater than that of pure convection because radiation heat transfer is involved. As the wall temperature increases the radiation contribution becomes greater. Therefore, radiation heat transfer cannot be ignored for this particular application. On the other hand, the turbulent convection Nusselt number for the same configuration is about 300, if Reynolds number is 7,000 (Seo et al., 2000). Based on the comparison of the order of magnitude of radiation and turbulent convection Nusselt numbers, it can be said roughly that the radiation contribution to total heat transfer in a circular duct with circumferential fins and circular disks is less than 10% when the Reynolds number is about 7,000.

In addition, it is known from Fig. 9 and Fig. 10 that the total Nusselt number increases with the Reynolds number. The amount of the total Nusselt number increase due to the Reynolds number is greater than that of the radiation Nusselt number. This means that the convection Nusselt number is more sensitive to the Reynolds number that the radiation Nusselt number.

5. Conclusion

Pure convection and combined convection and radiation in a circular duct with circumferential fins and circular disk are numerically investigated for developing and fully developed laminar flow. Based on the pure convection calculation, it is shown that the flow is fully developed after passing the third module. Convection heat transfer is significantly affected by the Reynolds number and the sizes of the circumferential fin and the circular disk. Therefore, these should be carefully designed for the optimal operation of the compact heat exchanger. On the other hand, the fin-disk spacing is relatively less important to heat transfer than the other design variables. From the combined convection and radiation calculation, it is shown that radiation contribution to the total heat transfer is significant if the flow is laminar. The total Nusselt number increases by 14~84% if radiation is considered. However, the role of radiation decreases significantly when the flow becomes turbulent. It is because convection becomes very strong due to the circumferential fins and the circular disks. Therefore, the amount of radiation contribution should be carefully estimated before designing the heat exchangers.

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