한국 퍼지 및 지능시스템 학회 2002, Vol. 12, No. 6, pp. 583-588

On entropy for intuitionistic fuzzy sets applying the Euclidean distance

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ABSTRACT

Recently, Szmidt and Kacprzyk[Fuzzy Sets and Systems 118(2001) 467-477] proposed a non-probabilistic-type entropy measure for intuitionistic fuzzy sets. It is a result of a geometric interpretation of intuitionistic fuzzy sets and uses a ratio of distances between them. They showed that the proposed measure can be defined in terms of the ratio of intuitionistic fuzzy cardinalities: of $F\cap F^c$ and $F\cup F^c$, while applying the Hamming distances. In this note, while applying the Euclidean distances, it is also shown that the proposed measure can be defined in terms of the ratio of some function of intuitionistic fuzzy cardinalities: of $F\cap F^c$ and $F\cup F^c$.

Key Words : Intuitionistic fuzzy sets; Entropy; Cardinality; Distance

1. Introduction

There have been several typical methods being used to measure the fuzziness(entropy) of fuzzy sets[Zadeh[18], Shannon entropy[7], De Luca and Termini[12], Pedrycz[13], Kaufmann[8], Yager[16], Kosko[9-11], Burillo and Bustince[6]], since Zadeh in 1965 first mentioned about it.

Recently, Szmidt and Kacprzyk[14] proposed a measure of fuzziness for intuitionistic fuzzy sets introduced by Atanassov[1-5]. The measure of entropy is a result of a geometric interpretation of intuitionistic fuzzy sets and uses a ratio of distances between them. They showed that the proposed measure can be defined in terms of the ratio of intuitionistic fuzzy cardinalities: of $F\cap F^c$and $F\cup F^c$, while applying the Hamming distances.

In this note, while applying the Euclidean distances, it is also shown that the proposed measure can be defined in terms of the ratio of some function of intuitionistic fuzzy cardinalities: of $F\cap F^c$and $F\cup F^c$.

2. Definitions

In this section, we briefly review related definitions.

Definition 1. A fuzzy set A' in $X=\{x\}$ may be given as [17]

$$A'=\{\langle x, \mu_{A'}(x)\rangle | x\in X\}, \tag{1}$$

where $\mu_{A'}:x\to[0,1]$ is the membership function of A'; $\mu_{A'}(x)\in[0,1]$ is the degree of membership of $x\in X$ in A'.

Definition 2. An intuitionistic fuzzy set A in X is given by [1-5]

$$A'=\{\langle x, \mu_A(x), \nu_A(x)\rangle | x\in X\}, \tag{2}$$

where

$$\mu_A: X\to[0,1] \qquad \nu_A: X\to[0,1]$$

with the condition

$$0\le\mu_A(x)+\nu_A(x)\le 1 \quad \forall x\in X.$$

The numbers $\mu_A(x), \nu_A(x)\in[0,1]$ denote the degree of membership and non-membership of x to A, respectively.

For each intuitionistic fuzzy set in X, we will call

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x), \tag{3}$$

the intuitionistic index of x in A. It is a hesitancy degree of x to A [1-5].

Definition 3. Let A be an intuitionistic fuzzy set in X and $f:[0,1]\to[0,1]$ be a function. We define the following two cardinalities of a function of an intuitionistic fuzzy set:

· the least (:sure") cardinality of A is equal to the so-called sigma-count (cf. [19, 20]), and is called here

접수일자 : 2002년 5월 2일
완료일자 : 2002년 11월 8일
본 연구는 2002학년도 대구가톨릭대학교 일반연구비에 의해 지원받았습니다.

the $\min \Sigma Count$(min-sigma-count):

$$\min \Sigma Count(f(A)) = \sum_{i=1}^{n} f(\mu_A(x_i)) \tag{4}$$

· the biggest cardinality of $f(A)$, which is possible due to π_A, is called the $\max \Sigma Count$(max-sigma-count), and is equal to

$$\max \Sigma Count(f(A)) = \sum_{i=1}^{n} (f(\mu_A(x_i)) + f(\pi_A(x_i))) \tag{5}$$

and, clearly, for A^c we have

$$\min \Sigma Count(f(A^c)) = \sum_{i=1}^{n} f(\nu_A(x_i)) \tag{6}$$

$$\max \Sigma Count(f(A^c)) = \sum_{i=1}^{n} (f(\nu_A(x_i)) + f(\pi_A(x_i))) \tag{7}$$

Then the cardinality of a function of an intuitionistic fuzzy set is defined as the interval

$$\text{card } f(A) = [\min \Sigma Count(f(A)), \max \Sigma Count(f(A))]. \tag{8}$$

Remark. In the above formulas (4)-(8), for $i=0$, we will use later, for simplicity, the following symbols: $\min Count(f(A))$ instead of $\min \Sigma Count(f(A))$, $\max Count(f(A))$, instead of $\max \Sigma Count(f(A))$, $\min Count(f(A^c))$ instead of $\min \Sigma Count(f(A^c))$, $\max Count(f(A^c))$, instead of $\max \Sigma Count(f(A^c))$.

As it was shown in [15], distances between intuitionistic fuzzy set should be calculated taking into account three parameters describing an intuitionistic fuzzy set.
The most popular distances between intuitionistic fuzzy sets A, B in $X = \{x_1, x_2, \cdots, x_n\}$ are [15]:

· The Hamming distance :

$$d_{IFS}(A, B) = \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|). \tag{9}$$

· The Euclidean distance :

$$e_{IFS}(A, B) = (\sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2)^{\frac{1}{2}}. \tag{10}$$

3. Entropy

An intuitionistic fuzzy set is represented by the triangle ABD and its interior(Fig. 1). All points which are above the segment AB have a hesitancy margin greater than 0. The most undefined is point D. As the hesitancy margin for D is equal to 1, we cannot tell if this point belongs or does not belong to the set. The distance from

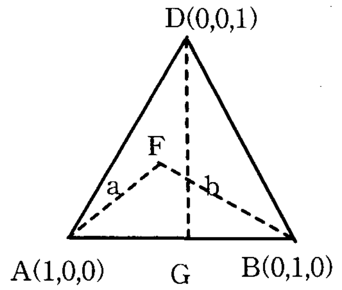


Fig. 1 The triangle ABD explaining a ratio-based measure of fuzziness.

D to A (full belonging) is equal to the distance to B (full non-belonging). So, the degree of fuzziness for D is equal to 100%. But the same situation occurs for all points x_i on the segment DG. For DG we have $\mu_{DG}(x_i) = \nu_{DG}(x_i)$, $\pi_{DG}(x_i) \geq 0$ (equality only for point G), and certainly

$$\mu_{DG}(x_i) + \nu_{DG}(x_i) + \pi_{DG}(x_i) = 1.$$

For every $x_i \in DG$ we have:

$$distance(A, x_i) = distance(B, x_i):$$

This geometric representation of an intuitionistic fuzzy set motivates a ratio-based measure of fuzziness (a similar approach was proposed in [11] to calculate the entropy of fuzzy sets):

$$E(F) = \frac{a}{b}, \tag{11}$$

where a is a $distance(F, F_{near})$ from F to the nearer point F_{near} among A and B, and b is the $distance(F, F_{far})$ from F to the farther point F_{far} among A and B.

An interpretation of entropy (11) can be as follows. This entropy measures the whole missing information which may be necessary to have no doubts when classifying the point F to the area of consideration, i.e. to say that F fully belongs (point A) or fully does not belong to our set (point B).
Formula (11) describes the degree of fuzziness for a single point belonging to an intuitionistic fuzzy set.
For n points belonging to an intuitionistic fuzzy set we have

$$E = \frac{1}{n} \sum_{i=1}^{n} E(F_i). \tag{12}$$

Applying the Hamming distance in Eq. (11), Szmidt and Kacprzyk[14] showed that the entropy of intuitionistic fuzzy sets is the ration of the biggest cardinalities ($\max \Sigma Counts$) involving only F and F^c.

Theorem 1. [14] A generalized entropy measure of an intuitionistic fuzzy set F of n elements is

$$E(F)=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\max Count(F_i\cap F_i^c)}{\max Count(F_i\cup F_i^c)}\right), \tag{13}$$

where [1-5]

$$F_i\cap F_i^c=\langle \min(\mu_{F_i},\mu^c_{F_i}), \max(\nu_{F_i},\nu^c_{F_i})\rangle,$$
$$F_i\cup F_i^c=\langle \max(\mu_{F_i},\mu^c_{F_i}), \min(\nu_{F_i},\nu^c_{F_i})\rangle.$$

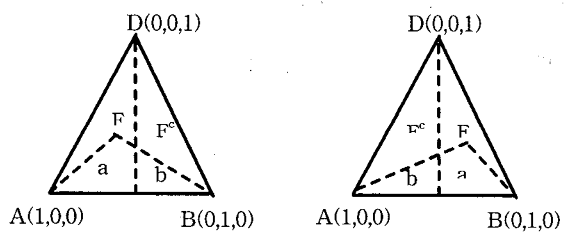
Applying the Euclidean distance in Eq. (10), we will have the following result.

Theorem 2. A generalized entropy measure of an intuitionistic fuzzy set F of n elements is

$$E(F)=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{(\max Count(F_i\cap F_i^c))^2+\max Count(F_i^2\cap(F_i^c)^2)}{(\max Count(F_i\cup F_i^c))^2+\max Count(F_i^2\cup(F_i^c)^2)}\right)^{\frac{1}{2}}, \tag{14}$$

where

$$f(F_i)\cap f(F_i^c)$$
$$=\langle \min(f(\mu_{F_i}),f(\mu^c_{F_i})), \max(f(\nu_{F_i}),f(\nu^c_{F_i}))\rangle,$$
$$f(F_i)\cup f(F_i^c)$$
$$=\langle \max(f(\mu_{F_i}),f(\mu^c_{F_i})), \min(f(\nu_{F_i}),f(\nu^c_{F_i}))\rangle.$$



(a) (b)

Fig. 2 (a) A case when point A is the nearest non-fuzzy neighbor, point B is the farthest non-fuzzy neighbor of F. (b) A case when point B is the nearest non-fuzzy neighbor, point A is the farthest non-fuzzy neighbor of F.

Proof. Let

- F - a point having coordinates $\langle \mu_F, \nu_F, \pi_F\rangle$.
- $F^c=\neg F$, a point having coordinates $\langle \mu_F^c, \nu_F^c, \pi_F^c\rangle=\langle \mu_F, \nu_F, \pi_F\rangle$.
- $\overline{F}$ - the nearest non-fuzzy neighbor of F(i.e. point A for Fig. 2a, or point B for Fig. 2b),
- $\underline{F}$ - the farthest non-fuzzy neighbor of F(i.e. point B for Fig. 2a, or point A for Fig. 2b).

Due to Eq. (11), we have

$$E(F)=\frac{a}{b}=\frac{d_{IFS}(F,\overline{F})}{d_{IFS}(F,\underline{F})} \tag{15}$$

and for the situation in Fig.2a we have [using the Euclidean distance (10)]

$$E(F)=\frac{(1-\mu_F)^2+(0-\nu_F)^2+(0-\pi_F)^2}{(0-\mu_F)^2+(1-\nu_F)^2+(0-\pi_F)^2}. \tag{16}$$

Having in mind that $\mu_F+\nu_F+\pi_F=1$, from Eq. (16) we obtain

$$\begin{aligned}E(F)&=\left(\frac{(1-\mu_F)^2+\nu_F^2+\pi_F^2}{\mu_F^2+(1-\nu_F)^2+\pi_F^2}\right)^{\frac{1}{2}}\\&=\left(\frac{(\nu_F+\pi_F)^2+\nu_F^2+\pi_F^2}{(\mu_F+\pi_F)^2+\mu_F^2+\pi_F^2}\right)^{\frac{1}{2}}\\&=\left(\frac{(\max Count(F^c))^2+\max Count((F^c)^2)}{(\max Count(F))^2+\max Count(F^2)}\right)^{\frac{1}{2}}\end{aligned} \tag{17}$$

For multiple elements F_i $(i=1,\cdots,n)$ whose point A is their nearest fuzzy neighbor, Eq. (17) becomes owing to Eqs. (4), (6), and (12)

$$\begin{aligned}E&=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{(\nu_{F_i}+\pi_{F_i})^2+\nu_{F_i}^2+\pi_{F_i}^2}{(\mu_{F_i}+\pi_{F_i})^2+\mu_{F_i}^2+\pi_{F_i}^2}\right)^{\frac{1}{2}}\\&=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{(\max Count(F_i^c))^2+\max Count((F_i^c)^2)}{(\max Count(F_i))^2+\max Count(F_i^2)}\right)^{\frac{1}{2}}\end{aligned} \tag{18}$$

For the situation in Fig. 2b we have

$$E(F)=\left(\frac{(0-\mu_F)^2+(1-\nu_F)^2+(0-\pi_F)^2}{(1-\mu_F)^2+(0-\nu_F)^2+(0-\pi_F)^2}\right)^{\frac{1}{2}}, \tag{19}$$

i.e. by following the previous line of reasoning, we obtain

$$\begin{aligned}E(F)&=\left(\frac{\mu_F^2+(1-\nu_F)^2+\pi_F^2}{(1-\mu_F)^2+\nu_F^2+\pi_F^2}\right)^{\frac{1}{2}}\\&=\left(\frac{(\mu_F+\pi_F)^2+\mu_F^2+\pi_F^2}{(\nu_F+\pi_F)^2+\nu_F^2+\pi_F^2}\right)^{\frac{1}{2}}\end{aligned}. \tag{20}$$

Therefore, for multiple elements F_i $(i=1,\cdots,n)$, we have

$$\begin{aligned}E&=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{(\mu_{F_i}+\pi_{F_i})^2+\mu_{F_i}^2+\pi_{F_i}^2}{(\nu_{F_i}+\pi_{F_i})^2+\nu_{F_i}^2+\pi_{F_i}^2}\right)^{\frac{1}{2}}\\&=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{(\max Count(F_i))^2+\max Count(F_i^2)}{(\max Count(F_i^c))^2+\max Count((F_i^c)^2)}\right)^{\frac{1}{2}}\end{aligned} \tag{21}$$

In Eqs. (18) and (21) the numerator and the denominator are changed. If we take into account our assumptions, i.e. the fact that for a point F we have:

- in Fig. 2a : $\mu_A(x)>\nu_A(x)$,
- in Fig. 2b : $\mu_A(x)<\nu_A(x)$,

(certainly, for $\mu_A(x)=\nu_A(x)\Rightarrow E=1$), so for the situation in Fig. 2a we have

$$\max Count(F \cap F^c) = \max Count(F^c), \tag{22}$$

$$\max Count(F \cup F^c) = \max Count(F), \tag{23}$$

and hence

$$\max Count(F^2 \cap (F^c)^2) = \max Count((F^c)^2), \tag{24}$$

$$\max Count(F^2 \cup (F^c)^2) = \max Count(F^2). \tag{25}$$

A similar consideration for the situation in Fig. 2b gives

$$\max Count(F \cap F^c) = \max Count(F), \tag{26}$$

$$\max Count(F \cup F^c) = \max Count(F^c), \tag{27}$$

$$\max Count(F^2 \cap (F^c)^2) = \max Count(F^2), \tag{28}$$

$$\max Count(F^2 \cup (F^c)^2) = \max Count((F^c)^2). \tag{29}$$

Formula (24)-(29) lead to formulas (18) and (21) as

$$E(F) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{(\max Count(F_i \cap F_i^c))^2 + \max Count(F_i^2 \cap (F_i^c)^2)}{(\max Count(F_i \cup F_i^c))^2 + \max Count(F_i^2 \cup (F_i^c)^2)} \right)^{\frac{1}{2}}, \tag{30}$$

We next consider the same examples as in [14] and compare the result.

Example 1. Let us calculate the entropy for an element F_1 with the coordinates

$$F_1 = \left\langle \frac{3}{4}, \frac{1}{6}, \frac{1}{12} \right\rangle. \tag{31}$$

Thus,

$$d(A, F_1) = \left((1-\frac{3}{4})^{\frac{1}{2}} + (0-\frac{1}{6})^{\frac{1}{2}} + (\frac{0-1}{12})^{\frac{1}{2}} \right)^{\frac{1}{2}}, = \frac{\sqrt{14}}{12}$$

$$d(B, F_1) = \left((0-\frac{3}{4})^{\frac{1}{2}} + (1-\frac{1}{6})^{\frac{1}{2}} + (0-\frac{1}{12})^{\frac{1}{2}} \right)^{\frac{1}{2}} = \frac{\sqrt{182}}{12}$$

and

$$E(F_1) = \frac{d(A, F_1)}{d(B, F_1)} = \frac{1}{\sqrt{13}}. \tag{32}$$

We can obtain the same result using formula (30) and having in mind that

$$F_1^c = \left\langle \frac{1}{6}, \frac{3}{4}, \frac{1}{12} \right\rangle,$$

$F_1 \cap F_1^c = \langle \frac{1}{6}, \frac{3}{4}, \frac{1}{12} \rangle = F_1^c$, and

$$F_1^2 \cap (F_1^c)^2 = \left((\frac{1}{6})^2, (\frac{3}{4})^2, (\frac{1}{12})^2 \right),$$

$$\max Count(F_1 \cap F_1^c) = \frac{1}{6} + \frac{1}{12} = \frac{3}{12},$$

$$\max Count(F_1^2 \cap (F_1^c)^2) = (\frac{1}{6})^2 + (\frac{1}{12})^2 = \frac{5}{144},$$

$$F_1 \cup F_1^c = \langle \frac{3}{4}, \frac{1}{6}, \frac{1}{12} \rangle = F_1,$$

$$F_1^2 \cap (F_1^c)^2 = \left((\frac{3}{4})^2, (\frac{1}{6})^2, (\frac{1}{12})^2 \right),$$

$$\max Count(F_1 \cup F_1^c) = \frac{3}{4} + \frac{1}{12} = \frac{10}{12},$$

$$\max Count(F_1^2 \cup (F_1^c)^2) = (\frac{3}{4})^2 + (\frac{1}{12})^2 = \frac{82}{144}, \tag{33}$$

so that

$$E(F_1) = \left(\frac{(\max Count(F_1 \cap F_1^c))^2 + \max Count(F_1^2 \cap (F_1^c)^2)}{(\max Count(F_1 \cup F_1^c))^2 + \max Count(F_1^2 \cup (F_1^c)^2)} \right)^{\frac{1}{2}}$$

$$= \left(\frac{(\frac{3}{12})^2 + \frac{5}{144}}{(\frac{10}{12})^2 + \frac{82}{144}} \right)^{\frac{1}{2}} = \frac{1}{\sqrt{13}}, \tag{34}$$

i.e. the same value as from Eq. (32).

Let us consider another element F_2 with the coordinates

$$F_2 = \left\langle \frac{1}{2}, 0, \frac{1}{2} \right\rangle.$$

from Eq. (11) we have

$$E(F_2) = \frac{d(A, F_2)}{d(B, F_2)} = \left(\frac{(1-\frac{1}{2})^2 + (0-0)^2 + (\frac{0-1}{2})^2}{(0-\frac{1}{2})^2 + (1-0)^2 + (0-\frac{1}{2})^2} \right)^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \tag{35}$$

or having in mind that $F_2^c = \left\langle 0, \frac{1}{2}, \frac{1}{2} \right\rangle$, we obtain

$$F_2 \cap F_2^c = \langle 0, \frac{1}{2}, \frac{1}{2} \rangle = F_2^c,$$

$$F_2 \cup F_2^c = \langle \frac{1}{2}, 0, \frac{1}{2} \rangle = F_2,$$

$$F_2^2 \cap (F_2^c)^2 = \langle 0, (\frac{1}{2})^2, (\frac{1}{2})^2 \rangle,$$

$$F_2^2 \cup (F_2^c)^2 = \langle (\frac{1}{2})^2, 0, (\frac{1}{2})^2 \rangle,$$

and

$$E(F_2) = \left(\frac{(\max Count(F_2 \cap F_2^c))^2 + \max Count(F_2^2 \cap (F_2^c)^2)}{(\max Count(F_2 \cup F_2^c))^2 + \max Count(F_2^2 \cup (F_2^c)^2)} \right)^{\frac{1}{2}}$$

$$= \left(\frac{(\frac{1}{2})^2 + \frac{1}{4}}{1^2 + \frac{1}{2}} \right)^{\frac{1}{2}} = \frac{1}{\sqrt{3}}. \quad (36)$$

i.e. the same value as from Eq. (35)

For another point F_3 with the coordinates $F_3 = \langle \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \rangle$, we obtain due to Eq. (11)

$$\begin{aligned} E(F_3) &= \frac{d(A, F_3)}{d(B, F_3)} \\ &= \left(\frac{(1-\frac{1}{2})^2 + (0-\frac{1}{4})^2 + (\frac{0-1}{4})^2}{(0-\frac{1}{2})^2 + (1-\frac{1}{4})^2 + (0-\frac{1}{4})^2} \right)^{\frac{1}{2}}, \quad (37) \\ &= \sqrt{\frac{3}{7}} \end{aligned}$$

or taking into account that $F_3^c = \langle \frac{1}{4}, \frac{1}{2}, \frac{1}{4} \rangle$,

$$F_3 \cap F_3^c = \langle \frac{1}{4}, \frac{1}{2}, \frac{1}{4} \rangle = F_3^c,$$

$$F_3 \cup F_3^c = \langle \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \rangle = F_3,$$

$$F_3^2 \cap (F_3^c)^2 = \langle (\frac{1}{4})^2, (\frac{1}{2})^2, (\frac{1}{4})^2 \rangle,$$

$$F_3^2 \cup (F_3^c)^2 = \langle (\frac{1}{2})^2, (\frac{1}{4})^2, (\frac{1}{2})^2 \rangle,$$

we obtain from Eq. (30)

$$E(F_3) = \left(\frac{(\max Count(F_3 \cap F_3^c))^2 + \max Count(F_3^2 \cap (F_3^c)^2)}{(\max Count(F_3 \cup F_3^c))^2 + \max Count(F_3^2 \cup (F_3^c)^2)} \right)^{\frac{1}{2}}$$

$$= \left(\frac{(\frac{1}{2})^2 + (\frac{1}{4})^2 + \frac{(1}{4})^2}{(\frac{3}{4})^2 + (\frac{1}{2})^2 (\frac{1}{4})^2} \right)^{\frac{1}{2}} = \sqrt{\frac{3}{7}}, \quad (38)$$

i.e. the same value as from Eq. (37).

From Eq. (12) we can calculate the entropy of an intuitionistic fuzzy set $Z \subseteq X = \{F_1, F_2, F_3\}$.
Taking into account Eqs. (32), (36) and (37) we have

$$\begin{aligned} E(Z) &= \frac{1}{3} \{E(F_1) + E(F_2) + E(F_3)\}. \\ &= \frac{1}{3} (\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \sqrt{\frac{3}{7}}) = 0.50 \end{aligned}$$

Comparing values of the entropy for elements and intuitionistic fuzzy set, while applying the Hamming distance in [14], we see that most values are close and the order of norm of entropy for elements are same.

Table 2. entropy for elements and intuitionistic fuzzy set

	$F_1 = \langle \frac{3}{4}, \frac{1}{6}, \frac{}{1}$	$F_2 = \langle \frac{1}{2}, 0, \frac{1}{2}$	$F_3 = \langle \frac{1}{2}, \frac{1}{4},$	$\frac{1}{n} \sum_{i=1}^{n} E(F$ $= E(Z)$
Using Hamming Distance	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{2}{3}$	0.49
Using Euclidean Distance	$\frac{1}{\sqrt{13}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{3}{7}}$	0.50

4. Conclusion

We have shown that the entropy measure proposed by Szmidt and Kacprzyk [Fuzzy Sets and Systems 118 (2001) 467-477] can be also defined in terms of the ration of some function of intuitionistic fuzzy cardinality of $F \cap F^c$ and $F \cup F^c$, while applying the Euclidean distance as well as the hamming distance. Same example in [14] is treated to compare values of the entropy for elements and intuitionistic fuzzy set.

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