

## A Study on the Camera Calibration Algorithm of Robot Vision Using Cartesian Coordinates

Yong-Joong Lee\*

(논문접수일 2002. 3 26, 심사완료일 2002. 10. 28)

### Abstract

In this study, we have developed an algorithm by attaching a camera at the end-effector of industrial six-axis robot in order to determine position and orientation of the camera system from cartesian coordinates. Cartesian coordinate as a starting point to evaluate for suggested algorithm, it was easy to confront increase of orientation vector for a linear line point that connects two points from coordinate space applied by recursive least square method which includes previous data result and new data result according to increase of image point. Therefore, when the camera attached to the end-effector has been applied to production location, with a calibration mask that has more than eight points arranged, this simulation approved that it is possible to determine position and orientation of cartesian coordinates of camera system even without a special measuring equipment.

**Key Words :** Industrial six-axis robot, Camera system, Cartesian coordinates, Recursive least square method, Imaginary point, End-effector, Calibration mask, Position and orientation.

### 1. Introduction

If a camera system used as an image input unit contains error or its accurate position is unknown, position compensation of industrial six-axis robot can not be done through this camera system. Therefore, it is necessary to measure accurate external parameter reflecting position and orientation of camera, and internal parameter and cartesian coordinates which show

error of camera. As for position and orientation determination of a camera system, Sobel suggested an algorithm interpreting the camera system which represented by non-linear equation<sup>(1)</sup>. Liu presented method for computing camera location using 2-D to 3-D straight line or point correspondences. A linear algorithm is described which required at least eight line correspondences or six point correspondences, and a nonlinear algorithm is described which required at least

\* 주저자, 창원기능대학 생산자동화과 (leeyj@chollian.net)  
주소: 641-772 경남 창원시 중앙동 110번지

three line correspondences or three point correspondences<sup>(2)</sup>. Also Liu made a decision of external parameter by using correspondence relation of line and point from perspective transform model based on pin hole geometry, and there are many cases such algorithm as been applied to a robot<sup>(2,3)</sup>. Tsai divided internal parameter by lens error, effective focal length, and image factor from the suggested algorithm<sup>(4)</sup>. Meanwhile, in order to decide internal parameter of camera system, modeling of camera system should be done in advance. Also as for modeling of camera system, Puskorius compensated the error by suggesting the perspective translation error model, which divided parameter, about camera system, into internal and external parameter<sup>(5)</sup>. Therefore, setting up the geometrical perspective transform pin hole model excluding lens error from camera system, makes a linear line of space and linear line of a image plane from opposite relation of image coordinates and camera coordinates<sup>(3,6)</sup>. Next, find a linear equation that gives a focal length and a rotation matrix of camera system, using the result, it was possible to find the position translation of 6th axis coordinates robot which is camera system<sup>(6,7)</sup>. Also in the method of simulation, as a method that is different from least square method, which deals with all equation following input of new data, includes result of the new data and data which have found in the previous sentence<sup>(6,8)</sup>. This increased simulation efficiency by applying the recursive least square method which gives new result<sup>(8)</sup>. This study developed an algorithm that allows identification of the position and orientation of camera system when camera has been attached to the end-effector of the robot with a calibration mask with eight more arranged points.

## 2. Main Subject

Fig. 1 Shows a geometric perspective translation pin hole model excluding lens error from the camera system as reported in<sup>(5)</sup>.

where

$C_s$  : camera coordinates

$C_w$  : cartesian coordinates

$C_i$  : image coordinates

$p_o$  : coordinates value of imaginary point which is seen from image coordinates

$p_i$  : coordinates value of  $p_o$

$f$  : valid focal length

One point that's shown from camera coordinate which opposite from image coordinate value is as following.

$$\frac{x_i}{x} = \frac{f}{z}, \quad \frac{y_i}{y} = \frac{f}{z} \quad (1)$$

Also, the location of computer frame buffer which opposite from image coordinates value is as following<sup>(2,3)</sup>.

$$\begin{aligned} x_f &= S_x dx^{-1} x_i + c_x \\ y_f &= dy^{-1} y_i + c_y \\ dx &= dx' \frac{N_{cx}}{N_{fx}} \end{aligned} \quad (2)$$

where

$S_x$  : image scale parameter of  $x_i$  orientation

$(c_x, c_y)$  : column and row of computer memory center

$dx'$  : arranged scan orientation of camera system's CCD sensor element distance

$dy$  : scan direction rectangular camera system's CCD sensor element distance

$dx$  : computer buffer distance in scan direction

$N_{cx}$  : CCD sensor element number of camera system in scan direction

$N_{fx}$  : sampling of pixel numbers done by camera

Calculated image scale is the one which shows ratio of computer frame buffer and holder of CCD sensor<sup>(3)</sup>. The CCD sensor element of camera system which is being used commonly these days is produced at a regular distance. It is not necessary to decide calculated image scale specially because pixel distance matches exactly 1:1

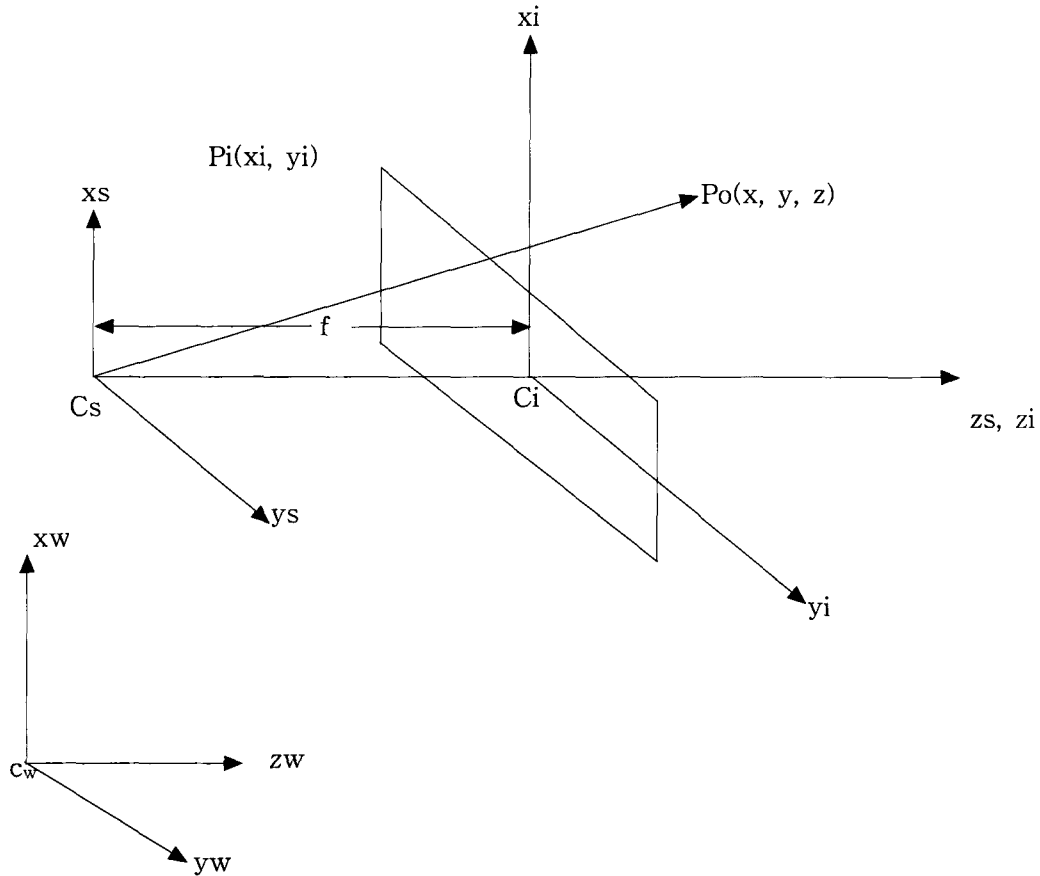


Fig. 1. Geometric perspective translation pin hole model excluding lens distortion

in column and row of computer frame buffer<sup>(3)</sup>. Also for industrial CCD camera which has lens distortion error below  $\pm 0.01$ , it have been approved that distortion error does not effect system<sup>(2,3)</sup>. Therefore, in this research, we did not regard the errors as internal parameter. Point correspondence relation in cartesian coordinates is as following<sup>(6,7)</sup>.

where

- $C_w$  : standard cartesian coordinates
- $C_i$  : image coordinates
- $C_s$  : coordinates of attachment of camera at the end-effector of industrial six-axis robot
- $C_6$  : 6th robot coordinates

- $r$  : linear line for connecting the two points space
- $L$  : linear line that has been created by projecting on a plane
- $\vec{N}$  : normal vector regarding to projecting on a plane of linear line  $r$
- $\vec{r}_w$  : vector of orientation of linear line  $r$
- $\vec{p}_i$  : vector of orientation of point  $p_i$
- $T_w^6$  : transforming of six-axis robot coordinates as a basic of three dimension standard coordinates
- $T_s^6$  : transforming of camera coordinates which is based on six-axis robot coordinates

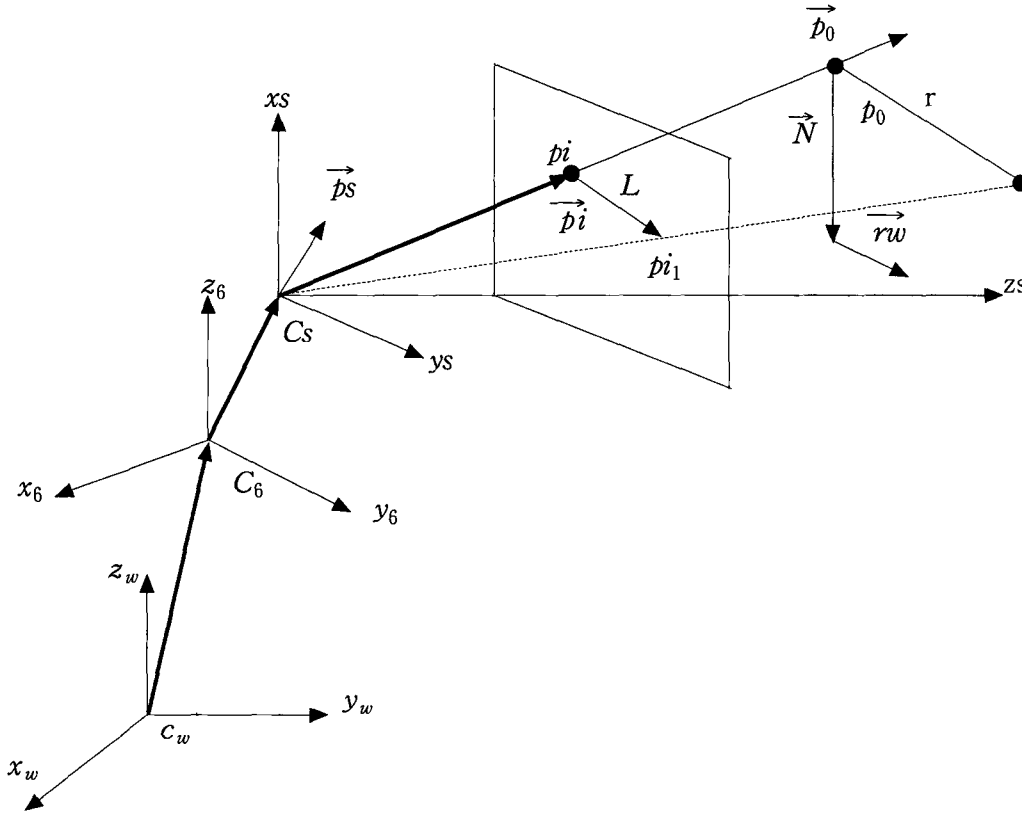


Fig. 2 Point correspondence relation in cartesian coordinates

Lead relation of linear line  $L$  of image plane and linear line  $r$  of cartesian coordinates which has been known from this figure and use it to decide position and orientation of camera system which is attach to end-effector of industrial robot<sup>(3,6)</sup>.

$$\vec{p}_i \cdot \vec{N} = 0 \quad (3)$$

Therefore, using equation (3), we can show relation of linear line  $r$ , linear line  $L$  and rotation convert matrix  $R$  of camera system which is based on the cartesian coordinates

$$(\vec{r}_w^T R) \vec{N} = 0 \quad (4)$$

To make a decision of rotation convert matrix  $R$  and

available focal length  $f$  of camera system, we find construction of equation (4) using algorithm<sup>(3)</sup>.

[Theory 1] : Rotation convert matrix  $R$

Amongst 24 angles of Euler, we apply tilt, swing and spin as followings.

$$\begin{aligned} R_{Z'Y'X'}(\alpha, \beta, \gamma) &= R_{Z'}(\alpha)R_{Y'}(\beta)R_{X'}(\gamma) \\ &= \begin{bmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta \sin \gamma - \sin \beta \cos \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma \\ -\sin \beta & \cos \beta \sin \gamma & \cos \beta \cos \gamma \end{bmatrix} \\ &= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \end{aligned} \quad (5)$$

[Theory 2] : Normal vector  $\vec{N}$

$$Dx_i + Ey_i + F = 0 \quad (6)$$

$$Dx + Ey + f^{-1}F = 0 \quad (7)$$

$$[l \ m \ n] \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} D \\ E \\ f^{-1}F \end{bmatrix} = 0 \quad (8)$$

Linear equation solved by equation (8) is as followings.

$$[lD \ mD \ nD \ lE \ nE \ lF \ mF \ nF] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix} = [-mE] \quad (9)$$

where

$$\begin{aligned} q_1 &= r_{11} r_{22}^{-1}, & q_2 &= r_{21} r_{22}^{-1} \\ q_3 &= r_{31} r_{22}^{-1}, & q_4 &= r_{12} r_{22}^{-1} \\ q_5 &= r_{32} r_{22}^{-1}, & q_6 &= f^{-1} r_{13} r_{22}^{-1} \\ q_7 &= f^{-1} r_{23} r_{22}^{-1}, & q_8 &= f^{-1} r_{33} r_{22}^{-1} \end{aligned}$$

However, in Figure 2, orientation vector  $\vec{p}_o$  according to the linear line  $r$ , rotation convert matrix  $R$ , orientation vector  $\vec{p}_s$  and position convert vector  $\vec{p}$  of camera system which is about cartesian coordinates's basis  $C_w$  can be shown as following<sup>(6)</sup>.

$$\vec{p}_o = R \vec{p}_s + \vec{p} \quad (10)$$

where

$$\begin{aligned} \vec{p}_o &= (x, y, z)^T \\ \vec{p}_s &= (x_s, y_s, z_s)^T \\ \vec{p} &= (p_x, p_y, p_z)^T \end{aligned}$$

$$\vec{p}_o' = \vec{p}_s + \vec{p}' \quad (11)$$

where

$$\vec{p}_0' = R^{-1} \vec{p}_w = (x', y', z')^T$$

$$\vec{p}' = R^{-1} \vec{p} = (p_x', p_y', p_z')^T$$

If solve equation (11)

$$\begin{aligned} x' &= x_s + p_x' \\ y' &= y_s + p_y' \\ z' &= z_s + p_z' \end{aligned} \quad (12)$$

When equation (1) is applied to equation (12), it turns out to be following equation.

$$\begin{aligned} fx' &= (z' - p_z')x_i + fp_x' \\ fy' &= (z' - p_z')y_i + fp_y' \end{aligned} \quad (13)$$

$$\begin{bmatrix} -f & 0 & x_i \\ 0 & -f & y_i \end{bmatrix} \begin{bmatrix} p_x' \\ p_y' \\ p_z' \end{bmatrix} = \begin{bmatrix} x_i z' - fx' \\ y_i z' - fy' \end{bmatrix} \quad (14)$$

[Algorithm 1] : Existence following by point opposition relation

$$det(A^T A) = 2f^4 [(x_i - x_{il})^2 + (y_i - y_{il})^2] \quad (15)$$

$$\vec{p} = R \vec{p}' \quad (16)$$

[Algorithm 2] : Position convert matrix  ${}^6_s T$  of camera coordinates for 6th axis coordinates.

By using equation (16), it is possible to organize convert matrix  ${}^w_s T$  of camera coordinates which uses cartesian coordinates as a base

$${}^w_s T = \begin{bmatrix} R & \vec{p} \\ 0 & 1 \end{bmatrix} \quad (17)$$

Also, position convert matrix of camera coordinates for 6th coordinates robot is as following<sup>(6)</sup>.

$${}^6_s T = ({}^w_s T)^{-1} \cdot {}^w_s T \quad (18)$$

### 3. Simulation and Result Inquiry

In Appendix, depending on the input of new data, the least square, which has been applied in this research, will give a new data set which includes results of both new and previous data<sup>(6)</sup>. Therefore, if the unknown number  $q_1 - q_8$  of equation (9), to find regarding to  $q_1 - q_8$ , there should be more than 5 known points, and minimizing the error by increasing those points, than it is possible to find  $q_1 - q_8$  by applying the least square method<sup>(8)</sup>.

$$x(k) = [A(k)^T A(k)]^{-1} A(k)^T b(k) \quad (19)$$

$$\begin{aligned} x(k+1) \\ = x(k) + g(k+1)[b_{k+1} - a(k+1)^T x(k)] \end{aligned} \quad (20)$$

This simulation that used a specialized mask revision is a work of monitor coordinates value measurement and point distance measurement. So, in this particular research, we applied Tsai's researched data<sup>(4,6)</sup>. This result is almost the same as Liu's simulation result<sup>(2,3)</sup>. Therefore, it's been approved that even without special detecting device, camera system of cartesian coordinates's position and orientation is possible to be known accurately by attaching a camera at the end-effector of industrial six-axis robot, set up the robot application mode as cartesian coordinates, only with calibration mask of 8 more points layed out in regular distance<sup>(6,7)</sup>.

### 4. Conclusion

This study developed an algorithm to decide position and orientation of the camera system from cartesian coordinates by an attaching camera at the end-effector of industrial six-axis robot. Cartesian coordinates as a starting point to evaluate the suggested algorithm, it was easy to confront increase of orientation vector for a linear line point that connects two points from coordinates space applied by recursive least square method which includes

previous data result and new data result according to increase of image point. This study developed an algorithm that allows identification of the position and orientation of camera system when camera has been attached to the end-effector of the robot with a calibration mask with eight more arranged points.

### Reference

- (1) Sobel, I., "On Calibrating Computer Controlled Cameras for Perceiving 3-D Scene," *Artificial. Intell.*, Vol. 5, pp. 185-198, 1974.
- (2) Yuncai, L., Thomas S., and Olivier D. F., "Determination of Camera Location from 2-D to 3-D Line and Point Correspondences," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 12, No. 1, January 1990.
- (3) Kim, J. K. and Lee, B. H., "A Study on the Camera Calibration Method Using Line Correspondence," *The Transactions of the Korean Institute of Electrical Engineers*, Vol. 40, No.11, November 1991.
- (4) Tsai, R. Y., "A Versatile Camera Calibration Technique for High-Accuracy 3-D Machine Vision Metrology Using Off-the-Shelf TV Cameras and Lens," *IEEE Journal Robotics and Automation*, Vol. 13, No. 4, pp. 323-344, 1988.
- (5) Puskorius, G. V. and Feldkamp, L. A., "Camera Calibration Methodology Base on a Linear Perspective Transformation Error Model," *IEEE Conf. on Robotics and Automation*, pp. 1858-1860, 1988.
- (6) Lee, Y. J., Shin, S. B., and Lee, Y. B., "Determination of Camera System Orientation and Translation in Cartesian coordinates," *Proceeding of the KSMTE Spring Conference 2000*, pp. 109-114, April 2000.
- (7) Lee, Y. J. and Lee, Y. B., "An Implementation of the Automatic Labeling Rolling-Coil Using Robot Vision System," *Journal of Control, Automation and Systems Engineering*, Vol.3, No.5, pp. 497-502, 1997.
- (8) Curtis, F. G., *Applied Numerical Analysis*, Addison-Wesley Publishing Company, Inc. 1978.

APPENDIX :

Table 1 Assign data for camera system

$p_x = 152.1942, p_y = -70.6095, p_z/f = 24.4105$ [mm]			
$R$			
	0.0346	-0.9376	0.3457
	0.9952	0.0637	0.0730
	-0.0905	0.3415	0.9354

Table 3 6 points

1.000e+01	1.000e+01	0.000e+00	5.191e+02	9.450e+01
1.000e+01	1.900e+02	0.000e+00	2.126e+02	1.180e+02
3.000e+01	1.000e+01	0.000e+00	5.199e+02	1.309e+02
3.000e+01	1.900e+02	0.000e+00	2.140e+02	1.533e+02
5.000e+01	1.000e+01	0.000e+00	5.208e+02	1.674e+02
5.000e+01	1.900e+02	0.000e+00	2.152e+02	1.883e+02
$p_x = 157.8409, p_y = -69.3086, p_z/f = 24.6423$ [mm]				
$R$				
	0.0157	-0.9427	0.3331	
	0.9997	0.0103	0.0131	
	-0.0136	0.3333	0.9426	

Table 5 8 points

1.000e+01	1.000e+01	0.000e+00	5.191e+02	9.450e+01
1.000e+01	1.900e+02	0.000e+00	2.126e+02	1.180e+02
3.000e+01	1.000e+01	0.000e+00	5.199e+02	1.309e+02
3.000e+01	1.900e+02	0.000e+00	2.140e+02	1.533e+02
5.000e+01	1.000e+01	0.000e+00	5.208e+02	1.674e+02
5.000e+01	1.900e+02	0.000e+00	2.152e+02	1.883e+02
7.000e+01	1.000e+01	0.000e+00	5.222e+02	2.033e+02
7.000e+01	1.900e+02	0.000e+00	2.164e+02	2.235e+02
$p_x = 153.9213, p_y = -70.3640, p_z/f = 24.1946$ [mm]				
$R$				
	0.02825	-0.9395	0.3412	
	0.9966	0.0524	0.0619	
	-0.0761	0.3384	0.9379	

Table 2 5 points

1.000e+01	1.000e+01	0.000e+00	5.191e+02	9.450e+01
1.000e+01	1.900e+02	0.000e+00	2.126e+02	1.180e+02
3.000e+01	1.000e+01	0.000e+00	5.199e+02	1.309e+02
3.000e+01	1.900e+02	0.000e+00	2.140e+02	1.533e+02
5.000e+01	1.000e+01	0.000e+00	5.208e+02	1.674e+02
$p_x = 157.1337, p_y = -69.6096, p_z/f = 23.8157$ [mm]				
$R$				
	0.0180	-0.9421	0.3346	
	0.9998	0.0179	0.0034	
	-0.0027	0.3347	0.9423	

Table 4 7 points

1.000e+01	1.000e+01	0.000e+00	5.191e+02	9.450e+01
1.000e+01	1.900e+02	0.000e+00	2.126e+02	1.180e+02
3.000e+01	1.000e+01	0.000e+00	5.199e+02	1.309e+02
3.000e+01	1.900e+02	0.000e+00	2.140e+02	1.533e+02
5.000e+01	1.000e+01	0.000e+00	5.208e+02	1.674e+02
5.000e+01	1.900e+02	0.000e+00	2.152e+02	1.883e+02
7.000e+01	1.000e+01	0.000e+00	5.222e+02	2.033e+02
$p_x = 154.2696, p_y = -70.2823, p_z/f = 24.0352$ [mm]				
$R$				
	0.0273	-0.9401	0.3396	
	0.9970	0.0498	0.0578	
	-0.0713	0.3370	0.9387	

Table 6 9 point

1.000e+01	1.000e+01	0.000e+00	5.191e+02	9.450e+01
1.000e+01	1.900e+02	0.000e+00	2.126e+02	1.180e+02
3.000e+01	1.000e+01	0.000e+00	5.199e+02	1.309e+02
3.000e+01	1.900e+02	0.000e+00	2.140e+02	1.533e+02
5.000e+01	1.000e+01	0.000e+00	5.208e+02	1.674e+02
5.000e+01	1.900e+02	0.000e+00	2.152e+02	1.883e+02
7.000e+01	1.000e+01	0.000e+00	5.222e+02	2.033e+02
7.000e+01	1.900e+02	0.000e+00	2.164e+02	2.235e+02
9.000e+01	1.000e+01	0.000e+00	5.236e+02	2.396e+02
$p_x = 153.4234, p_y = -70.4349, p_z/f = 24.3839$ [mm]				
$R$				
	0.0297	-0.9389	0.3428	
	0.9959	0.0569	0.0696	
	-0.0849	0.3394	0.9367	