

# Minimization of Hidden Area Using Genetic Algorithm in 3D Terrain Viewing

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**Abstract :** Optimal allocation of viewers on a terrain in such a way that the hidden area would be minimized has many practical applications. However, it is impossible in practical sense to evaluate all the possible allocations. In this paper, we propose an optimal allocation of viewers based on genetic algorithm that enables probabilistic search of huge solution space. An experiment for one and three viewers was performed. The algorithm converges to good solutions. Especially, in one viewer case, the algorithm found the best solution.

**Key Words :** Visibility Analysis, Terrain Information, Genetic Algorithm.

## 1. Introduction

Given a terrain surface and a viewer, the visibility analysis is calculating the portion of the terrain that is visible from the viewer. Visibility analysis on terrain is fundamental problem in computer graphics, navigation, and other engineering applications. Many researchers including Cole and Sharir(1989) and Floriani and Magillo(1994) dealt with efficient calculation of visibility. There are many applications of visibility analysis. For example, Choo and Koo(1998), and Choo(1998) dealt with the problem of finding optimal curve through shadow volume, which is the set of points in 3D space visible to none of the viewers, under a certain performance index. It can be applied to navigation of the unmanned plane which penetrates over the terrain avoiding observation from the viewers(Choo,

1998).

One of the most important applications of visibility analysis is optimal allocation of the viewers. Some practical applications include placement of antennas for cellular telephone company and placement of camera for security purpose on banks. Marengoni *et al.*(1996) dealt with the optimal allocation problem. They tried to find how many viewers are needed to cover the whole terrain and where to place them. In their work polyhedral terrain was assumed, and an approach based on computational geometry was used. Their work is theoretical but less useful in practical sense. For example, they requires 18 viewer for a simple data of 32 by 24 points. It is too much compared to the size of the area.

To overcome such a problem, we suggest a different approach: Given 3D terrain map and the number of

viewers, where should we place them to attain maximal coverage? In view of military applications, the problem is how to minimize shadow volume and chance of penetration by allocating the viewers effectively.

There are many interesting optimization problems for which no reasonably fast algorithms have been developed. The problem posed in this study is one of these problems. If there are  $n$  viewers and  $M$  candidate locations for each viewer,  $M^n$  allocations of viewers are possible. Usually  $M$  is a large number, and it is impossible in practical sense to evaluate all the possible allocations.

Genetic algorithm is a stochastic search algorithm developed by Holland(1975), whose search methods model the natural phenomena of genetic inheritance and Darwinian strife for survival. It has been applied successfully to a variety of optimization problems, including combinatoric optimizations, scheduling and CAD(Gen and Cheng, 1997). This paper proposes an optimal allocation of viewers using genetic algorithm.

## 2. Visibility Test and Genetic Algorithm

### 1) Visibility Test

Terrain can be represented as a function called height field  $z = H(x, y)$ . Assume that a test  $\mathbf{q} = [q_x, q_y, q_z]^T$  point and viewing point  $\mathbf{w} = [w_x, w_y, w_z]^T$  are given. In order for  $\mathbf{q}$  to be visible from  $\mathbf{v}$ , every point  $\mathbf{w} = [w_x, w_y, w_z]^T$  on the line segment  $\overline{\mathbf{v}\mathbf{q}}$  should satisfy  $w_z > H(w_x, w_y)$ . Following is the visibility test algorithm.

#### Algorithm Visibility( $\mathbf{v}, \mathbf{q}$ )

1)  $\alpha = 0$

$\Delta\alpha = d/D$  //  $D$ : Distance between  $\mathbf{v} = [v_x, v_y, v_z]^T$  and  $\mathbf{q} = [q_x, q_y, q_z]^T$  in  $xy$  plane. It's the distance between  $[v_x, v_y]^T$  and  $[q_x, q_y]^T$   
 //  $d$ : Incremental distance in  $xy$  for

visibility test.

2) Calculate the following. Last two terms represent integer and fraction part respectively.

$$\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = (1-\alpha) \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} + \alpha \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} + \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

3) Calculate  $h_w = H(w_x, w_y)$  by bilinear interpolation.

$$h_w = (1-r_x)(1-r_y)H(n_x, n_y) + r_x(1-r_y)H(n_x + 1, n_y) + (1-r_x)r_yH(n_x, n_y + 1) + r_xr_yH(n_x + 1, n_y + 1)$$

4) If  $w_z < h_w$ , **return(FALSE)**

5)  $\alpha = \alpha + \Delta\alpha$ ; If  $\alpha \geq 1$ , **return(TRUE)**

6) Go to step 2.

Let  $Coverage(\mathbf{v})$  be the set of points visible from  $\mathbf{v}$ . The set of visible points from the set of viewing points  $\mathbf{V} = \{\mathbf{v}_1 \dots \mathbf{v}_n\}$ , can be defined as

$$Coverage(\mathbf{V}) = \bigcup_{i=1}^n Coverage(\mathbf{v}_i),$$

where  $\bigcup$  is the set union operator. Then our goal is to find  $\mathbf{V}$  that maximizes  $Coverage(\mathbf{V})$ .

### 2) Genetic Algorithm

Genetic algorithm is a computational algorithm that utilizes the notion of 'survival of the fittest' and 'natural selection'. The fitter the individual to the environment, the higher its probability of survival. The algorithm is started with a set of solutions called population. Each solution is considered as a chromosome, and is usually coded by a bit string. Population of one generation evolves to the next generation through the process of selection, crossover, and mutation. These are basic operations of genetic algorithm.

Selection is the process of constructing the population for the next generation based on the fitness of individuals. In roulette selection, individuals with higher relative fitness have more chance to be selected. Probability of selection for the  $i$ th individual is

$$P_i = \frac{f_i}{\sum_k f_k},$$

where  $f_i$  is the fitness of the  $i$ th individual (Michalewicz, 1992; Mitchell, 1996).

Crossover is the process of creating new offsprings by combining parent chromosomes. In simple crossover, a random split point for two chromosomes is selected and the parents exchange their parts right to that point, making two offsprings. The expected number  $p_c \times popsize$  of chromosomes undergo the crossover operation, where  $p_c$  is predefined crossover probability and  $popsize$  the size of population.

Mutation is the process of randomly changing bits in chromosome. Randomly selected bit with value 1 is changed to value 0, and vice versa. The predefined mutation probability is  $p_m$  the ratio of bits that undergo mutation to total number of bits.

### 3. Maximization of Visibility using Genetic Algorithm

#### 1) Preprocessing

A ridge is a high edge along a terrain, and can be used as candidate location for the viewers. Ridge points are extracted from the terrain by calculating uphill flow field and generating ridge graph (Guo, 1998). They are sorted in descending order of height and stored in  $C[M]$  as the following.



Fig. 1. Terrain and ridge points.

$$C[M] = \{(x_0, y_0), (x_1, y_1), \dots, (x_{M-1}, y_{M-1})\}$$

where  $M$  denotes the number of ridge points.

Fig. 1. shows the terrain and the extracted ridge points, which are displayed in white.

#### 2) Genetic Algorithm in Shadow Volume Minimization

##### (1) Representation of Individual

The  $k$ th individual,  $s^k$  is represented as shown in Fig. 2, where  $n$  is the number of viewers. The index of the  $i$ th viewpoint in  $C[M]$ ,  $v_i^k$  is encoded in binary number. The number of bits,  $b$ , which is needed to represent each  $v_i^k$ , is the integer that satisfies

$$2(b-1) < M \leq 2^b.$$

Then the total length of individual  $nb$  is bits. The index value in  $[0, M-1]$  is mapped to a binary number in  $[0, 2^b-1]$ . To recalculate the index value,  $i$  from the binary number,  $g$  for the individual, the following relation is used

$$i = \left[ g \times \frac{M-1}{2^b-1} + 0.5 \right],$$

where  $[ \ ]$  is Gauss operator.

##### (2) Evaluation

The evaluation function for the  $k$ th individual reflects the area visible from the viewing points, and defined as

$$f(s^k) = \frac{\bigcup_{i=0}^{n-1} Coverage(v_i^k)}{TotalArea},$$

where  $Coverage(v_i^k)$  is the area in  $xy$  plane viewed from the  $i$ th viewpoint in the  $k$ th individual, and  $TotalArea$  is the total area of given terrain in  $xy$  plane.

Using the **visibility** algorithm presented in Section 2,

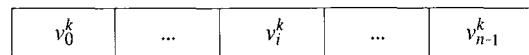


Fig. 2. Representation of the  $k$ th individual  $s^k$  with  $n$  view points.

Coverage( $v_i^k$ ) is computed as the following,

Coverage( $v_i^k$ ) = { } // Initialization.

**for each q in I // I** : Set of all points on the terrain.

**if (visibility( $v_i^k$ , q) is TRUE)** Coverage( $v_i^k$ ) = Coverage( $v_i^k$ )  $\cup$  {q}.

(3) Selection

Good individuals are selected from population to construct the next generation. In this paper, roulette selection is used, and the best individual is always selected in each generation. Following is the procedure.

1. For each individual  $s^k$ ,  $k = 1, 2, \dots, popsize$ , compute the fitness  $f(s^k)$ .
2. Calculate the total fitness for the population:

$$F = \sum_{k=1}^{popsize} F(s^k)$$

3. For each individual  $s^k$ , compute the selection probability  $p_k$ :

$$p_k = \frac{f(s^k)}{F}, k = 1, 2, \dots, popsize$$

4. For each individual  $s^k$ , calculate the cumulative probability  $q_k$ :

$$q_k = \sum_{j=1}^k p_j, k = 1, 2, \dots, popsize$$

5. Select the individuals using random number:

Generate a random number  $r$  in the interval [0,1]. If  $r \leq q_1$ , select the first individual  $s^1$ , else if  $q_{k-1} < r \leq q_k$  select  $k$ th individual  $s^k$ .

As the algorithm evolves, combination of viewpoints with high coverage gets higher fitness and survives, and the process converges to the final solution.

(4) Crossover

In the simple crossover that we use, a random split point for two chromosomes is selected and the parents exchange their parts right to that point, making two offsprings. If individuals  $s^i$  and  $s^j$  undergo crossover, and the  $k$ th position were chosen as the crossover point, gene exchange takes place as shown in Fig. 3. The

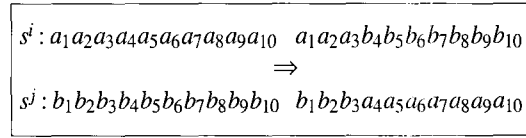


Fig. 3. Simple crossover(crossover point = 3).

individuals undergo crossover with probability  $p_c$ , and the crossover point is selected by generating random number in the interval [1,  $nb$ ], the length of the individual is  $nb$ .

Intuitively, each split part of chromosomes represents combination of view points, combined chromosome from two parts representing good combinations of viewers gets higher fitness and probability of survival.

(5) Mutation

Mutation is used to avoid trapping in the local optima. By randomly changing a bit in chromosome, new solution is searched and better one could be found. Some bits in the chromosome are reversed with the probability of mutation probability  $p_m$ . There are  $nb \times popsize$  bits in the whole population. We expect, on average,  $p_m \times nb \times popsize$  mutations per generation. For every bit in the population, we generate a random number  $r$  from the range [0,1]. If  $r < p_m$  we mutate the bit.

(6) Termination Condition

We measure the progress made by the algorithm in a predefined number of generations  $N$ , and stop the algorithm if the progress is smaller than some predefined number.

## 4. Experimental Results

The  $200 \times 200$  Digital elevation model(DEM) was used in the experiment. In this study, 3942 ridge points were extracted and used as candidate viewpoints. The experiment was performed for 1 and 3 viewers. Fig. 4

shows terrain of the two test areas overlapped with contour lines for the two cases. In both cases, numbers of individual and generation were 10 and 100, the probability of crossover and mutation are predefined

constants, and are 0.25, and 0.01, respectively. Fig. 5 and Fig. 6 show the evolution process. In these figures,  $j$  is the generation number,  $r$  is the coverage,  $x, y$  are coordinate of viewpoints,  $h$  is height of the viewpoints,

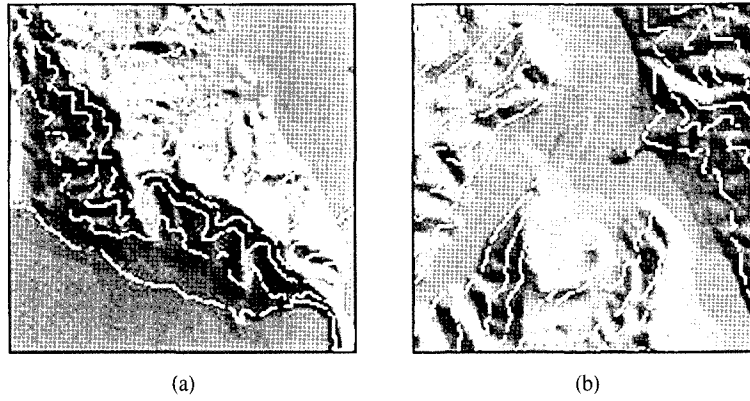


Fig. 4. Two test areas for 1 viewer(a) and 3 viewers(b).

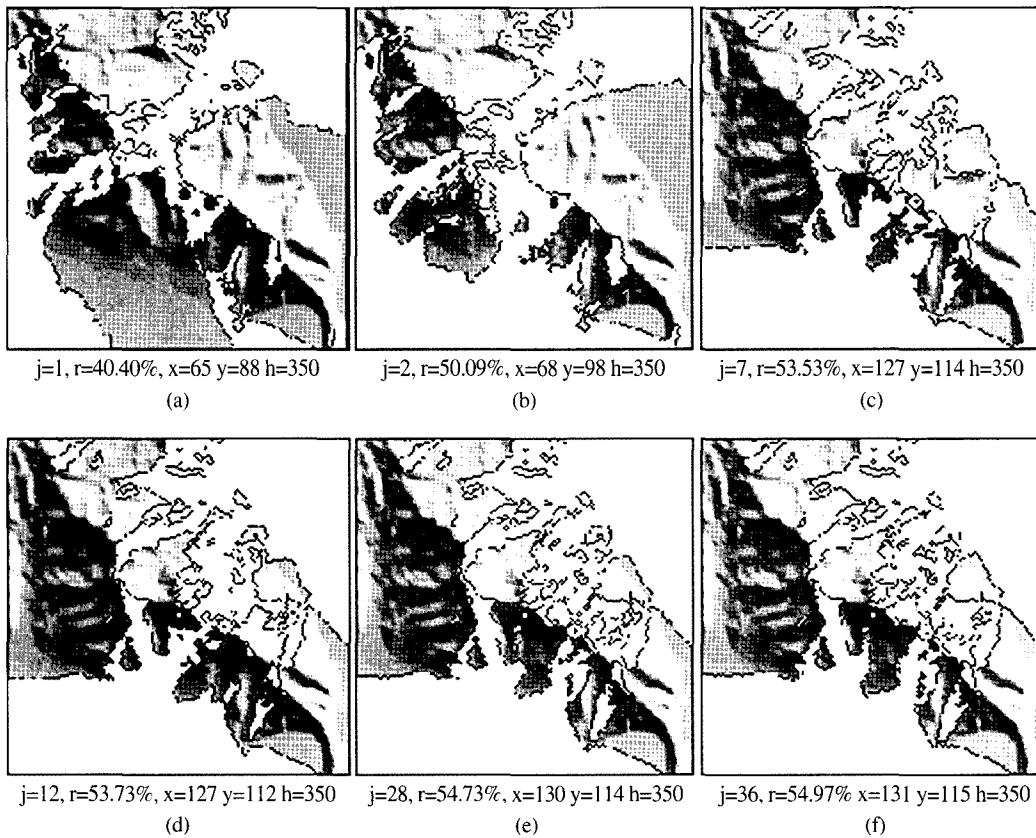


Fig. 5. Evolution process in 1-point search.

Table 1. Top 10 View Points and Visibilities.

rank	index	x	y	visibility
1	7	131	115	54.97%
2	8	130	114	54.74%
3	3	129	113	53.88%
4	5	126	113	53.74%
5	4	127	112	53.74%
6	2	127	114	53.54%
7	6	128	115	52.94%
8	48	132	115	52.30%
9	60	133	116	51.48%
10	20	68	100	50.44%

and  $\oplus$  is the location of viewpoints. Darker part is invisible area and brighter part is visible area. The darker part is overlapped on the terrain map to emphasize the effect of terrain on the visibility. In 1-point case, all the possible locations were tested for visibility and top 10 results of the exhaustive calculation is shown in Table 1. This shows the genetic algorithm found the best location successfully(Fig. 4(f) x131 y=115 r=54.97%).

Three point search case requires about  $(4000)^3$  times more calculation than one point case. So the result of the algorithm cannot be compared with the best solution.

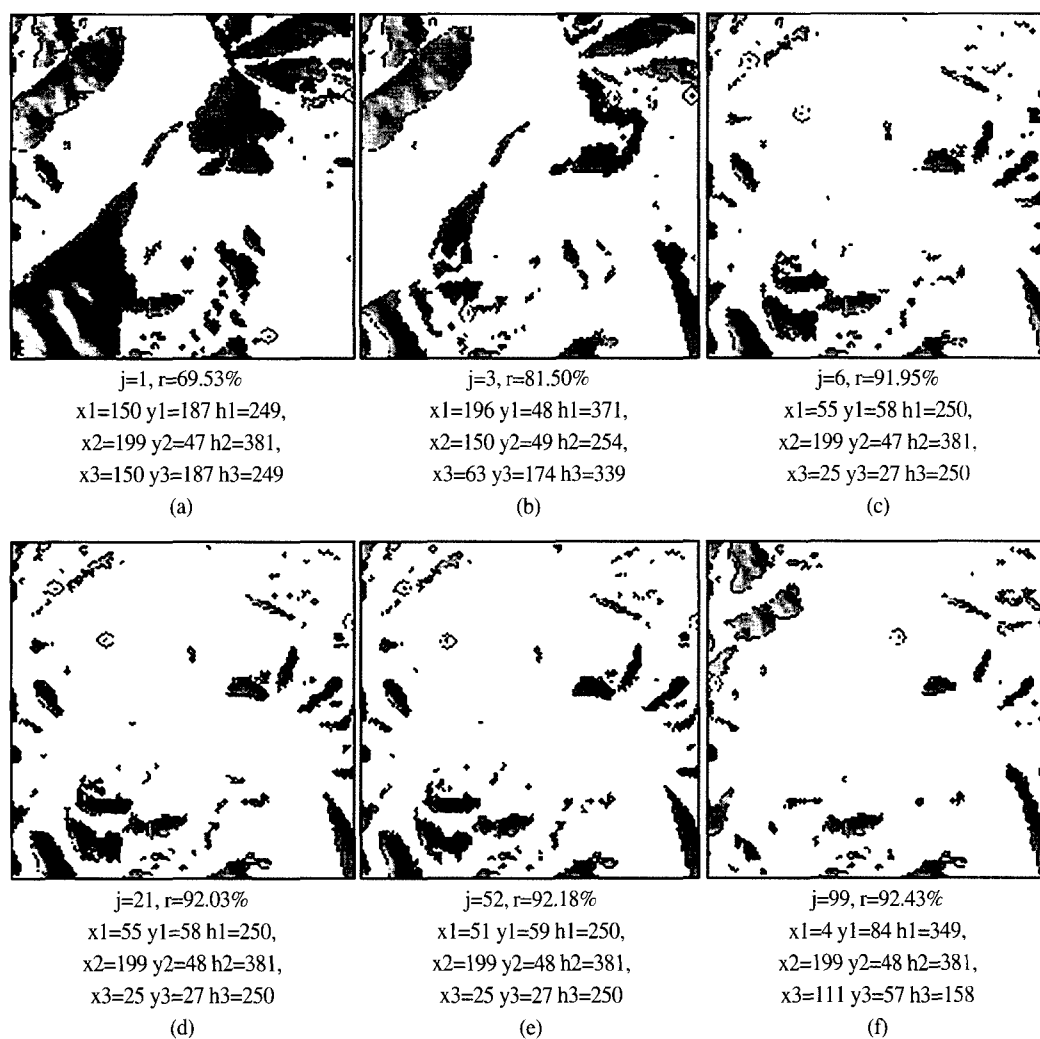


Fig. 6. Evolution process in 3-point search.

But the algorithm converged to a good solution with the visibility of 92.43% at the 99th generation.

## 5. Conclusions

When terrain of interest is given and some viewers can be located on it, a part of the terrain would be seen from the viewers and the other part would be hidden by the terrain itself, and visible to none of the viewers. We present an idea about how to solve the problem of placing given number of viewers on a terrain such that visible area of the terrain is maximized. For there are too many possible allocations of the viewers, the exhaustive search was impossible and a method based on genetic algorithm was developed.

The algorithm proposed in this study was applied to an  $200 \times 200$  DEM, and two cases for one and three viewers were tested. In one viewer case, the algorithm found the best solution. In three viewer case, comparison with the best solution was impossible, but the algorithm showed to converge to a good solution with coverage of 92.43%.

We are now expanding the algorithm presented here to the problem of placing the relay stations for mobile telecommunication. The key factor would be redefinition of the function defined in section 2 for the radio frequency wave. In parallel we are looking for a better implementation of the algorithm visibility defined in section 2 to speed up the processing time.

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## References

- Choo, Y.W., 1988. *A Study on the Curve Generation for Optimal Path*, Ph.D. Dissertation, Dankook Univ.
- Choo, Y.W. and J.Y. Koo, 1998. Optimization of Curves through Polygon, *Journal of Korea Information Processing Society*, 5(9): 2388-2394.
- Cole, R. and M. Sharir, 1989. Visibility problems for polyhedral terrains, *Journal of Symbolic Computation*, 7: 11-30.
- Floriani, L. and P. Magillo, 1994. Visibility algorithms on triangulated digital terrain models, *International Journal of GIS*, 8(1): 13-41.
- Gen, M. and R. Cheng, 1997. *Genetic Algorithm and Engineering Design*, Wiley & Sons, Inc.
- Guo, D., 1998. [www.cs.brown.edu/people/dbg/publication/embs98/ivus\\_speckle.pdf](http://www.cs.brown.edu/people/dbg/publication/embs98/ivus_speckle.pdf).
- Holland, J.H., 1975. *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor.
- Marengoni, M., B. Draper, A. Hanson and R. Sitaramen, 1996. Placing Observers to Cover a Polyhedral Terrain in Polynomial Time, *Proc. IEEE Workshop on Application of Computer Vision*, Sarasota, Florida.
- Michalewicz, Z., 1992. *Genetic Algorithms + Data Structures = Evolution Programs*, Springer-Verlag.
- Mitchell, M., 1996. *An Introduction to Genetic Algorithms*, MIT Press.